Exchange Forces :→
(Saturation character of Nuclear Forces)

The properties of nuclear matter are :-

(i) The nuclear density is approximately constant which leads to the relation $R = R_0 A^{1/3}$ for the nuclear radius. and

(ii) The constant binding energy per nucleon.

These facts implies that the binding energy and volume of nuclei are proportional to A, the mass number. This statement is not consistent with the result. In the nucleus with mass no. A, there are

$A(A-1)/2$ pairs and hence binding energy will be proportional to $A(A-1)/2$ – a contradiction to the above statement.

The properties (i) and (ii) are referred to as saturation properties of nuclear ~~f~~ matter and consequently of nuclear forces. To account the saturation character of nuclear forces, Heisenberg proposed that nuclear forces are of exchange type. The physical exchange force arises from the exchange of mesons. Meson exchange leads to exchange forces.

The wave func. describing a system of two nucleons is written as $\psi(r_1, r_2, S_1, S_2)$, where, r_1 and r_2 are position coordinates and S_1 and S_2 are spin coordinates of the two particles 1 and 2 respectively. The wave eqn for the two body system is

$$\left[\frac{\hbar^2}{2M}\nabla^2 + E\right]\psi(r_1, r_2, S_1, S_2) - V_{12}\psi(r_1, r_2, S_1, S_2) \quad (1)$$

where V_{12} refers to the two body potential including the distance dependence one and one that depends on exchange concept.

Now splitting wave func. into two parts

$$\psi(r_1, r_2, S_1, S_2) = \psi(r)\chi_{12} \quad (2)$$

There are four types of exchange ~~force~~ interactions

(I) Wigner force (no exchange) :-

The interaction betn two nucleons is such that it does not exchange the coordinates of the two particles. The corresponding potential is known as Wigner potential and written as

$$\hat{V}_{12} = V(r)\,\hat{W}_{12} \quad \text{--- (3)}$$

Where, $\hat{W}_{12}$ is unit operator. This force are attractive and $V(r)$ include (Square well, exponential, Gaussian and Yukawa). The wave eq^n is

$$\left(\frac{\hbar^2}{M}\nabla^2 + E\right)\psi(r) = V(r)\psi(r) \quad \text{--- (4)}$$

2) Majorana force (Space exchange) :-

The interaction bet^n two nucleon exchanges the space coordinates i.e.,

$$\hat{V}_{12} = \hat{V}(r)\,\hat{M}_{12} \quad \text{--- (5)}$$

Where, $\hat{M}_{12}$ is Majorana operator defined as

$$\hat{M}_{12}\psi(r) = \psi(-r) \quad \text{--- (6)}$$

$$\left(\text{for } (r_1 - r_2) = -(r_2 - r_1)\right)$$

This is equivalent to parity operation and the eigen value of parity operator are as $P = \pm 1$, hence.

$$\psi(-r) = (-1)^{1/2(1-P)}\psi(r) \quad \text{--- (7)}$$

The wave eq^n becomes.

$$\left(\frac{\hbar^2}{M}\nabla^2 + E\right)\psi(r) = (-1)^{1/2(1-P)}V(r)\psi(r) \quad \text{--- (8)}$$

This means that Majorana forces are attractive for even parity states (s, d, ..) and repulsive for odd parity states. So eq^n (8) becomes.

$$\left(\frac{\hbar^2}{M}\nabla^2 + E\right)\psi(r) = (-1)^{L}V(r)\psi(r) \quad \text{--- (9)}$$

(3). Barlett Forces (Spin exchange) :-

$$\hat{V}_{12} = V(r)\,\hat{B}_{12} \qquad (10)$$

Where,

$$\hat{B}_{12}\chi_{12} = \chi_{21} \qquad (11)$$

i.e, Barlett potential is such that when it operates on a wave func. it exchange spin coordinates. The wave eq^n is

$$\left(\frac{\hbar^2}{M}\nabla^2 + E\right)\psi(r)\,\chi_{12} = V(r)\,\psi(r)\,\chi_{21} \qquad (12)$$

For triplet states χ_{12} is symmetric ($\chi_{12} = \chi_{21}$) and for Singlet state it is antisymmetric ($\chi_{12} = -\chi_{21}$) and hence eq^n 12 reduces to

$$\left(\frac{\hbar^2}{M}\nabla^2 + E\right)\psi(r)\,\chi_{12} = (-1)^{S+1}\,V(r)\,\psi(r) \qquad (13)$$

This implies Barlett forces are attractive for triplet states and repulsive for Singlet states.

(4). Heisenberg force (Space and Spin exchange) :-
In This interaction both type of coordinate and spin exchanged i.e.

$$\hat{V}_{12} = V(r)\,\hat{H}_{12} \qquad (14)$$

&

$$\hat{H}_{12}\,\psi(r)\,\chi_{12} = \psi(-r)\,\chi_{21}$$
$$= \hat{M}_{12}\,\hat{B}_{12} \qquad (15)$$

The wave eq^n is

$$\left(\frac{\hbar^2}{M}\nabla^2 + E\right)\psi(r) = (-1)^{l+S+1}\,V(r)\,\psi(r) \qquad (16)$$

The Heisenberg potential is attractive for even triplet states and odd singlet states i.e, for the states for which $\psi(r_1, r_2, S_1, S_2)$ is symmetric and repulsive when $\psi(r_1, r_2, S_1, S_2)$ is antisymmetric under Heisenberg exchange.