Excited State of Deutron -

Que ÷ Prove that is no any bound excited state of deutron nuclei ? or

Excited state of Deuteron ?

→ In the ground state solution ($l=0$) for extreme situation when B was equated to zero KR_0 was found to be slightly larger than π, The first excited state will correspond to the case when KR_0 will be slightly larger than $3\pi/2$ as the wave function $U(r)$ is required to have a radial node in the first excited sta[te]. However, the case of +ve binding energies on will permitted to define an excited state in the nuclear potential well and in these cases KR_0 must be less than π. Therefore excited states in $l=0$ case will all lie in an unbounded state and these are the state in which neutron and proton will be in a fre[e] state. To examine the possibility of the excit[ed] states of deutron in the higher quantum states, the wave eqn in the generalized from can be express[ed] as-

$$\frac{d^2u}{dr^2} + \frac{M}{\hbar^2}\left[-B - V(r)\right]U(r) - \frac{l(l+1)}{r^2}U(r) = 0 \quad —(1)$$

In the inside of the range of the nuclear forc[e] eqn (1) can be written as

$$\frac{d^2U_l(r)}{dr^2} + \left[K^2 - \frac{l(l+1)}{r^2}\right]U_l(r) = 0 \quad \{r \leq 0\}$$

where $K^2 = \frac{M}{\hbar^2}(V_0 - B)$

and outside the range of nuclear force eqn ① become,

$$\frac{d^2U_l(r)}{dr^2} + \left[\gamma^2 + \frac{l(l+1)}{r^2}\right]U_l(r) = 0 \quad r > 0 \quad —(3)$$

The general solution of these eqns involve spheric[al]

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Bessel function j_l and spherical Neumann functions n_l. The latter approches $-\infty$ as $r \to 0$, thus the solution of eq^n. (2) is

$$U_l(r) = A j_l(Kr \qquad r \leq r_0 \quad \text{—— (4)}$$

where the spherical bessel function j_l is given by

$$j_l(\rho) = \left(\frac{\pi}{2\rho}\right)^{1/2} J_{l+1/2}(\rho) \quad \text{—— (5)}$$

Here, $J_{l+1/2}(\rho)$ being the bessel function of odd integer order parallel the solution of eq^n (3) is given as.

$$U_l(r) = B h_l(i\gamma r) \quad \text{—— (6)}$$

where

$$h_l = j_l + i n_l \quad \text{—— (7)}$$

The n_l spherical neumann function is given by

$$n_l(\rho) = (-1)^{l+1}\left(\frac{\pi}{2\rho}\right)^{1/2} J_{-l-1/2}(\rho) \quad \text{—— (8)}$$

The boundary conditions shows that $U_l(r)$ and its first derivative be continous at edge of the well $r=0$ may be combined into one condition i.e. the lograthmic derivative are continuous at $r = R_0$. This gives the relationship between the binding energy and the depth of the potential is

$$\left(\frac{1}{U_l}\frac{dU_l(r)}{dr}\right)_{inner} = \left(\frac{1}{U_l}\frac{dU_l(r)}{dr}\right)_{outter} \quad r = r_0 \quad \text{—— (9)}$$

We now using the relation

$$\frac{d j_l(\rho)}{d\rho} = j_{l-1}(\rho) - \frac{l+1}{\rho} j_l(\rho) \quad \text{—— (10)}$$

Frist we with calculate $\left(\frac{1}{U_l}\frac{dU_l(r)}{dr}\right)_{inner}$ by eq^n (4) and (10)

$$= \frac{1}{A j_l(Kr)}\left[A K j_{l-1}(Kr) - AK\frac{l+1}{Kr} j_l(Kr)\right]_{r=r_0}$$

$$= k\left[\frac{j_{l-1}(kr)}{j_l(kr)} - \frac{l+1}{kr}\right]_{r=r_0} \quad —⑪$$

and now RHS by eqⁿ ⑥ using eqⁿ ⑩

$$\left(\frac{1}{U_l}\frac{dU_l(r)}{dr}\right)_{outer} = \frac{1}{B h_l(i\gamma r)}\left[B i\gamma h_{l-1}(i\gamma r) - \frac{l+1}{i\gamma r}(B i\gamma) h_l(i\gamma r)\right]_{r=r_0}$$

$$= i\gamma\left[\frac{h_{l-1}(i\gamma r)}{h_l(i\gamma r)} - \frac{l+1}{i\gamma r}\right]_{r=r_0} \quad —⑫$$

Subsitituting eqⁿ ⑪ and ⑫ in eqⁿ ⑨ we get.

$$\Rightarrow k\left[\frac{j_{l-1}(kr)}{j_l(kr)} - \frac{l+1}{kr}\right]_{r=R_0} = i l\left[\frac{h_{l-1}(i\gamma r)}{h_l(i\gamma r)} - \frac{l+1}{i\gamma r}\right]_{r=R_0}$$

multiply by k in L.H.S and l on RHS $r = R_0$

$$\frac{j_{l-1}(kR_0)}{j_l(kR_0)} = \frac{\gamma}{k}\left[\frac{i h_{l-1}(i\gamma R_0)}{h_l(i\gamma R_0)}\right] \quad —⑬$$

Taking $R_0 (1.43 \times 10^{-13}\ cm)$ we have $\gamma R_0 < 1$ and $\gamma << k$. The expression in the braket on the RHS is less than 1 and thus it should be equal to zero.

we get $j_{l-1}(kR_0) \approx 0$

∴ from eqⁿ ⑬ $j_{l-1}(kR_0) = 0$

This is condition holds for all angular momentum accept $l = 0$ for $l = 1$

$$j_0(kr_0) \approx 0 \quad —⑭$$

using ~~ralion~~ relation $j_0(\rho) = \frac{\sin\rho}{\rho}$

we obtain by eqⁿ ⑭ $\frac{\sin kR_0}{kR_0} \approx 0$

or $\sin kR_0 \approx 0$ —⑮

$kR_0 = \pm\pi, \pm 2\pi, \pm 3\pi \pm \ldots$

The minimum will depth from (15) is thus $KR_0 = \pi$

$$K^2 = \frac{M}{\hbar^2}(V_0 - B)$$

$$K = \sqrt{\frac{MV_0}{\hbar}}$$

$$K = \sqrt{\frac{mV_0}{\hbar^2}} \qquad R_0 = \lambda$$

$B = 0$

$$\frac{mV_0}{\hbar^2} R_0^2 = \pi^2$$

$$V_0 = \pi^2 \hbar^2 / m R_0^2 \quad \text{—(16)}$$

If we choose $R_0 = 2F = 2 \times 10^{-13}$ cm (2 Fermi)

Then $V_0 = V_0 = 144$ MeV
which is almost 4 times as large as the actual well depth is the ground state. Repeating this procedure for larger and larger values of l are find a deeper and deeper well depth is required to produce a bound state. Thus we conclude that no bound state exist of deutron for ~~ ~~ $l > 0$