M.Sc. III Sem
PHY- 302
U-3

1. Nuclear Forces:

Since nuclei contain charged particles, protons, the forces acting between the protons should be forces of repulsion. If there were no other forces acting between the particles of nuclei, the latter would not be able to retain a stable state, because the repelling protons would fly out in all directions. To have a stable state of a nucleus we have to assume that some other forces exceed the forces of repulsion. Even aside from the sign of the force, electrostatic forces are much too weak to account for the main effects, gravitational forces are weaker still and offer no assistance in the problem. It is therefore necessary to postulate an entirely new type of interaction, Known as nuclear force. Let us collect following **information on nuclear forces** from study of complex nuclei.

a) Saturation Property-We know that the nuclei have the same density ($R=R_oA^{1\backslash 3}$) and the binding energy per nucleon for nuclei with A>40 is constant. These facts imply that the nuclear force saturates. Nucleons attract each other strongly only if they are in the same orbital state. According to the Pauli principle only two neutrons and two protons will be found in the same orbital state. Therefore it is possible to find four nucleons strongly bound or α-particle structure, also confirmed by binding energy curve.

b) Charge Independence-We know that the atomic mass number A is approximately equal to twice the atomic number Z for the light and intermediate nuclei. It shows that light nuclei prefer to add nucleons in n-p pairs, i.e. there is a strong interaction between neutrons and protons. The neutron excess in heavy nuclei confirms that n-n force is attractive, but it is sufficiently attractive to lead to a stable di-neutron. For a good approximation one can write

n-n=p-p=n-p or the forces as charge independent.

C) Shape of the potential-As the nuclear interactions do not extend to very large distances beyond the nuclear radius, and $1\backslash r^n(r>1)$ character is not useful to solve the problem. To avoid this difficulty, the idea of cut –off in the potential is introduced. The parameters are taken to be the strength v_0 (the value of potential at the origin and the range α (the distance beyond which potential goes to zero rapidly). It is less than nuclear dimensions.

We use the principles of wave mechanics on nuclear problems and than by comparison with experimental data; find a consistent description of the nuclear forces acting between two nucleons. There are two general methods of investigation, the study of n-p and p-p scattering events over a wide range of energy and the study of deuteron.

Introduction of Deuteron: Let us first consider deuteron in order to exhibit some of the concepts involved in discussing nuclear potentials and the quantum states of nuclei. The deuteron does possess measurable properties which might serve as a guide in the search for the correct nuclear interaction. **These properties are:**

1) The extraordinary stability of the alpha particle shows that the most stable nuclei are those in which number of neutrons and photons are equal. The deuteron consists of two particles of roughly equal masses M, so that the reduced mass of the system is 1\2M.

2) The binding energy of the deuteron is very small. Its experimental value is 2.225±0.002 Mev. Since the energy needed to pull a nucleon out of a medium mass nucleus is about 8 MeV, we must regard the deuteron as loosely bound.

3)The angular momentum quantum number, often called the nuclear spin, of the ground state of the deuteron determined by a number of optical, radiofrequency and micro – wave methods is one. It suggests that the spins are parallel (triplet state) and the orbital angular momentum of the deuteron about their common center of mass is zero. Thus the ground state is ^3s state.

4) The parity of deuteron as measured, indirectly, by studies of nuclear disintegrations and reactions for which certain rules of parity changes exist, is even.

5) The sum of the magnetic dipole moments of the proton ($2.79275_{\mu N}$) and neutron ($-1.91315_{\mu N}$), do not exactly equal to magnetic moment of deuteron ($0.85735_{\mu N}$) measured by magnetic resonance absorption method.

6) The electric quadruple moment and the magnetic moment discrepancy can be explained if the ground state is a mixture of the triplet states $^3S_1$ and $^3D_1$ having even parity.

7) The force depends only on the separation of the nucleons not on the relative velocity or orientation of the nucleon spins with respect to the line.

Why we study deuteron problem: Two ~~neutrons~~ nucleons either are

a) Two protons (P,P)
b) Two neutrons (n,n)
c) One proton and one neutron (p,n) which is called deuteron

While studying two nucleon problems if is first decided whether

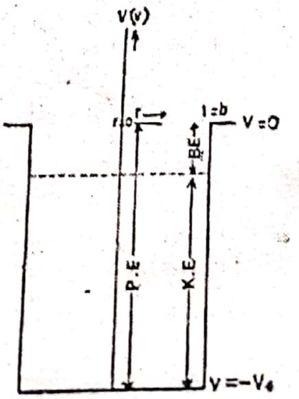
the nucleons are in **bound state and unbound state.**

Bounded state: When the nucleons are found in the form of a deuteron, they are said to be in bound state.

Unbound state: When the two nucleon are (all) independently moving i.e. they are in excited state (n-p) Scattering occurs, called unbound state.

The ground state of deuteron: To ascertain the many properties of nuclei and to deal with problems such as nuclear structure and binding energies, it is important to understand the nature of the forces existing between nucleons in the nucleus. However, there is not enough known about the internucleon forces, and it is difficult to predict the nuclear properties accurately. In spite of this, theoretical predictions. Consequently, we know a great deal about the nature of nuclear forces despite the fact that the form of the nuclear force law is not known as explicitly as, say, that of the coulomb law.

The three possible states of the two nucleon system, die-neutron (nn), die proton (pp) and deuteron (np), only the deuteron is known to be stable.



By assuming the fact that the central potential describe the deuteron in its ground state.

