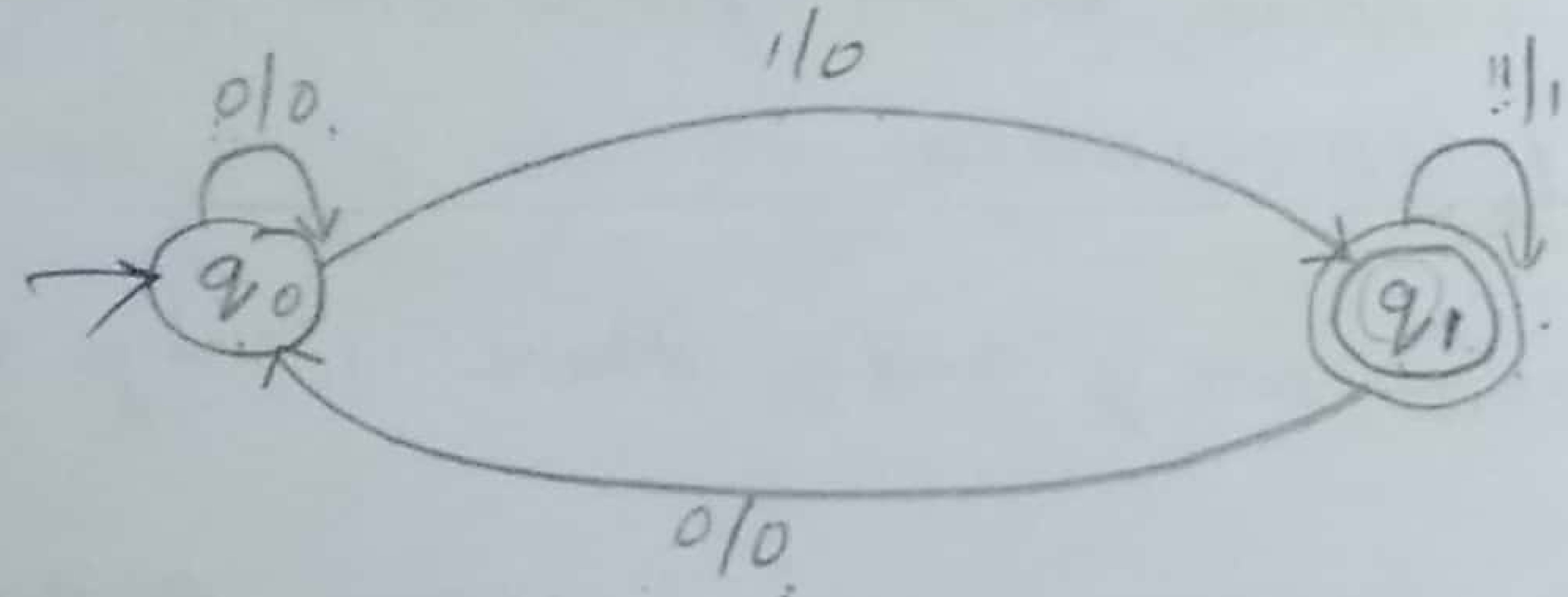
Transition Systems :-

A transition graphs or a transition system is a finite directed labelled graph in which each vertex (or node) represents a state and the directed edges indicates the transition of a state and the edges are labelled with input / output.

A typical transition system is shown in fig.



In the figure, the intial state is represented by a circle with an arrow pointing towards it, the final State by two concentric circles, and the other states are represented by just a circle. The edges are labelled by input / output (e.g by 1/0 or 1/1). for eg:- if the

System is in state q_0 and the input 1 is applied, the system moves to state q_1 as there is a directed edge from q_0 to q_1 with label 1/0. It outputs 0.

