

Biostatistics

Unit 7 – Hypothesis Testing

Testing Hypotheses

Hypothesis testing and estimation are used to reach conclusions about a population by examining a sample of that population. Hypothesis testing is widely used in medicine, dentistry, health care, biology and other fields as a means to draw conclusions about the nature of populations.

(continued)

Hypothesis testing

Hypothesis testing is to provide information in helping to make decisions. The administrative decision usually depends a test between two hypotheses. Decisions are based on the outcome.

Definitions

Hypothesis: A hypothesis is a statement about one or more populations. There are research hypotheses and statistical hypotheses.

Definitions

Research hypothesis: A research hypothesis is the supposition or conjecture that motivates the research. It may be proposed after numerous repeated observation. Research hypotheses lead directly to statistical hypotheses.

Definitions

Statistical hypotheses: Statistical hypotheses are stated in such a way that they may be evaluated by appropriate statistical techniques. There are two statistical hypotheses involved in hypothesis testing.

(continued)

Definitions

H_0 is the null hypothesis or the hypothesis of no difference.

H_A (otherwise known as H_1) is the alternative hypothesis or what we will believe is true if we reject the null hypothesis.

Rules for testing hypotheses

1. Your expected conclusion, or what you hope to conclude as a result of the experiment should be placed in the alternative hypothesis.
2. The null hypothesis should contain an expression of equality, either $=$, \leq or \geq .
3. The null hypothesis is the hypothesis that will be tested.

(continued)

Rules for testing hypotheses

4. The null and alternative hypotheses are complementary. This means that the two alternatives together exhaust all possibilities of the values that the hypothesized parameter can assume.

Note: Neither hypothesis testing nor statistical inference proves the hypothesis. It only indicates whether the hypothesis is supported by the data or not.

Stating statistical hypotheses

<u>Type of equation</u>	<u>H_0</u>	<u>H_A</u>	<u>Type of distribution</u>
=	$\mu = 50$	$\mu \neq 50$	Two tail, two rejection regions
\leq	$\mu \leq 50$	$\mu > 50$	One tail, rejection region on right
\geq	$\mu \geq 50$	$\mu < 50$	One tail, rejection region on left

Test statistic

The general formula for a test statistic is

$$\text{test statistic} = \frac{\text{relevant statistic} - \text{hypothesized parameter}}{\text{standard error of the relevant statistic}}$$

Example of test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

\bar{x} sample mean

μ_0 hypothesized parameter— population mean

σ / \sqrt{n} standard error of \bar{x} which is the relevant statistic

This all depends on the assumptions being correct.

Level of significance

The level of significance, α , is a probability and is, in reality, the probability of rejecting a true null hypothesis. For example, with 95% confidence intervals, $\alpha = .05$ meaning that there is a 5% chance that the parameter does not fall within the 95% confidence region. This creates an error and leads to a false conclusion.

Significance and errors

When the computed value of the test statistic falls in the rejection region it is said to be significant. We select a small value of α such as .10, .05 or .01 to make the probability of rejecting a true null hypothesis small.

Types of errors

- When a true null hypothesis is rejected, it causes a Type I error whose probability is α .
- When a false null hypothesis is not rejected, it causes a Type II error whose probability is designated by β .
- A Type I error is considered to be more serious than a Type II error.

Table of error conditions

	Condition of null hypothesis	
Possible Action	True	False
Fail to reject H_0	Correct (1- α)	Type II error β
Reject H_0	Type I error α	Correct (1- β)

Risk management

Since rejecting a null hypothesis has a chance of committing a type I error, we make α small by selecting an appropriate confidence interval. Generally, we do not control β , even though it is generally greater than α . However, when failing to reject a null hypothesis, the risk of error is unknown.

Procedure for hypothesis testing

In science, as in other disciplines, certain methods and procedures are used for performing experiments and reporting results. A research report in the biological sciences generally has five sections.

Procedure for hypothesis testing

I. Introduction

The introduction contains a statement of the problem to be solved, a summary of what is being done, a discussion of work done before and other basic background for the paper.

Procedure for hypothesis testing

II. Materials and methods

The biological, chemical and physical materials used in the experiments are described. The procedures used are given or referenced so that the reader may repeat the experiments if s/he so desires.

Procedure for hypothesis testing

III. Results

A section dealing with the outcomes of the experiments. The results are reported and sometimes explained in this section. Other explanations are placed in the discussion section.

Procedure for hypothesis testing

IV. Discussion

The results are explained in terms of their relationship to the solution of the problem under study and their meaning.

Procedure for hypothesis testing

V. Conclusions

Appropriate conclusions are drawn from the information obtained as a result of performing the experiments.

Procedure for hypothesis testing

This method can be modified for use in biostatistics. The materials and procedures used in biostatistics can be made to fit into these five categories. Alternatively, we will use an approach that is similar in structure but contains seven sections.

Procedure for hypothesis testing

- a. Given
- b. Assumptions
- c. Hypotheses
- d. Statistical test
- e. Calculations
- f. Discussion
- g. Conclusions

Explanation of procedure

a. Given

This section contains introductory material such as the measurements or counts given in the statement of the problem. It should contain enough information so as to solve the problem without further information

Explanation of procedure

b. Assumptions

This is another set of introductory material that is necessary for a correct solution of the problem. In biostatistics, problems are solved and conclusions drawn based on certain assumptions being true. If they are not true the results may be invalid.

Explanation of procedure

c. Hypotheses

The specific hypotheses being tested are written

H_0 : the null hypothesis

H_A : the alternative hypothesis

Explanation of procedure

d. Test statistic

In this section, several items are necessary.

First, decide on the test statistic that will be used. for example z or t . It must be possible to compute the test statistic from the data of the sample. How it is set up depends on how the hypotheses are stated.

Explanation of procedure

Second, the *distribution* of the test statistic must be known. For example, the z statistic is distributed as the standard normal distribution whereas the t statistic is distributed following Student's t distribution.

Explanation of procedure

Third, the criteria for making the decision are required. It is best to draw an actual distribution with rejection and non-rejection regions. The values of the test statistic that mark the regions should be calculated in advance so that when the problem is solved, the resulting figure can be compared immediately.

Explanation of procedure

e. Calculations

The test statistic is calculated using the data given in the problem. The result is compared with the previously specified rejection and non-rejection regions.

Explanation of procedure

f. Discussion

A statistical decision is made on whether to reject the null hypothesis or not.

