

Dr. Apurva Muley (Guest Lecturer)

School of Studies in Physics, Vikram University, Ujjain

Lecture for M. Sc. Physics IV Semester students

Paper-I: Condense matter physics-II

Unit-3 Carrier Concentration

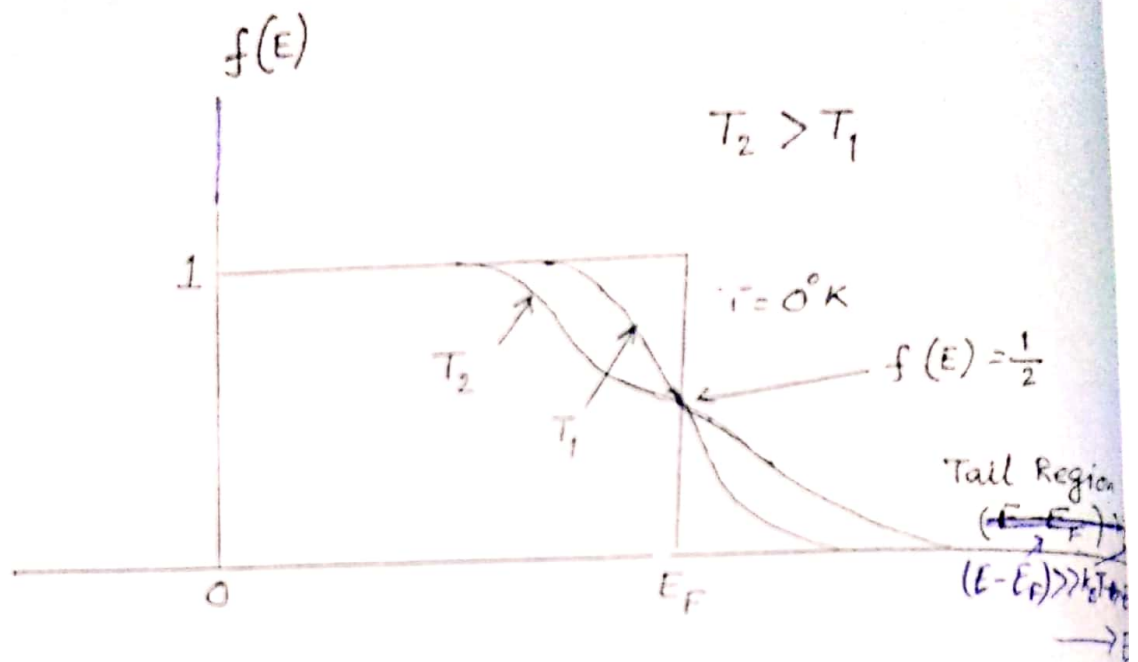
Carrier Concentration: Intrinsic semiconductors

In the field of semiconductors, electrons and holes are usually referred to as free carriers, or simply carriers, because it is these particles which are responsible for carrying the currents. The number of carriers is an important property of a semiconductor, as this determines its electrical conductivity. In order to determine the number of carriers, we need some of the basic results of statistical mechanics.

The most important result in this regard is the Fermi-Dirac (FD) distribution function

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1} \quad (1)$$

This function gives the probability that an energy level E is occupied by an electron when the system is at temperature T . The illustration of this function is as follows:



"The Fermi-Dirac distribution function"

Here we see that, as the temperature rises, the unoccupied region below the Fermi level E_F becomes longer, which implies that the occupation of high energy states increases as the temperature is raised, a conclusion which is most plausible, since increasing the temperature raises the overall energy of the system. Note also that $f(E) = \frac{1}{2}$ at the Fermi level ($E = E_F$) regardless of the temperature. That is, the probability that Fermi level is occupied is always equal to one-half.

In semiconductors, it is the tail region of the FD distribution which is of particular interest. In that region the inequality $(E-E_f) \gg k_B T$ holds true and, one may therefore neglect the term unity in the denominator of eq (1). The FD distribution then reduces to the form