Def - Analytical def. of a transition system:-

A transition System is a 5-tuple $(Q, \Sigma, \delta, Q_0, F)$ where,

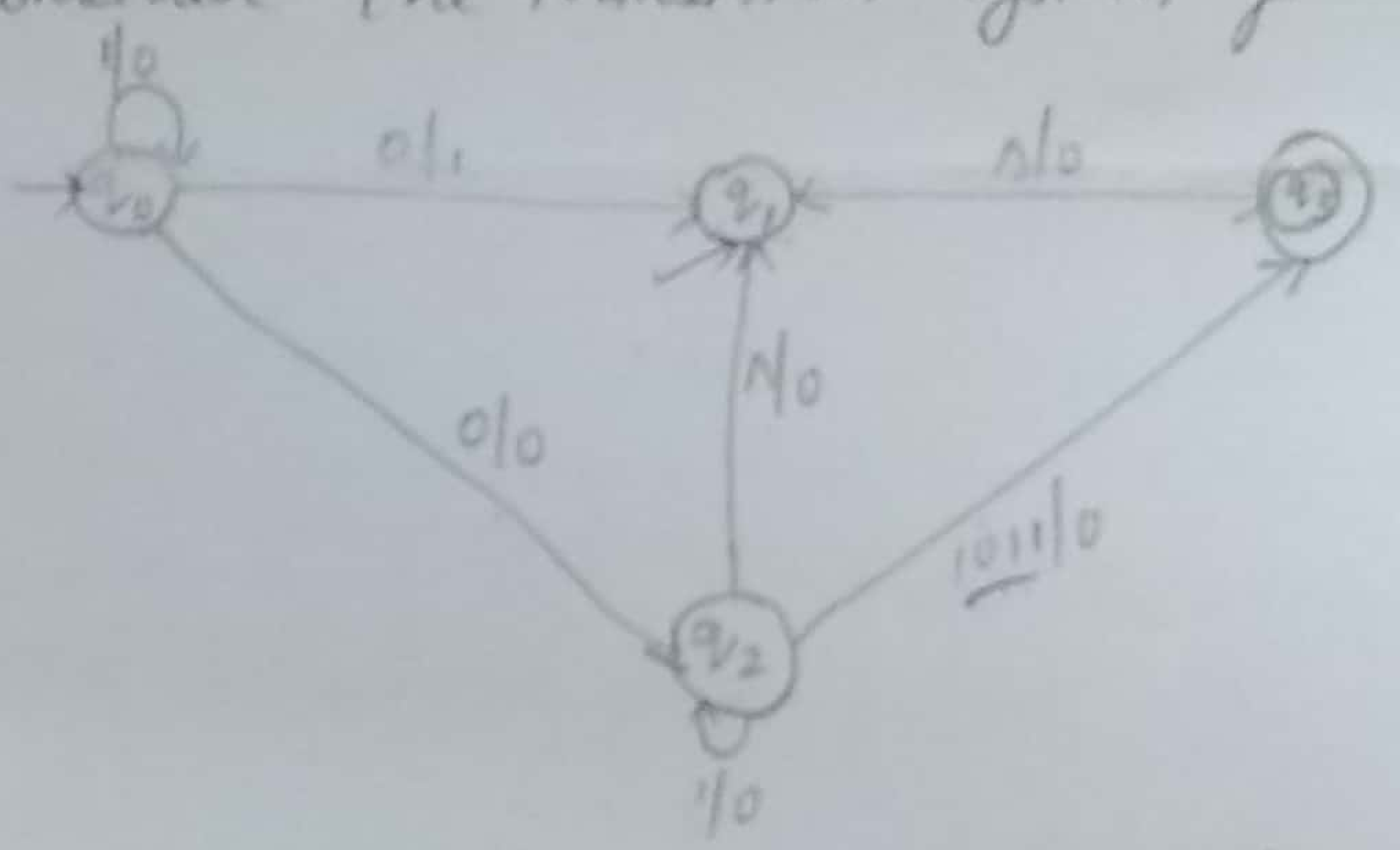
a> Q, Σ and F are the finite non empty set of states, the input alphabet, and the set of final states, respectively, as in the case of finite automata

b> $Q_0 \subseteq Q$, and Q_0 is non empty.

c> δ is a finite subset of $Q \times \Sigma \times Q$.

Exampl -

Consider the transition system given in fig.



Transition System for ex

Determine the initial states, final states and the acceptability of 101011, 111010

Solution:-

The initial states are q_0 and q_1. there is only one final state q_3.

The path-value of $q_0q_0q_2q_3$ is 101011. As q_3 is the final state, 101011 is accepted by the transition System. But, 111010 is not accepted by the transition System as there is no path value 111010.