Explanation of procedure

g. Conclusion

After the statistical decision is made, conclusions are drawn. What is reported depends on the statistical decision made in the Discussion section.

If H_0 is rejected we conclude that H_A is true. This conclusion can be stated as written.

Explanation of procedure

If H_0 is not rejected, then we conclude that H_0 may be true. Caution is required when reporting this outcome. One should not say " H_0 is true," because there is always a chance of making a Type II error. This would mean that a false null hypothesis was not rejected. It is better to say, " H_0 may be true." in this case.

Purpose of hypothesis testing

Hypothesis testing is to provide information in helping to make decisions. The administrative decision usually depends on the null hypothesis. If the null hypothesis is rejected, usually the administrative decision will follow the alternative hypothesis.

(continued)

Purpose of hypothesis testing

It is important to remember never to base a decision solely on the outcome of only one test. Statistical testing can be used to provide additional support for decisions based on other relevant information.

Topics

Hypothesis Testing of a Single Population Mean

Hypothesis Testing of the Difference Between Two Population Means

Hypothesis Testing of a Single Population Proportion

Hypothesis Testing of the Difference Between Two Population Proportions

Hypothesis Testing of a Single Population Variance

Hypothesis Testing of the Ratio of Two Population Variances

A) Hypothesis testing of a single population mean

A hypothesis about a population mean can be tested when sampling is from any of the following.

- A normally distributed population--variances known

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

Hypothesis testing of a single population mean

- A normally distributed population--variances unknown

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

Hypothesis testing of a single population mean

- A population that is not normally distributed
(assuming $n \geq 30$; the central limit theorem applies)

$$Z = \frac{\bar{X} - \mu_0}{s / \sqrt{n}}$$

p values

A p value is a probability that the result is as extreme or more extreme than the observed value if the null hypothesis is true. If the p value is less than or equal to α , we reject the null hypothesis, otherwise we do not reject the null hypothesis.

One tail and two tail tests

In a one tail test, the rejection region is at one end of the distribution or the other. In a two tail test, the rejection region is split between the two tails. Which one is used depends on the way the null hypothesis is written.

Confidence intervals

A confidence interval can be used to test hypotheses. For example, if the null hypothesis is: $H_0: \mu = 30$, a 95% confidence interval can be constructed. If 30 were within the confidence interval, we could conclude that the null hypothesis is not rejected at that level of significance.

Procedure

The procedure of seven steps is followed for hypothesis testing. It is very important to observe several items.

- Avoid rounding numbers off until the very end of the problem.
- Pay strict attention to details.
- Be very careful with regard to the way things are worded.
- Write down all the steps and do not take short cuts.

Sampling from a normally distributed population--variance known

- Example 7.2.1

A simple random sample of 10 people from a certain population has a mean age of 27. Can we conclude that the mean age of the population is not 30? The variance is known to be 20. Let $\alpha = .05$.

Solution

a. Data

$$n = 10 \quad \sigma^2 = 20$$

$$\bar{x} = 27 \quad \alpha = .05$$

Solution

b. Assumptions

- simple random sample
- normally distributed population

Solution

c. Hypotheses

$$H_0 : \mu = 30$$

$$H_A : \mu \neq 30$$

Solution

d. Statistical test

Test statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

Solution

d. Statistical test

Distribution of test statistic

If the assumptions are correct and H_0 is true, the test statistic follows the standard normal distribution.

Solution

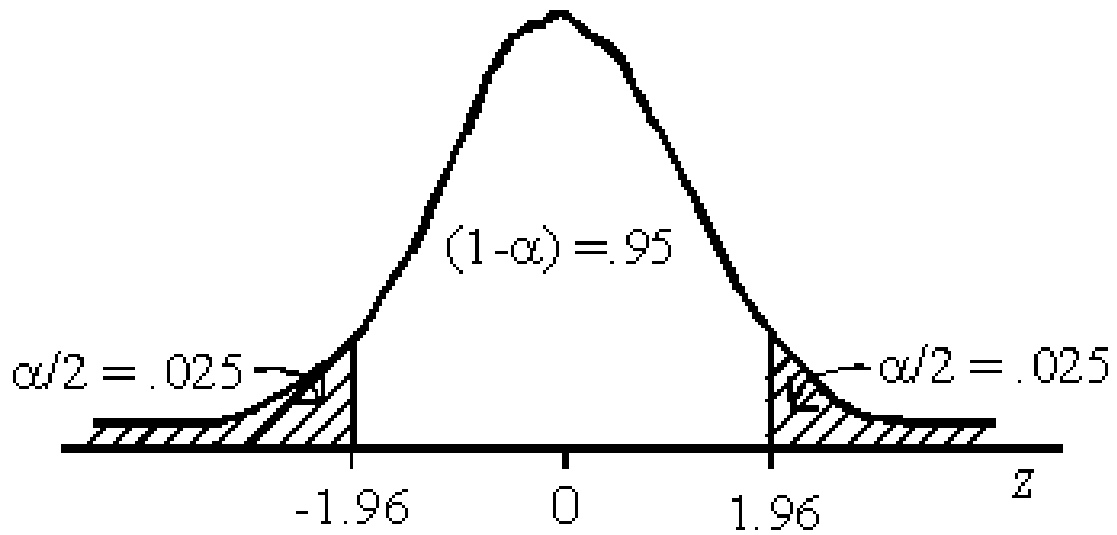
d. Statistical test

Decision criteria

Reject H_0 if the z value falls in the rejection region. Fail to reject H_0 if it falls in the nonrejection region.

Solution

d. Statistical test -- Decision criteria



Solution

d. Statistical test

Decision criteria

Because of the structure of H_0 it is a two tail test. Therefore, reject H_0 if $z \leq -1.96$ or $z \geq 1.96$.

Solution

e. Calculations

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

$$z = \frac{27 - 30}{\sqrt{20/10}} = \frac{-3}{1.4142} = -2.12$$

Solution

f. Discussion

We reject the null hypothesis because $z = -2.12$ which is in the rejection region. The value is significant at the .05 level.

Solution

g. Conclusions

We conclude that μ is not 30.

$$p = .0340$$

(continued)

Solution

g. Conclusions

A z value of -2.12 corresponds to an area of .0170. Since there are two parts to the rejection region in a two tail test, the p value is twice this which is .0340.

Confidence interval

$$\bar{x} \pm z \sigma \sqrt{n}$$

$$27 \pm 1.96 \sqrt{20/10}$$

$$27 \pm 1.96 (1.4142)$$

$$(24.228, 29.772)$$

Solution

Same example as a one tail test.