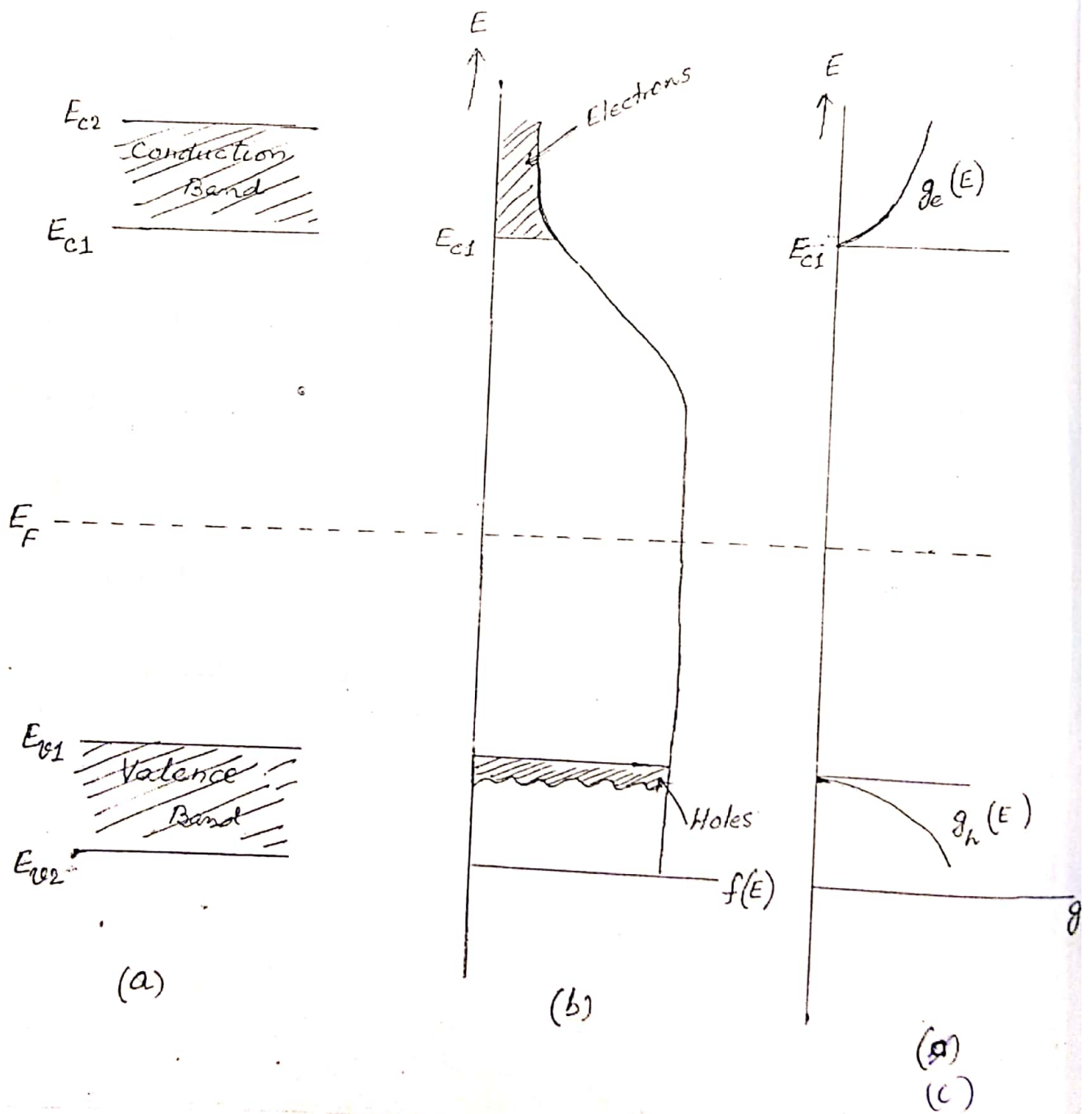
$$f(E) = e^{E_f/k_B T} e^{-E/k_B T}, \quad (2)$$

which is the familiar Maxwell-Boltzmann, or classical distribution. This simple distribution therefore suffices for the discussion of electron statistics in semiconductors.

We can calculate the concentration of electrons in the conduction band in the following manner. The number of states in the energy range $(E, E+dE)$ is equal to $g_e(E)dE$, where $g_e(E)$ is the density of electron states. Since each of these states has an occupation probability $f(E)$, the number of electrons actually found in this range is equal to $f(E)g_e(E)dE$. The concentration of electrons throughout the conduction band is thus given by the integral over the band

$$n = \int_{E_{c1}}^{E_{c2}} f(E) g_e(E) dE \quad (3)$$

where E_{c1} and E_{c2} are the bottom and the top of the band respectively, as shown below:



(L)

It may be observed that the entire conduction band falls in the tail region. Thus we may use the M-B function for $f(E)$ in figure.

We calculate the density of states where the expression appropriate to the standard band form is given by

$$g_c(E) = \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} (E - E_g)^{1/2} \quad (4)$$

where the zero energy level has been chosen to lie at the top of the valence band. Thus $g_c(E)$ vanishes for $E < E_g$, and is finite only for $E > E_g$ as shown in fig. (c).

When we substitute for $f(E)$ and $g_c(E)$ into eq. (3), we obtain

$$n = \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} e^{E_F/k_B T} \int_{E_g}^{\infty} (E - E_g)^{1/2} e^{-E/k_B T} dE \quad (5)$$

For convenience the top of conduction band is set equal to infinity.

Since the integrand decreases exponentially to high energies, the error

introduced by changing this limit from E_0 to ∞ is quite negligible. By changing the variables, and using the result

$$\int_0^{\infty} (x)^{1/2} \cdot e^{-x} dx = \frac{\pi^{1/2}}{2},$$

One can readily evaluate the integral in eq (5). The electron concentration then reduces to the expression

$$n = 2 \left(\frac{m_e k_B T}{2\pi\hbar^2} \right)^{3/2} e^{E_F/k_B T} e^{-E_g/k_B T} \quad (6)$$

The electron concentration is still not known explicitly because the Fermi energy is so far unknown. This can be calculated in the following manner. Essentially the same ideas employed above may be used to evaluate the number of holes in the valence band. The probability that a hole occupies a level E in this band is equal to $1-f(E)$ since $f(E)$ is the probability of electron occupation. Thus the probability of hole occupation f_h is

$$f_h = 1 - f(E) \quad (7)$$

Since the energy range involved here is much lower than E_F , the FD function of eq (1) must be used rather than eq (2). Thus

$$f_h = 1 - \frac{1}{e^{(E-E_F)/k_B T} + 1} = \frac{1}{e^{(E_F-E)/k_B T} + 1} \approx e^{-E_F/k_B T} e^{E/k_B T}, \quad (8)$$

where the approximation in the last expression follows as a result of the inequality $(E_F - E) \gg k_B T$. This validity of this inequality in turn can be seen by referring to Figure (b), which shows that $E_F - E$ is of the order of $E_g/2$ which is much larger than $k_B T$ at room temperature.

The density of states for the holes is

$$g_h(E) = \frac{1}{2\pi^2} \left(\frac{2m_h}{\hbar^2} \right)^{3/2} (-E)^{1/2} \quad (9)$$

which is approximate for an inverted band. Note that the term $(-E)$ in this equation is positive, because the zero-energy level is at the top of the valence band, and the energy is measured positive upward and negative downward from this level.