The Schrodinger wave equation for deuteron can be written as;

$$\nabla^2\varphi(r,\theta,\emptyset) + \frac{2\mu}{\hbar^2}(W-v(r))\,\varphi(r,\theta,\emptyset) = 0 \ldots\ldots\ldots\ldots (1)$$

Where w is the binding energy i.e. w=-B

And μ = reduced mass.

$\mu = M_p M_n / M_p + M_n = M/2$ (since, $M_n \approx M_p \approx M$)

$$\nabla^2\varphi(r,\theta,\emptyset) + \frac{M}{\hbar^2}\{-B - v(r)\}\varphi(r,\theta,\emptyset) = 0 \ldots\ldots (2)$$

Here $\varphi(r,\theta,\emptyset)$ can be split into a radial wave function and angular wave function $(\theta and\ \emptyset)$

$$\varphi(r,\theta,\emptyset) = \varphi_1(r)\,\varphi_2(\theta,\emptyset) \ldots\ldots\ldots (3)$$

Putting equation (3) in eq^n (2) we get an eq^n having left hand term of radial part and right hand term of angular function θ *and* $\emptyset$. *Both the sides* of resulting equation will be equal to constant which comes out to be $l(l+1)$ but the deuteron ground state is a 3_{s1} state and l=0

$$\frac{d^2\mu}{dr^2} + \frac{M}{\hbar^2}(-B - v(r))u(r) - \frac{l(l+1)}{r^2}u(r) = 0 \ldots\ldots (4)$$

Where, $\frac{l(l+1)}{r^2} \approx u(r)$

$$\varphi_1(r) = \frac{u(r)}{r}$$

Hence equation (4) can be written as;

$$\frac{d^2u}{dr^2} + \frac{M}{\hbar^2}(-B - v(r))u(r) = 0 \ldots\ldots\ldots\ldots (5)$$

The boundary condition of the rectangle potential in well v(r)=0 for $r>r_0$........(6)

v(r)=v_0 for $r\leq r_0$.......(7)

Thus equation (5) takes the form in this two region as;

$$\frac{d^2\mu}{dr^2}+\frac{M}{\hbar^2}(v_0-B)u=0 \; for \; r\leq r_0$$

or

$$\frac{d^2\mu}{dr^2}+K^2u=0 \text{........ (8)}$$

$$K^2=\frac{M}{\hbar^2}(v_0-B)$$

Similarly equation (5) and (6) we have

$$\frac{d^2\mu}{dr^2}-\frac{MB}{\hbar^2}u=0$$

or

$$\frac{d^2\mu}{dr^2}-r^2u=0 \text{....... (9)}$$

$$r^2=\frac{MB}{\hbar^2}$$

Equation (8) and (9) are two second order differential equation the general solution of (8) can be written as

$u(r)$= A sinkr + B coskr......... (10)

The usual condition on ψ(r) (finite at r=0 and zero at r-∞) demand that U= ψr vanishes at the origin thus, from equation (10) we get

$$u(r)=Asinkr \qquad For \; r\leq r_0 \text{..........(11)}$$

And the general solution of equation (9) is

$u(r) = Ce^{-\gamma r} + De^{\gamma r}$ $r>r_0$ … … . (12)

And the boundary condition at infinity demand that D=0 so that U(r) remains finite, Thus

$u(r) = Ce^{-\gamma r}$ $r>r_0$……….. (13)

Now at $r=r_0$ both the function given equation (12) and (13) should be continuous i.e.

$$A\sin Kr_0 = Ce^{-\gamma r_0} \quad \dots\dots\dots (14)$$

And

$$AK\cos Kr_0 = Ce^{-1/r_0} \quad \dots\dots\dots\dots (15)$$

Dividing equation (15) by equation (14) we obtain

$$K\cot Kr_0 = -\gamma$$

$$\cot Kr_0 = \frac{-r}{K} \quad \dots\dots\dots\dots (16)$$

Equation (16) represents the relation b/w depth of potential, nuclear distribution and binding energy. The value of γ *is*

Known from empirical value of B and M. Hence equation (16) given the relationship b/w ranges r_o and depth v_0 of potential.

In the about table some values of potential depth corresponding to nuclear distance are given

Range $r_o(10^{-13}m)$	Potential depth v_0 (1MeV)
1.0	120
1.5	**59**
2.0	**36**
2.5	**25**

From the experimental measurement it is known as the binding energy on deuteron in ground state is 1.113 MeV per nucleon compared with the average value of over and MeV per nucleon in nuclei. Thus it explain the fact that the deuteron is a loosely bound system.

Since V_0 is more greater than W by rewriting equation (16)

$$cotKr_0 = \frac{-r}{K}$$

$$K^2 = \frac{M}{\hbar^2} V_0$$

$$r^2 = \frac{MB}{\hbar^2}$$

$$\cot Kr_0 = -\frac{\sqrt{MB}}{\hbar^2} \sqrt{\hbar^2/mv_0}$$

$$\cot Kr_0 = -\sqrt{B}/v_0$$

$$\cos Kr_0/\sin Kr_0 = -\sqrt{B}/v_0$$

$$\cos Kr_0 = 0$$

$$Kr_0 \approx \pi/2,\ 3\pi/2 \ldots\ldots$$

$$V_0 r_0^2 \approx \frac{\hbar^2}{m}\ \pi^2/4,\ \frac{\hbar^2}{m} \qquad (9\pi^2/4) \ldots.$$

Hence the ground state wave function together with the range and depth of the potential is given by

$$V_0 r_0^2 \approx \pi^2\ \hbar^2/4m$$