Example 7.2.1 (reprise)

A simple random sample of 10 people from a certain population has a mean age of 27. Can we conclude that the mean age of the population is less than 30? The variance is known to be 20. Let $\alpha = .05$.

Solution

a. Given

$$n = 10 \quad \sigma^2 = 20$$

$$\bar{x} = 27 \quad \alpha = .05$$

Solution

b. Assumptions

- simple random sample
- normally distributed population

Solution

c. Hypotheses

$$H_0: \mu \geq 30$$

$$H_A: \mu < 30$$

Solution

d. Statistical test

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Solution

d. Statistical test

Distribution

If the assumptions are correct and H_0 is true, the test statistic follows the standard normal distribution. Therefore, we calculate a z score and use it to test the hypothesis.

Solution

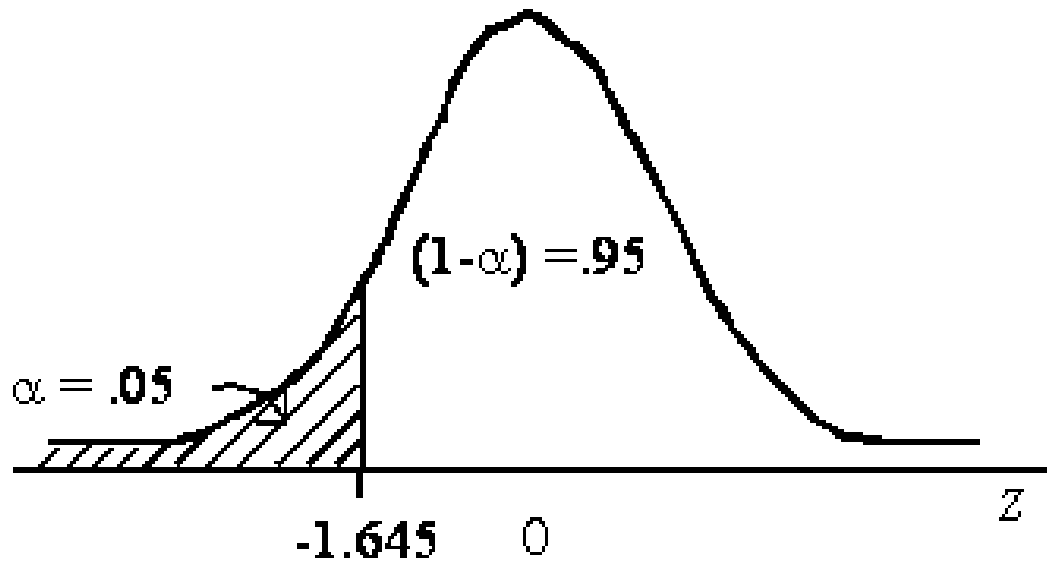
d. Statistical test

Decision criteria

Reject H_0 if the z value falls in the rejection region. Fail to reject H_0 if it falls in the nonrejection region.

Solution

d. Statistical test -- Decision criteria



Solution

d. Statistical test

Decision criteria

With $\alpha = .05$ and the inequality we have the entire rejection region at the left. The critical value will be $z = -1.645$. Reject H_0 if $z < -1.645$.

Solution

e. Calculations

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

$$z = \frac{27 - 30}{\sqrt{20/10}} = \frac{-3}{1.4142} = -2.12$$

Solution

f. Discussion

We reject the null hypothesis because
 $-2.12 < -1.645$.

Solution

g. Conclusions

We conclude that $\mu < 30$.

$p = .0170$ this time because it is only a one tail test and not a two tail test.

Sampling is from a normally distributed population--variance unknown.

Example 7.2.3 Body mass index

A simple random sample of 14 people from a certain population gives body mass indices as shown in Table 7.2.1. Can we conclude that the BMI is not 35? Let $\alpha = .05$.

Subject	BMI	Subject	BMI
1	23	8	24
2	25	9	32
3	21	10	57
4	37	11	23
5	39	12	26
6	21	13	31
7	23	14	45

Solution

a. Given

$$n = 14$$

$$s = 10.63918736$$

$$\bar{x} = 30.5$$

$$\alpha = .05$$

Solution

b. Assumptions

- simple random sample
- population of similar subjects
- normally distributed

Solution

c. Hypotheses

$$H_0: \mu = 35$$

$$H_A: \mu \neq 35$$

Solution

d. Test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Solution

d. Distribution

If the assumptions are correct and H_0 is true, the test statistic follows Student's t distribution with 13 degrees of freedom.

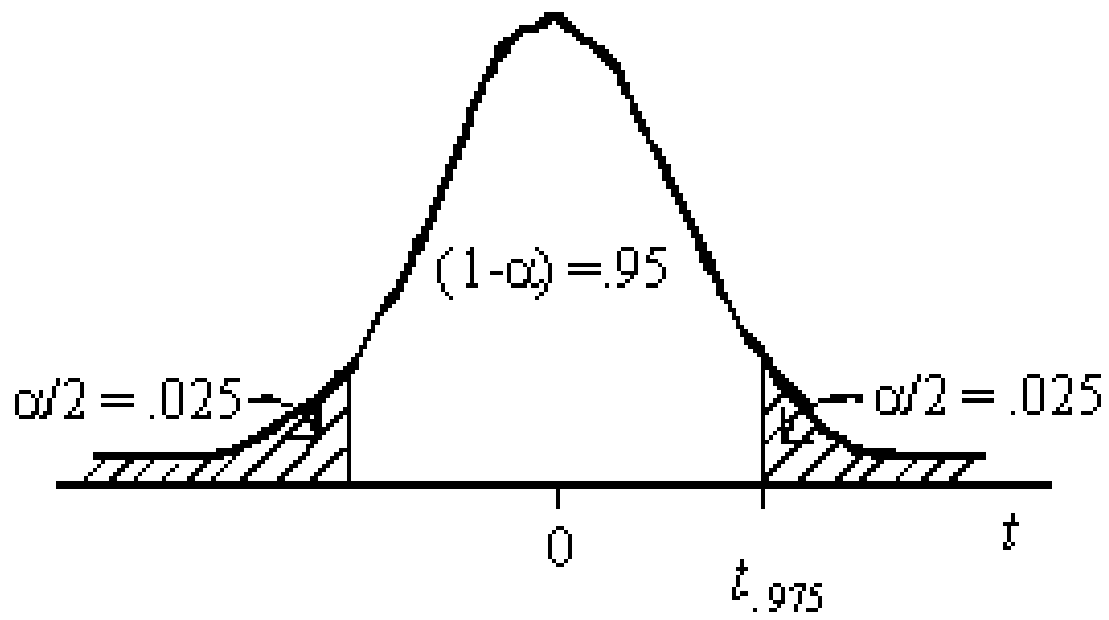
Solution

d. Decision criteria

We have a two tail test. With $\alpha = .05$ it means that each tail is 0.025. The critical t values with 13 df are -2.1604 and 2.1604.

