Internal Conversion

In the process of internal conversion the nuclear gamma ray ejects out an orbital electron. As K-shell is nearest to the nucleus, a K-shell electron to be ejected compared to the one in any higher shells. One therefore calcutates the transition for K-electron conversion only.

Initially, the electron is in a bound state, and a nuclear gamma ray Knocks it out of the nucleus. Thus, its final wave function is a free particle wave function with Coulomb function F(Z,E)

$$\psi_i = \sqrt{\frac{1}{\pi a^3}} \exp(-R/a) \quad \text{(1a)}$$

$$\psi_f = \sqrt{\frac{1}{V}} \exp(i \cdot \vec{k} \cdot \vec{R}) \quad \text{(1b)}$$

Where,

$a = a_0/Z$, $a_0 = \hbar^2/mc^2$ being the Bohr radius, and V is the volume of the box in which one normalises ψ_f.

Here, $\vec{R}$ is the position vector of the electron with respect to the nucleus, and $\vec{K}$ is the wave ~~position~~ vector of the ejected electron. If x_i and x_f are initial and final wave functions of the nucleus. ~~Then~~, the transition probability for the conversion process is given by.

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$$\lambda_{a\to b} = 2\,\frac{2\pi}{\hbar}\, Z(k) \int |H'_{if}|^2\, d\Omega \qquad (2)$$

where, the factor of 2 occurs because there are two K-shell electrons and.

$$Z(k) = V\,\frac{m\hbar k}{(2\pi\hbar)^3} \qquad (3)$$

Assume H' to be the electrostatic potential,

$$H' = \sum_{i=1}^{Z} \frac{e^2}{|\vec{R}-\vec{r}_i|} \qquad (4)$$

Where, $\vec{r}_i$ is the position coordinate of the i^{th} proton. Then, the matrix element in eq^n (2) is given by,

$$H'_{if} = \langle \psi_i \chi_i | H' | \psi_f \chi_f \rangle = \sqrt{\frac{1}{\pi a^3 V}} \sum_{i=1}^{Z} \int d^3R \int d\tau\, \exp(-i\vec{k}\cdot\vec{R})\, \chi_f^* \frac{e^2}{|\vec{R}-\vec{r}_i|} \exp(-R/a)\, \chi_i \qquad (5)$$

In order to solve this integral we expand the Coulomb function as,

$$\frac{1}{|\vec{R}-\vec{r}_i|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{4\pi}{2l+1} \frac{r_i^l}{R^{l+1}} Y_m^l(\Phi,\phi)\, Y_m^{l*}(\theta_i,\theta_l) \qquad (6)$$

Where

(Φ, Φ) are the polar angles of R, and (θ_i, ϕ_i) are those of r_i.

Substituting eq^n 6 into eq^n 5 we get.

$$H'_{if} = -\frac{e}{\sqrt{\pi a^3 V}} \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} \frac{4\pi}{2l+1} Q_{lm}(i,f) I_{lm}, \qquad (7)$$

Where $Q_{lm}(i,f)$ is the electric multipole matrix element given by eq^n 2 and I_{lm} is the integral

$$I_{lm} \equiv \int \exp(-R/a) \exp(-i\vec{k}\cdot\vec{R}) R^{-(l+1)} Y_m^l(\Phi, \Phi)\, d^3R \qquad (8)$$

On assuming that the internal conversion process takes place well above the threshold, i.e ⇒ $Ka \gg 1$, and $\exp(-R/a) \simeq 1$.

If θ and ϕ are the polar angles of the vector $\vec{K}$ with respect to some arbitrary z-direction in space, then,

$$I_{lm} = 4\pi i^{-1} \frac{K^{l-2}}{(2l-1)!!} Y_m^l(\theta, \phi) \text{ for } ka \gg 1. \qquad (9)$$

and eq^n 7 takes the form.

$$H'_{if} = \frac{(4\pi)^2}{\sqrt{\pi a^3 V}} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{i^l k^{l-2}}{(2l+1)!!} Q_{lm}(i,f) Y_m^l(\theta, \phi) \qquad (10)$$

Thus, the transition probability is given by

$$T_{i\to f}^{(IC)} = 128 \frac{mc^2}{\hbar^3 a^3} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{K^{2l-3}}{[(2l+1)!!]^2} \left| Q_{lm}(i,f) \right|^2 \quad (11)$$

Because of the selection rules, only certain l-values in the above sum are admissible.

The internal conversion coefficient 'α' is now defined as the ratio of the probability for internal conversion to the probability for multipole transitions

$$\alpha = \frac{T_{i\to f}^{(EL)}}{T_{i\to f}} \quad (12)$$

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