Example :-

Consider the finite state machine whose transition function δ is given in table in the form of a transition table. Here $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $F = \{q_0\}$. give the entries sequence of states for the input string 110101

11111
000000

Transition Function Table

	Inputs	
States	0	1
$\rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

Solution -

$$\begin{aligned}
\delta(q_0, 110101) &= \delta(q_1, 10101) \\
&= \delta(q_0, 0101) \\
&= \delta(q_2, 101) \\
&= \delta(q_3, 01) \\
&= \delta(q_1, 1) \\
&= \delta(q_0, \Lambda) \\
&= q_0
\end{aligned}$$

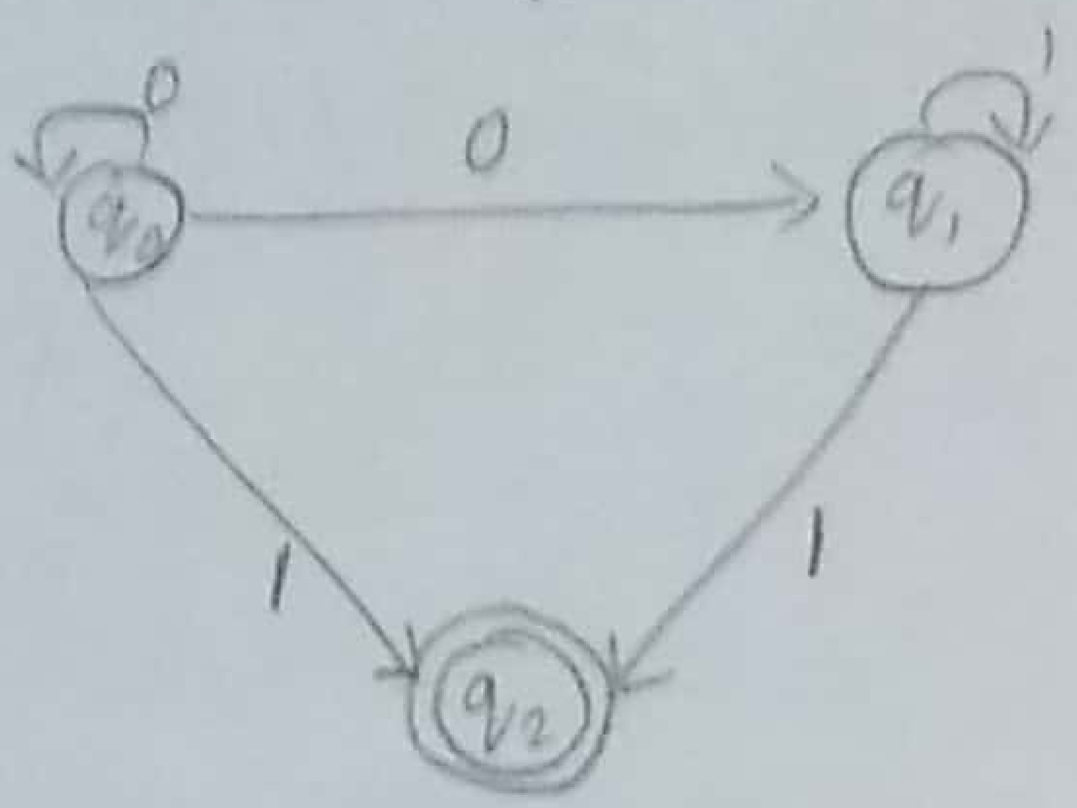
Hence $q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_2 \xrightarrow{1} q_3 \xrightarrow{0} q_1 \xrightarrow{1} q_0$

The symbol $\downarrow$ incidicates the current input symbol being processed by the machines.

Non Deterministic finite State machines :-

NDFA
NFA

we explain the concept of non deterministic finite automaton using a transition ~~dig~~ diagram



Transition system representing non deterministic automaton

If the automaton is in a State $\{q_0\}$ and the input symbol is 0. what will be the next state? From the fig. It is clear that the next State will be either $\{q_0\}$ or $\{q_1\}$, thus some moves of the machines cannot be determined uniquely by the input symbol and the present state. Such machines are called non deterministic automata.

Formal definition :-

A non deterministric finite automaton (NDFA) is a 5 - tuple $(Q, \Sigma, \delta, q_0, F)$, where.

i) Q is a finite non empty set of states.

ii) Σ is a finite non empty set of inputs.