We reject H_0 if the $t \leq -2.1604$ or $t \geq 2.1604$.

decision criteria



Solution

e. Calculation of test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$t = \frac{30.5 - 35}{10.639/\sqrt{14}} = \frac{-4.5}{2.8434} = -1.58$$

Solution

f. Discussion

Do not reject the null hypothesis because -1.58 is not in the rejection region.

Solution

g. Conclusions

Based on the data of the sample, it is possible that $\mu = 35$. The value of $p = .1375$

Sampling is from a population that is not normally distributed

Example 7.2.4

Maximum oxygen uptake data

Can we conclude that $\mu > 30$?

Let $\alpha = .05$.

Solution

a. Given

$$n = 242 \quad s = 12.14$$

$$\bar{x} = 33.3 \quad \alpha = .05$$

Solution

b. Assumptions

- simple random sample
- population is similar to those subjects in the sample
(cannot assume normal distribution)

Solution

c. Hypotheses

$$H_0: \mu \leq 30$$

$$H_A: \mu > 30$$

Solution

d. Test statistic

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Solution

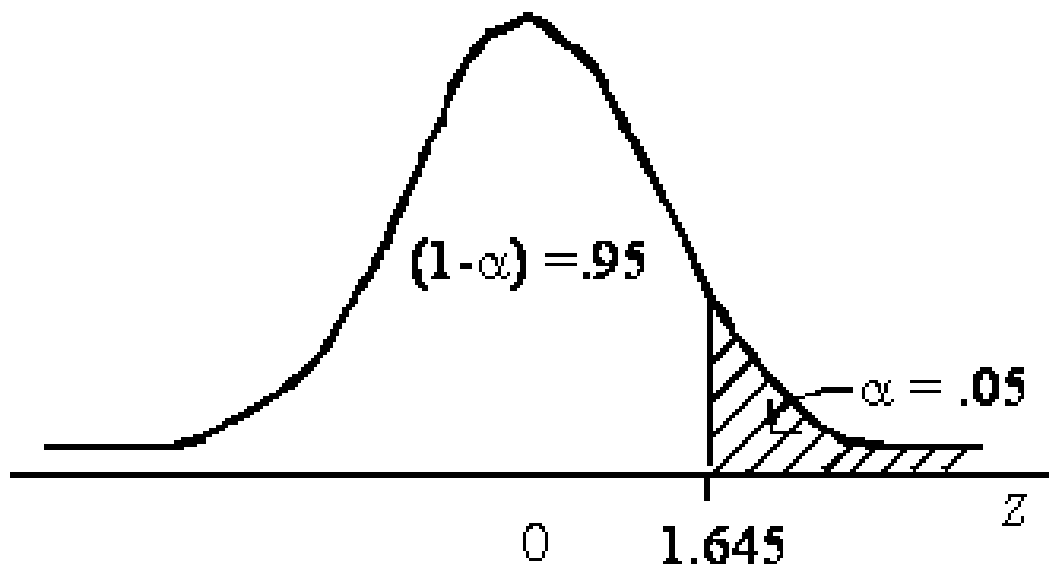
Distribution

By virtue of the Central Limit Theorem, the test statistic is approximately normally distributed with $\mu = 0$ if H_0 is true.

Solution

Decision criteria

This is a one tail test with $\alpha = .05$. The rejection region is at the right of the value $z = 1.645$.



Solution

e. Calculations

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$z = \frac{33.3 - 30}{12.14/\sqrt{242}} = \frac{3.3}{.7804} = 4.23$$

Solution

f. Discussion

Reject H_0 because $4.23 > 1.645$.

Solution

g. Conclusion

The maximum oxygen uptake for the sampled population is greater than 30.

The p value $< .001$ because 4.23 is off the chart ($p(3.89) < .001$).

Actual value $p=1.17 \times 10^{-5}$ (.0000117)

B) Hypothesis testing of the difference between two population means

This is a two sample z test which is used to determine if two population means are equal or unequal. There are three possibilities for formulating hypotheses.

(continued)

Possible hypotheses

1. $H_0: \mu_1 = \mu_2$ $H_A: \mu_1 \neq \mu_2$

2. $H_0: \mu_1 \geq \mu_2$ $H_A: \mu_1 < \mu_2$

3. $H_0: \mu_1 \leq \mu_2$ $H_A: \mu_1 > \mu_2$

Procedure

The same procedure is used in three different situations

Sampling is from normally distributed populations
with known variances

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}}$$

Sampling from normally distributed populations where population variances are unknown

population variances equal

This is with t distributed as Student's t distribution with $n_1 + n_2 - 2$ degrees of freedom and a pooled variance.

(continued)

Sampling from normally distributed populations
where population variances are unknown

population variances equal--distribution

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)_0}{\sqrt{s_p^2 / n_1 + s_p^2 / n_2}}$$

Sampling from normally distributed populations
where population variances are unknown

population variances equal—pooled variance

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

population variances unequal

When population variances are unequal, a distribution of t' is used in a manner similar to calculations of confidence intervals in similar circumstances.

Sampling from populations that are not normally distributed

If both sample sizes are 30 or larger the central limit theorem is in effect. The test statistic is z. If the population variances are unknown, the sample variances are used.

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}}$$

Sampling from normally distributed populations with population variances known

Example 7.3.1

Serum uric acid levels: Is there a difference between the means between individuals with Down's syndrome and normal individuals?

Solution

a. Given

$$\bar{x}_1 = 4.5 \quad n_1 = 12 \quad \sigma_1^2 = 1$$

$$\bar{x}_2 = 3.4 \quad n_2 = 15 \quad \sigma_2^2 = 1.5 \quad \alpha = .05$$

Solution

b. Assumptions

- two independent random samples
- each drawn from a normally distributed population

Solution

c. Hypotheses

$$H_0: \mu_1 = \mu_2$$

$$H_A: \mu_1 \neq \mu_2$$

Solution

d. Statistical test -- This is a two sample z test.

