16/4/20

Boolean Algebra

Boolean Algebra is the algebra of binary variables. Binary variables are also known as Boolean variables. Boolean variables have only two values 0 and 1.

The objectives of this use are as follows :-

1. To simplify the procedure necessary in the solution of logical problems.
2. To simplify any circuit to its fewest components necessary to perform the function.

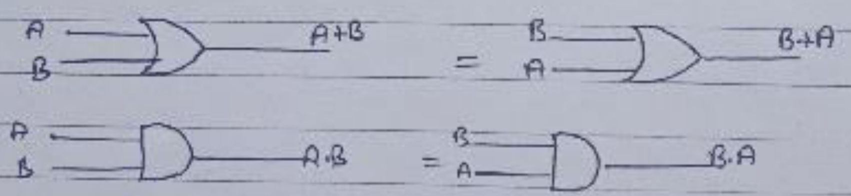
Basic Laws

1. Commutative Law :-

The commutative laws allow the change in position of an AND or OR variable.

(Law 1) $A+B = B+A$

(Law 2) $A.B = B.A$

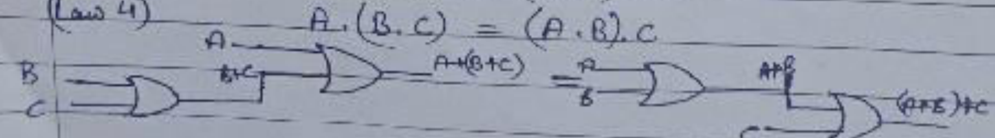


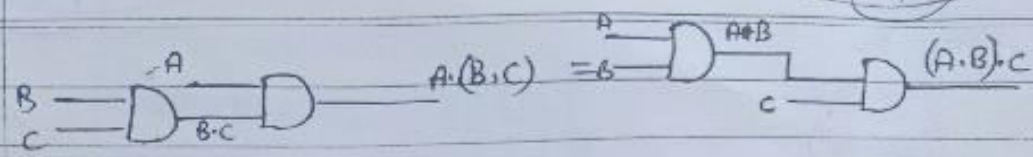
2. Associative Law :-

The associative law allow the grouping of variables.

(Law 3) $A+(B+C) = (A+B)+C$

(Law 4) $A.(B.C) = (A.B).C$

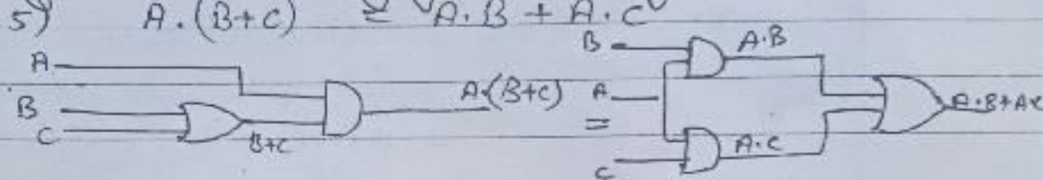




③ <u>Distributive Law :—</u>

The distributive laws allow the factoring or multiplying out of expressions.

(Law 5) $A.(B+C) = A.B + A.C$



④ <u>Law of Complementation :—</u> [NOT Law]

The term complement simply means to invert, to change 1's to 0's and 0's to 1's. The five laws of complementation are;

(Law 6) $\bar{0} = 1$

(Law 7) $\bar{1} = 0$

(Law 8) If $A = 0$ then $\bar{A} = 1$

(Law 9) If $A = 1$ then $\bar{A} = 0$

(Law 10) $\bar{\bar{A}} = A$

⑤ <u>OR Law :—</u>

The four OR Laws are as follows :—

(Law 11) $A + 0 = A$

(Law 12) $A + 1 = 1$

(Law 13) $A + A = A$

(Law 14) $A + \bar{A} = 1$

⑥ <u>AND Law :—</u>

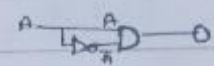
The four AND Laws are as follows :—

(Law 15) $A.0 = 0$

(Law 16) $A.1 = A$

(Law 17) $A.A = A$

(Law 18) $A.\bar{A} = 0$



(Law 19) $A+(BC) = (A+B)(A+C)$

Proof:- $A+BC = A.1+BC$ (Law 16)

$= A(1+B) + BC$ (Law 12)

$= A.1 + AB + BC$ (Law 5)

$= A(1+C) + AB + BC$ (Law 12 & 16)

$= A.1 + AC + AB + BC$ (Law 5)

$= A.A + AC + AB + BC$ (Law 17)

$= A(A+C) + AB + BC$

$= A(A+C) + B(A+C)$

$= (A+C)A + (A+C)B$ (Law 2)

$= (A+C)(A+B)$

$\boxed{A+BC = (A+B)(A+C)}$ hence verify

(Law 20) $A+(\bar{A}.B) = A+B$

$A + \bar{A}B = A.1 + \bar{A}B$

$= A(1+B) + \bar{A}B$

$= A.1 + AB + \bar{A}B$

$= A + AB + \bar{A}B$

$= A + BA + B\bar{A}$

$= A + B(A+\bar{A})$

$= A + B.1$

$\boxed{A+\bar{A}B = A+B}$

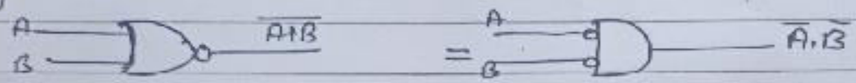
Demorgan's Theorem

① Demorgan's First Theorem :-

$$\boxed{\overline{A+B} = \overline{A}\cdot\overline{B}}$$

for n input $\overline{A+B+C+\cdots} = \overline{A}\cdot\overline{B}\cdot\overline{C}\cdots$

~~This~~ In words, it says the complementation of a sum is equals to the product of the complements.



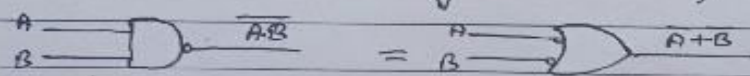
Bubbled AND gate

② Demorgan's Second Theorem :-

$$\overline{A.B} = \overline{A}+\overline{B}$$

for n inputs $\overline{A.B.C\cdots} = \overline{A}+\overline{B}+\overline{C}\cdots$

It says the complement of product equals the sum of the complements.



Using Demorgan's Theorem :-

The transformation is more easily performed by following these steps :-

① Complement the all variables.
② Change all AND into OR gate.
③ Change all OR into AND gate.
④ Complement both sides of the equation.

for eg:- $\quad Y = \overline{A.\overline{B}+C}$

$\overline{y} = A\overline{B}+C \Rightarrow \overline{\overline{y}} = (A+\overline{B})\, C$

Demorganisation →

$= y = \overline{(A+\overline{B}).C}$

Theorem of Duality :-

The ~~convert~~ theorem convert the Boolean relation into another Boolean realation

1. Change AND into OR sign
2. Change OR into AND sign
3. Complement 0's and 1's of expression

eg.:- $A+0=A$ Boolean relation

$\Rightarrow$ $A.1=A$ another Boolean relation