expression for the transmitivity and ~~its~~ is called Gamow's formula.

- The Gamow's theory of α-decay is well verified experimentally.

- <u>Selection rules for α-particle</u>

Since the α-particle has spin zero, the selection rule is simply

$$|J_i - J_f| \leq L \leq J_i + J_f$$

If the parity of the parent and the daughter nuclei are the same, then only even L-values are allowed otherwise L is odd.

- **Q-Value** - The Q value for the α-decay is given by

$$Q = (\underset{P.N}{M_{A+4}} - \underset{D.N}{M_A} - \underset{\alpha\text{ particle}}{M_{He}})c^2 \quad \text{—①}$$

For the spontaneous emission of α-particle, the momentum conservation gives:

$$M_A V_A = M_{He} V_{He} \quad \text{—②}$$

Thus, the decay constant λ can be written as

$$\lambda = C_1 \left(\frac{Z}{\sqrt{Q}} - C_3 Z^{2/3} \right) - C_2 \quad \text{—— (10)}$$

where C_1, C_2, C_3 are constants and may be obtained from experimental data.

The values of the constant from theory are

$C_1 = 1.70\,(MeV)^{1/2}$, $C_2 = 28.0$
$C_3 = 1.13\,(MeV)^{-1/2}$

However, if we take $C_1 = 1.61$, $C_2 = 28.9$ & $C_3 = 1$ in same units gives satisfactory agreement with the experimental data (fig (1)).

- <u>Gamow's theory of α-particle</u>

The wave mechanical explanations point out that there is a finite probability that the particle can leak through the barrier even though its kinetic energy is less than the hight of the barrier. The leakage of a α-particle through the barrier is called the tunnel effect and the process is quantum mechanical tunneling. The wave mechanical treatment gives the

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and $V(r) = -V_0$ for $r < R_0$ [as shown in fig①].

The regular solution of this problem has the form.

$$F(r, z, Q) = \frac{1}{2}\sqrt{\frac{K}{q(r)}}\ \exp\left[-\int_r^{R_t} q(r_1)\, dr_1\right] \quad \text{——⑤}$$

where,

$$q(r) = \frac{1}{\hbar}\left[2m\left(\frac{2Ze^2}{r} - Q\right)\right]^{1/2} \quad \text{——⑥}$$

The integral of eq. ⑤ gives.

$$I = g(r) F(r)$$

where,

$$g(r) = \frac{2e}{\hbar}(mZr)^{1/2}$$

and,

$$F(r) = \frac{\pi}{2\sqrt{r}} - \frac{1}{\sqrt{r}} \arcsin(\sqrt{r}) - (1-r)^{1/2} \quad \text{——⑦}$$

The function is singular at $r=0$ and has the limiting value of ~~ratio~~.

$$\lim_{r\to 0} F(r) = \frac{\pi}{2\sqrt{r}} - 2 \quad \text{——⑧}$$

If $1 \approx$ approximate

$$F(r) = \frac{\pi}{2\sqrt{r}} - \frac{\pi}{3} - \frac{\sqrt{3}}{2} \quad \text{——⑨}$$

If we plot the potential energy in terms of this distance, the following curve is obtained.

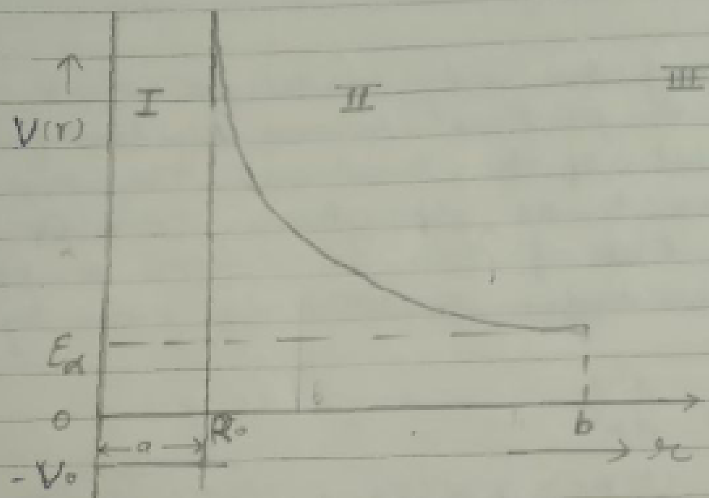

Figure: (1): Potential seen by an α-particle. ($l=0$).

The size of the curve from A to B shows the increase of repulsive force as the particle approach to the nucleus. The shape of the potential energy curve inside the nucleus can be represented by a constant attractive potential exerted over a distance a. The distance a is called the effective radius & this type of potential is known as potential well of depth V_0 and width a.

The potential $V(r) = \frac{2Ze^2}{r}$

for $r > R$.

It is assume in eq. ① that an α - particle is already formed, it is the process of emission.

Considering one body model in which an already formed α-particle moves in the field of the rest of the nucleons. The nucleus is taken to be spherical. Consider a simple case of $l=0$ (s-wave) α-particles. The schrödinger equation is given by

$$\frac{-\hbar^2}{2m}\frac{d^2U}{dr^2} + [V(r) - Q]\,U = 0$$

where $U(r) = r\psi(r)$, $\psi(r)$ is the α-particle wave function and m is reduced mass of α-particle. $m = \frac{m_\alpha m_d}{m_\alpha + m_d}$ → daughter nuclear system.

Upto distance 2×10^{-12} cm the potential energy $V(r)$ is given by the relation

$$V(r) = \frac{2Ze^2}{r}$$

For distances smaller than this, the coulombs law becomes invalid. The repulsive potential prevents (उत्पन्न) the incident α-particles from penetrating the nucleus and thus form a potential barrier.

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Calculation of α-decay rates
or
α-decay and barrier penetration

The α-decay rate depends upon the α-particle energy and the nuclear charge. The decay constant λ_i for a state i (in which an α-particle are formed) is therefore defined as the product of a penetration factor θ_i^2 and the reduced transition probability λ_0 then

$$\lambda_i = \lambda_0 \theta_i^2 \quad \text{—— (1)}$$

The penetration factor θ_i^2 gives the fraction of α-particles penetrating to the Coulomb barrier. The quantity λ_0 is essentially the no of collisions per second of the α-particle flux with the Coulomb barrier. The quantity θ_i^2 is known as reduced width of the i^{th} state and represent the square of the amplitude of the state. [$|\bar{A}-4, Z-2\rangle |\alpha\rangle$]. Therefore,

$$\sum \theta_i^2 = 1 \quad \text{—— (2)}$$

Therefore, the total probability of α-decay is