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}}$$

Solution

Distribution

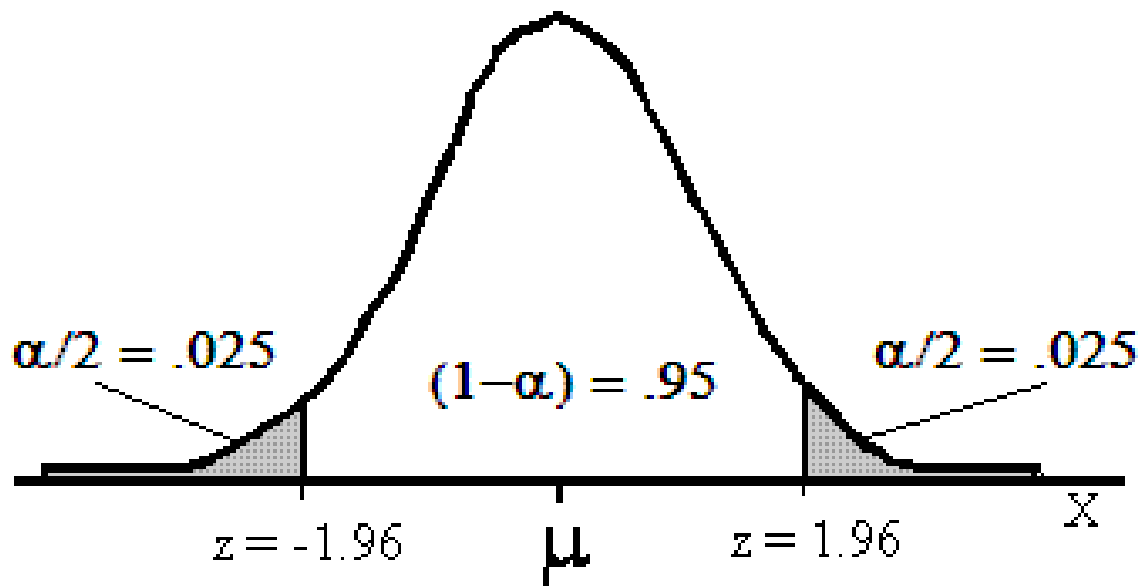
If the assumptions are correct and H_0 is true, the test statistic is distributed as the normal distribution.

Solution

Decision criteria

With $\alpha = .05$, the critical values of z are -1.96 and $+1.96$. We reject H_0 if $z < -1.96$ or $z > +1.96$.

Solution



Solution

e. Calculations

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}}$$

$$z = \frac{(4.5 - 3.4) - 0}{\sqrt{1/12 + 1.5/15}} = \frac{1.1}{.4282} = 2.57$$

Solution

f. Discussion

Reject H_0 because $2.57 > 1.96$.

Solution

g. Conclusion

From these data, it can be concluded that the population means are not equal. A 95% confidence interval would give the same conclusion.

$$p = .0102.$$

Sampling from normally distributed populations with unknown variances

With equal population variances, we can obtain a pooled value from the sample variances.

(continued)

Example 7.3.2 -- Lung destructive index

We wish to know if we may conclude, at the 95% confidence level, that smokers, in general, have greater lung damage than do non-smokers.

Solution

a. Given

$$\begin{array}{l} \text{Smokers:} \quad \bar{x}_1 = 17.5 \quad n_1 = 16 \quad s_1 = 4.4752 \\ \text{Non-Smokers:} \quad \bar{x}_2 = 12.4 \quad n_2 = 9 \quad s_2 = 4.8492 \\ \alpha = .05 \end{array}$$

Calculation of Pooled Variance

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$s_p^2 = \frac{(15)(4.4711) + (8)(4.8492)}{16 + 9 - 2}$$

$$s_p^2 = \frac{299.86 + 188.12}{23}$$

$$s_p^2 = 21.2165$$

Solution

b. Assumptions

- independent random samples
- normal distribution of the populations
- population variances are equal

Solution

(3) Hypotheses

$$H_0: \mu_1 \leq \mu_2$$

$$H_A: \mu_1 > \mu_2$$

Solution

d. Statistical test -- Test statistic

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)_0}{\sqrt{s_p^2 / n_1 + s_p^2 / n_2}}$$

Distribution of test statistic

If the assumptions are met and H_0 is true, the test statistic is distributed as Student's t distribution with 23 degrees of freedom.

Decision criteria

With $\alpha = .05$ and $df = 23$, the critical value of t is 1.7139. We reject H_0 if $t > 1.7139$.

Solution

e. Calculations

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{s_p^2 / n_1 + s_p^2 / n_2}}$$

$$t = \frac{(17.5 - 12.4) - 0}{\sqrt{21.2165/16 + 21.2165/9}} = \frac{5.1}{1.92} = 2.6563$$

Solution

f. Discussion

Reject H_0 because $2.6563 > 1.7139$.

Solution

g. Conclusions

On the basis of the data, we conclude that

$$\mu_1 > \mu_2 .$$

Actual values

$$t = 2.6558$$

$$p = .014$$

Sampling from populations that are not normally distributed -- Example 7.3.4

These data were obtained in a study comparing persons with disabilities with persons without disabilities. A scale known as the Barriers to Health Promotion Activities for Disabled Persons (BHADP) Scale gave the data. We wish to know if we may conclude, at the 99% confidence level, that persons with disabilities score higher than persons without disabilities.

Solution

a. Given

Disabled: $\bar{x}_1 = 31.83$ $n_1 = 132$ $s_1 = 7.93$
Nondisabled: $\bar{x}_2 = 25.07$ $n_2 = 137$ $s_2 = 4.80$
 $\alpha = .01$

Solution

b. Assumptions

- independent random samples

Solution

c. Hypotheses

$$H_0: \mu_1 \leq \mu_2$$

$$H_A: \mu_1 > \mu_2$$

Solution

d. Statistical test -- Test statistic

Because of the large samples, the central limit theorem permits calculation of the z score as opposed to using t . The z score is calculated using the given sample standard deviations.

Solution

Distribution

If the assumptions are correct and H_0 is true, the test statistic is approximately normally distributed

Solution

Decision criteria

With $\alpha = .01$ and a one tail test, the critical value of z is 2.33. We reject H_0 if $z > 2.33$.

Solution

e. Calculation of test statistic

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{s_1^2 / n_1 + s_2^2 / n_2}}$$

$$z = \frac{(31.83 - 25.07) - 0}{\sqrt{(7.93)^2 / 132 + (4.80)^2 / 137}} = \frac{6.76}{.8029} = 8.42$$

Solution

f. Discussion

Reject H_0 because $8.42 > 2.33$.