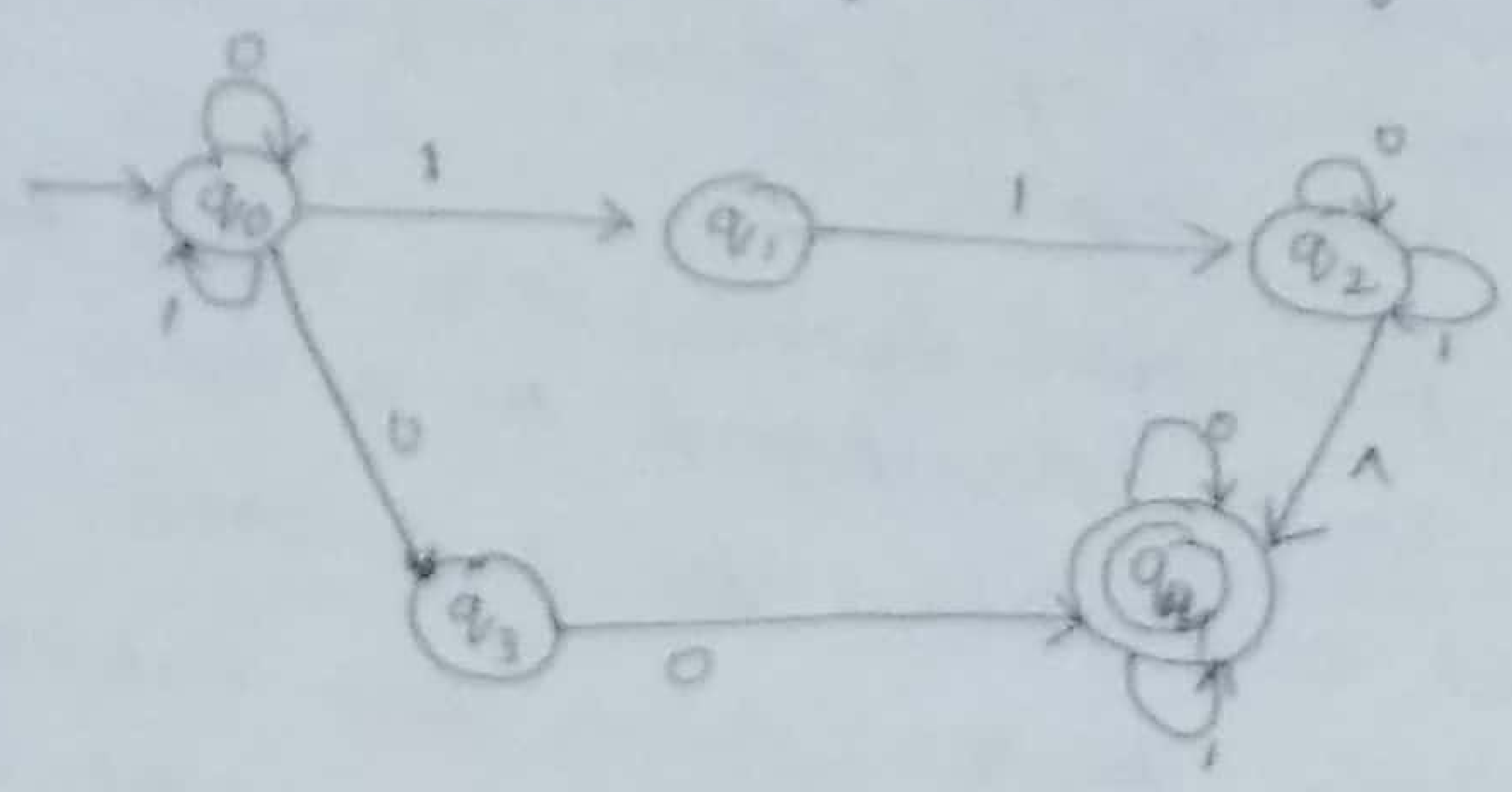
iii) δ is the transition function mapping from $Q \times \Sigma$ into 2^Q which is the power set of Q, the set of all subsets of Q.

iv) $q_0 \in Q$ is the initeal state.

v) $F \subseteq Q$ is the set of final states.

we note that the difference between the deterministic and non deterministic automata is only in δ. For deterministic automaton (DFA), the outcome is a state, i.e, an element of Q, for non-deterministic automaton in out come is a subset of Q.