Solution

g. Conclusions

On the basis of these data, the average persons with disabilities score higher on the BHADP test than do the nondisabled persons.

Actual values

$$z = 8.42$$

$$p = 1.91 \times 10^{-17}$$

Paired comparisons

Sometimes data come from nonindependent samples. An example might be testing "before and after" of cosmetics or consumer products. We could use a single random sample and do "before and after" tests on each person.

(continued)

Paired comparisons

A hypothesis test based on these data would be called a *paired comparisons test*. Since the observations come in pairs, we can study the difference, d , between the samples. The difference between each pair of measurements is called d_i .

Test statistic

With a population of n pairs of measurements, forming a simple random sample from a normally distributed population, the mean of the difference, μ_d , is tested using the following implementation of t .

(continued)

$$t = \frac{\bar{d} - \mu_{d_0}}{s_{\bar{d}}}$$

\bar{d} is the sample mean difference

μ_{d_0} is the hypothesized mean difference

$$s_{\bar{d}} = \frac{s_d}{\sqrt{n}} \text{ -- the standard error}$$

n is the number of sample differences

s_d is the standard deviation of the sample differences

Example 7.4.1

Very-low-calorie diet (VLCD) Treatment

- Table gives B (before) and A (after) treatment data for obese female patients in a weight-loss program.
- We calculate $d_i = A - B$ for each pair of data resulting in negative values meaning that the participants lost weight.

(continued)

Example 7.4.1

We wish to know if we may conclude, at the 95% confidence level, that the treatment is effective in causing weight reduction in these people.

Data table

	A	B
1	83.3	117.3
2	85.9	111.4
3	75.8	98.6
4	82.9	104.3
5	82.3	105.4
6	77.7	100.4
7	62.7	81.7
8	69.0	89.5
9	63.9	78.2

Solution

a. Given

$$A \rightarrow L_1 \quad B \rightarrow L_2$$

$$d_i = A - B \quad (L_1 - L_2 \rightarrow L_3)$$

$$n = 9 \quad \alpha = .05$$

Solution

b. Assumptions

- the observed differences are a simple random sample from a normally distributed population of differences

Solution

c. Hypotheses

$$H_0: \mu_d \geq 0$$

$H_A: \mu_d < 0$ (meaning that the patients lost weight)

Solution

d. Statistical test -- Test statistic

The test statistic is t
which is calculated as

$$t = \frac{\bar{d} - \mu_{d_0}}{s_d}$$

Solution

Distribution

The test statistic is distributed as Student's t with 8 degrees of freedom

Solution

Decision criteria

With $\alpha = .05$ and 8 df the critical value of t is -1.8595. We reject H_0 if $t < -1.8595$.

Solution

e. Calculations

$$\bar{d} = \frac{\sum d_i}{n} = \frac{-203.3}{9} = -22.5889$$

$$s_d^2 = 28.2961$$

$$t = \frac{\bar{d} - \mu_{d_0}}{s_d} = \frac{-22.5889 - 0}{\sqrt{28.2961/9}} = -12.7395$$

Solution

f. Discussion

Reject H_0 because $-12.7395 < -1.8595$

$p = 6.79 \times 10^{-7}$

Solution

g. Conclusions

- On the basis of these data, we conclude that the diet program is effective.
- Other considerations
 - a confidence interval for μ_d can be constructed
 - z can be used if the variance is known or if the sample is large

C) Hypothesis testing of a single population proportion

When the assumptions for using a normal curve are achieved, it is possible to test hypotheses about a population proportion. When the sample size is large to permit use of the central limit theorem, the statistic of choice is z .

(continued)

Example 7.5.1

IV drug users and HIV

We wish to know if we may conclude that fewer than 5% of the IV drug users in the sampled population are HIV positive.

Solution

a. Given

$n = 423$ with 18 possessing the characteristic of interest

$$\hat{p} = 18/423 = .0426$$

$$p_0 = .05$$

$$\alpha = .05$$

Solution

b. Assumptions

- because of the central limit theorem, the sampling distribution of \hat{p} is normally distributed

Solution

c. Hypotheses

$$H_0: p \geq .05$$

$$H_A: p < .05$$

(continued)

Solution

If H_0 is true, $p = .05$ and the standard error is

$$\sigma_{\hat{p}} = \sqrt{\frac{(.05)(.95)}{423}}$$

Note that the standard error is not based on the value of \hat{p}

Solution

d. Statistical test -- Test statistic

The test statistic is z which is calculated as

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Solution

Distribution

Because the sample is large we can use z distributed as the normal distribution if H_0 is true.

Solution

Decision criteria

With $\alpha = .05$ the critical z score is -1.645. We reject H_0 if $z \leq -1.645$.

e. Calculations

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$z = \frac{.0426 - .05}{\sqrt{\frac{.05(.95)}{423}}} = \frac{-.0074}{.010597} = -.6983$$

Solution

f. Discussion

Do not reject H_0 because $-.6983 > -1.645$

Solution

(7) Conclusions

We conclude that p may be $.05$ or more.

D) Hypothesis testing of the difference between two population proportions

It is frequently important to test the difference between two population proportions. Generally we would test $p_1 = p_2$. This permits the construction of a pooled estimate which is given by the following formula.

(continued)

D) Hypothesis testing of the difference between two population proportions

The pooled estimate for calculating the standard error of the estimator is:

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

(continued)

D) Hypothesis testing of the difference between two population proportions

The standard error of the estimator is:

$$\hat{\sigma}_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}$$

Two population proportions -- Example 7.6.1

In a study of patients on sodium-restricted diets, 55 patients with hypertension were studied. Among these, 24 were on sodium-restricted diets. Of 149 patients without hypertension, 36 were on sodium-restricted diets. We would like to know if we can conclude that, in the sampled population, the proportion of patients on sodium-restricted diets is higher among patients with hypertension than among patients without hypertension.

a. Given

Patients with hypertension:

$$n_1 = 55 \quad x_1 = 24 \quad \hat{p}_1 = .4364$$

Patients without hypertension:

$$n_2 = 149 \quad x_2 = 36 \quad \hat{p}_2 = .2416$$

$$\alpha = .05$$

Solution

b. Assumptions

independent random samples from the populations

Solution

c. Hypotheses

$$H_0: p_1 \leq p_2$$

$$H_A: p_1 > p_2$$

d. Statistical test – Test statistic

The test statistic is z which is calculated as

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)_0}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}}$$

Solution

Distribution

If the null hypothesis is true, the test statistic approximately follows the standard normal distribution.

Solution

Decision criteria

With $\alpha = .05$ the critical z score is 1.645. We reject H_0 if $z > 1.645$.

e. Calculations

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)_0}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}}$$
$$z = \frac{(.4364 - .2614) - 0}{\sqrt{\frac{(.2491)(.7059)}{55} + \frac{(.2491)(.7059)}{149}}} = 2.71$$

Solution

f. Discussion

Reject H_0 because $2.71 > 1.645$

Solution

g. Conclusions

The proportion of patients on sodium restricted diets among hypertensive patients is higher than in nonhypertensive patients.

$$p = .0034$$

E) Hypothesis testing of a single population variance

Where the data consist of a simple random sample drawn from a normally distributed population, the test statistic for testing hypotheses about a single population variance is $\chi^2 = (n-1) s^2 / \sigma^2$

which, when H_0 is true, is distributed as χ^2 with $n-1$ degrees of freedom.

Example 7.7.1

Response to allergen inhalation in allergic primates. In a study of 12 monkeys, the standard error of the mean for allergen inhalation was found to be .4 for one of the items studied. We wish to know if we may conclude that the population variance is not 4.

Solution

a. Given

$$n = 12$$

$$\text{standard error} = .4$$

$$\alpha = .05$$

$$df = 11$$

Solution

b. Assumptions

- simple random sample
- normally distributed population

Solution

c. Hypotheses

$$H_0: \sigma^2 = 4$$

$$H_A: \sigma^2 \neq 4$$

Solution

d. Statistical test -- Test statistic

The test statistic is $\chi^2 = (n-1) s^2 / \sigma^2$

Solution

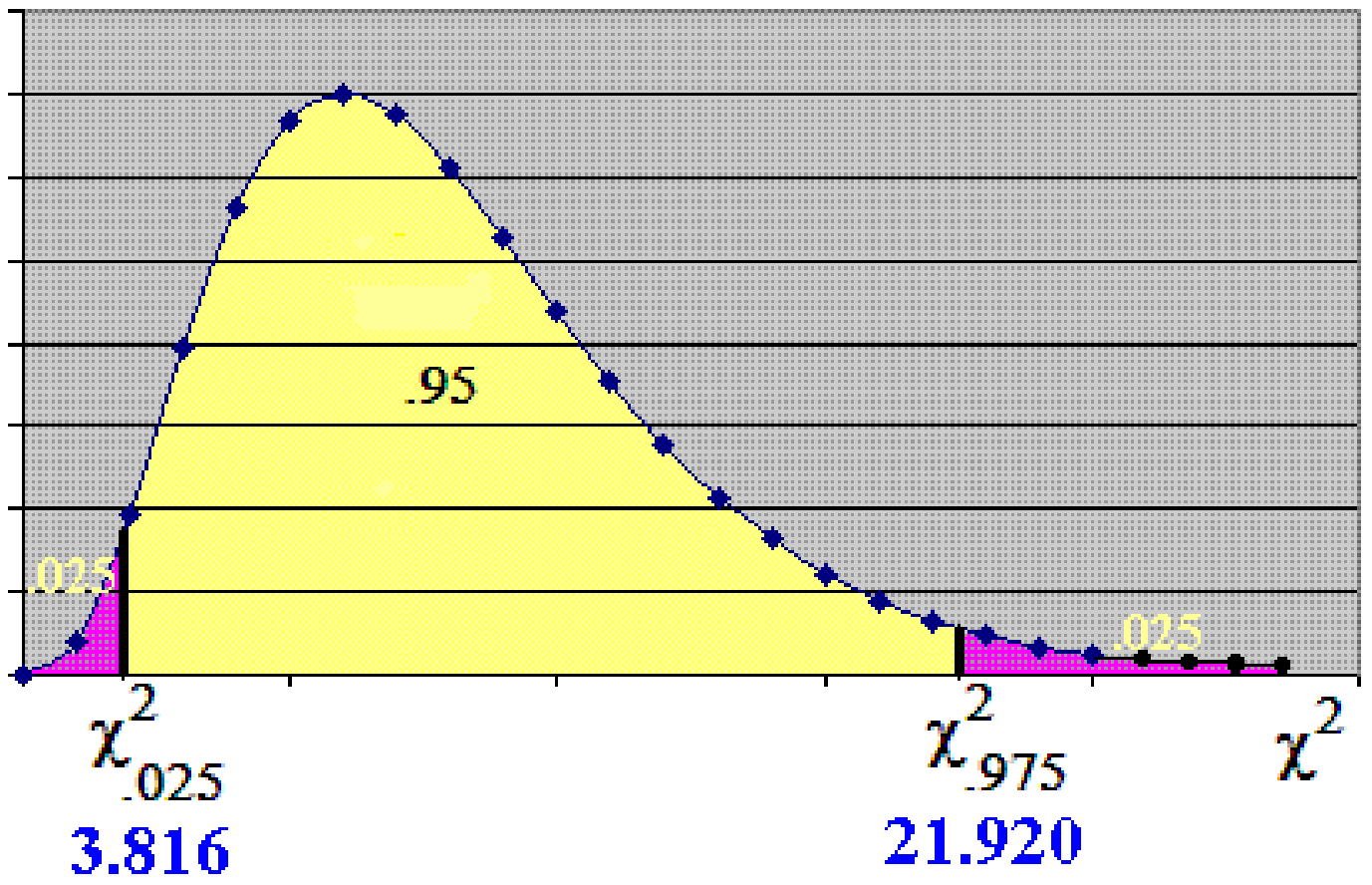
Distribution

When the null hypothesis is true, the distribution is χ^2 with 11 df.

Solution

Decision criteria

With $\alpha = .05$ and 11 df, the critical values are 3.816 and 21.920. Reject H_0 if $\chi^2 < 3.816$ or $\chi^2 > 21.920$.



Solution

e. Calculations

$$s^2 = .4 * 12 = 4.8$$

$$\chi^2 = [11(4.8)]/4 = 13.2$$

Solution

f. Discussion

Do not reject H_0 because $3.816 < 13.2 < 21.920$
(calculated value of χ^2 is not in rejection region)

Solution

g. Conclusions

Based on these data, we cannot conclude that the population variance is not 4.

$p > .05$ because $\chi^2 = 13.2$ is not in the rejection region.