Consider for ex, the non deterministic automaton, whose transition diagram is given in fig



Transition system for a nondeterministic automaton

The sequence of states for the input string 0100 is given in fig. Hence

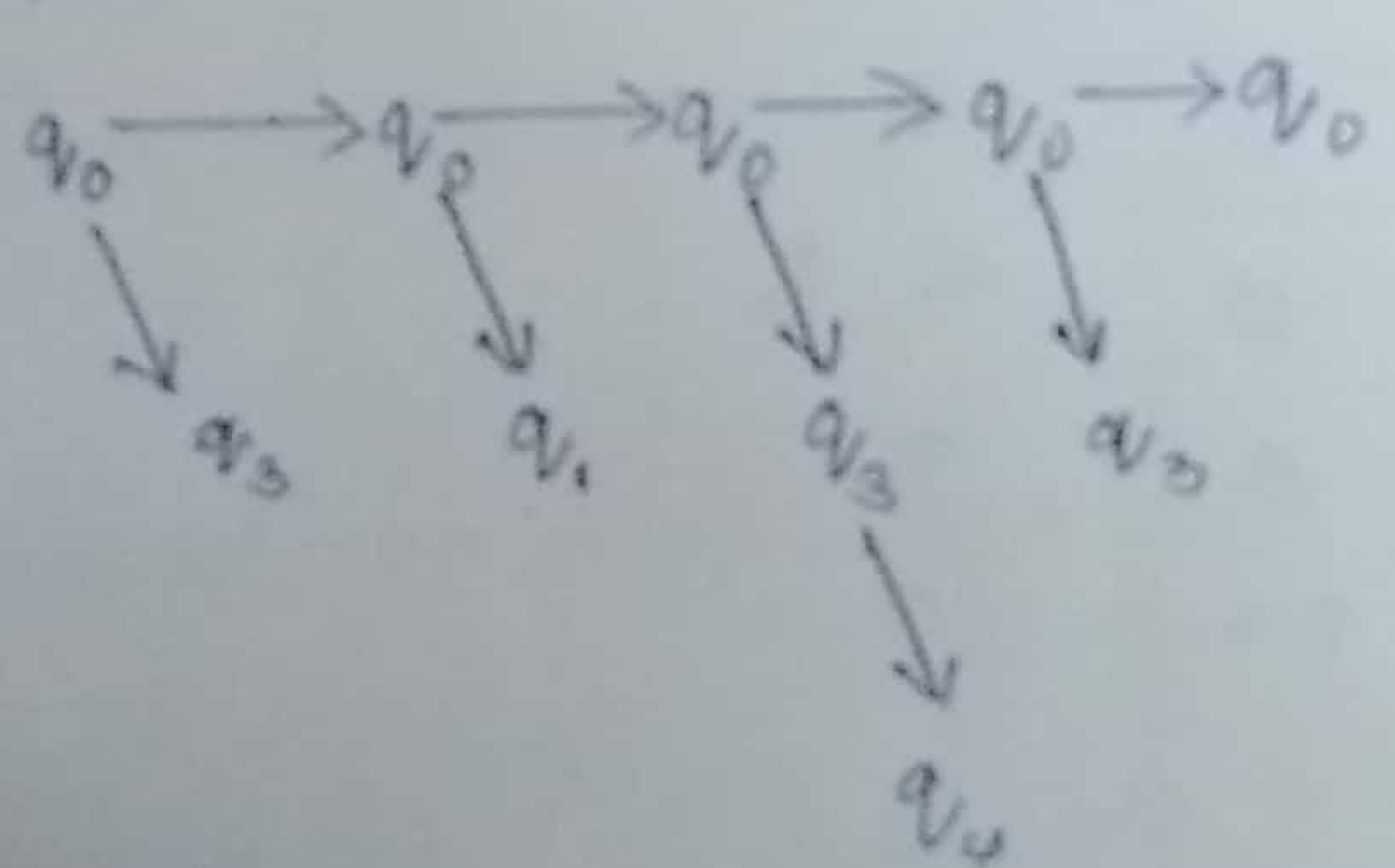
$$\delta(q_0, 0100) = \{q_0, q_3, q_4\}$$

Since q_4 is an accepting state, the input string 0100 will be accepted by the non deterministic automaton.

Def :-

A string $w \in \Sigma^*$ is accepted by NDFA M if $\delta(q_0, w)$ contains some final state.

NOTE- As M is non deterministic, $\delta(q_0, W)$ may have more that one state, so W is accepted by M if a final state is one among the possible states M can reach on application of W



States reached while processing 0100.