F) Hypothesis testing of the ratio of two population variances

This test is used to determine if there is a significant difference between two variances. The test statistic is the variance ratio

$$V.R. = \frac{s_1^2}{s_2^2}$$

(continued)

F) Hypothesis testing of the ratio of two population variances

The statistic follows the F distribution with $n_1 - 1$ numerator degrees of freedom and $n_2 - 1$ denominator degrees of freedom.

How V. R. is determined

- two tail test

The larger variance is put in the numerator.

How V. R. is determined

- one sided test $H_0: \sigma_1^2 \leq \sigma_2^2$

In a one sided test where $H_0: \sigma_1^2 \leq \sigma_2^2$ and $H_A: \sigma_1^2 > \sigma_2^2$ we put s_1^2 / s_2^2 . The critical value of F is determined for α using the appropriate degrees of freedom.

How V. R. is determined

- one sided test $H_0: \sigma_1^2 \geq \sigma_2^2$

In a one sided test where $H_0: \sigma_1^2 \geq \sigma_2^2$

and $H_A: \sigma_1^2 < \sigma_2^2$ we put s_2^2 / s_1^2 . The critical value of F is determined for α using the appropriate degrees of freedom.

Example 7.8.1 – Pituitary adenomas

A study was performed on patients with pituitary adenomas. The standard deviation of the weights of 12 patients with pituitary adenomas was 21.4 kg. A control group of 5 patients without pituitary adenomas had a standard deviation of the weights of 12.4 kg. We wish to know if the weights of the patients with pituitary adenomas are more variable than the weights of the control group.

a. Given

Pituitary adenomas:

$$n_1 = 12 \quad s_1 = 21.4 \text{ kg}$$

Control:

$$n_2 = 5 \quad s_2 = 12.4 \text{ kg}$$

$$\alpha = .05$$

Solution

b. Assumptions

- each sample is a simple random sample
- samples are independent
- both populations are normally distributed

Solution

c. Hypotheses

$$H_0: \sigma_1^2 \leq \sigma_2^2$$

$$H_A: \sigma_1^2 > \sigma_2^2$$

Solution

Statistical test -- Test statistic

The test statistic is

$$V.R. = \frac{s_1^2}{s_2^2}$$

Solution

Distribution

When H_0 is true, the test statistic is distributed as F with 11 numerator degrees of freedom and 4 denominator degrees of freedom.

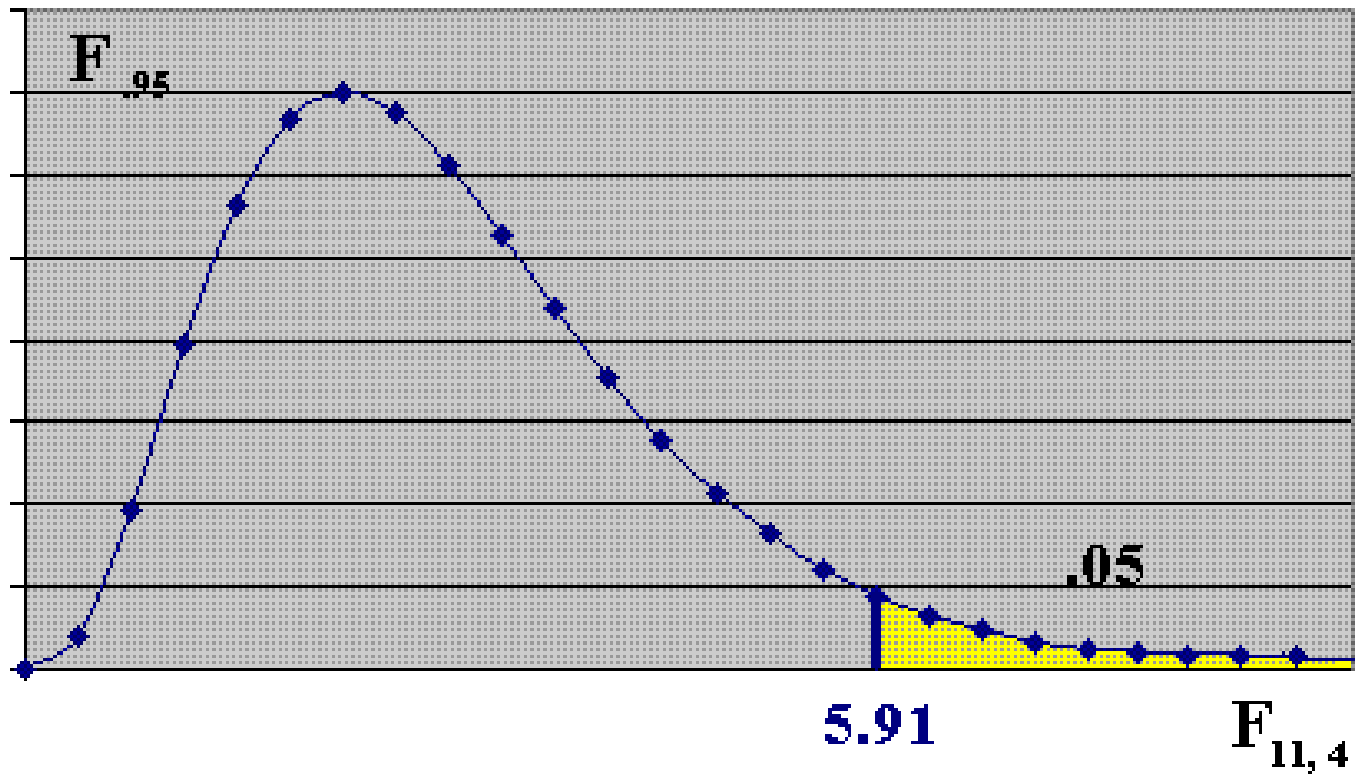
Solution

Decision criteria

The critical value is $F_{11,4} = 5.91$. Reject H_0 if

V.R. > 5.91

From the $F_{.95}$ table



Solution

e. Calculations

$$V.R. = \frac{s_1^2}{s_2^2} = \frac{(21.4)^2}{(12.4)^2} = 2.98$$

Solution

f. Discussion

We cannot reject H_0 because $2.98 < 5.91$. The calculated value for V.R. falls in the nonrejection region.

Solution

g. Conclusions

The weights of the population of patients may not be any more variable than the weights of the control subjects. Since V.R. of $2.98 < 3.10$, it gives $p > .10$
Actual $p = .1517$

fin