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Lecture for M. Sc. Physics II Semester students

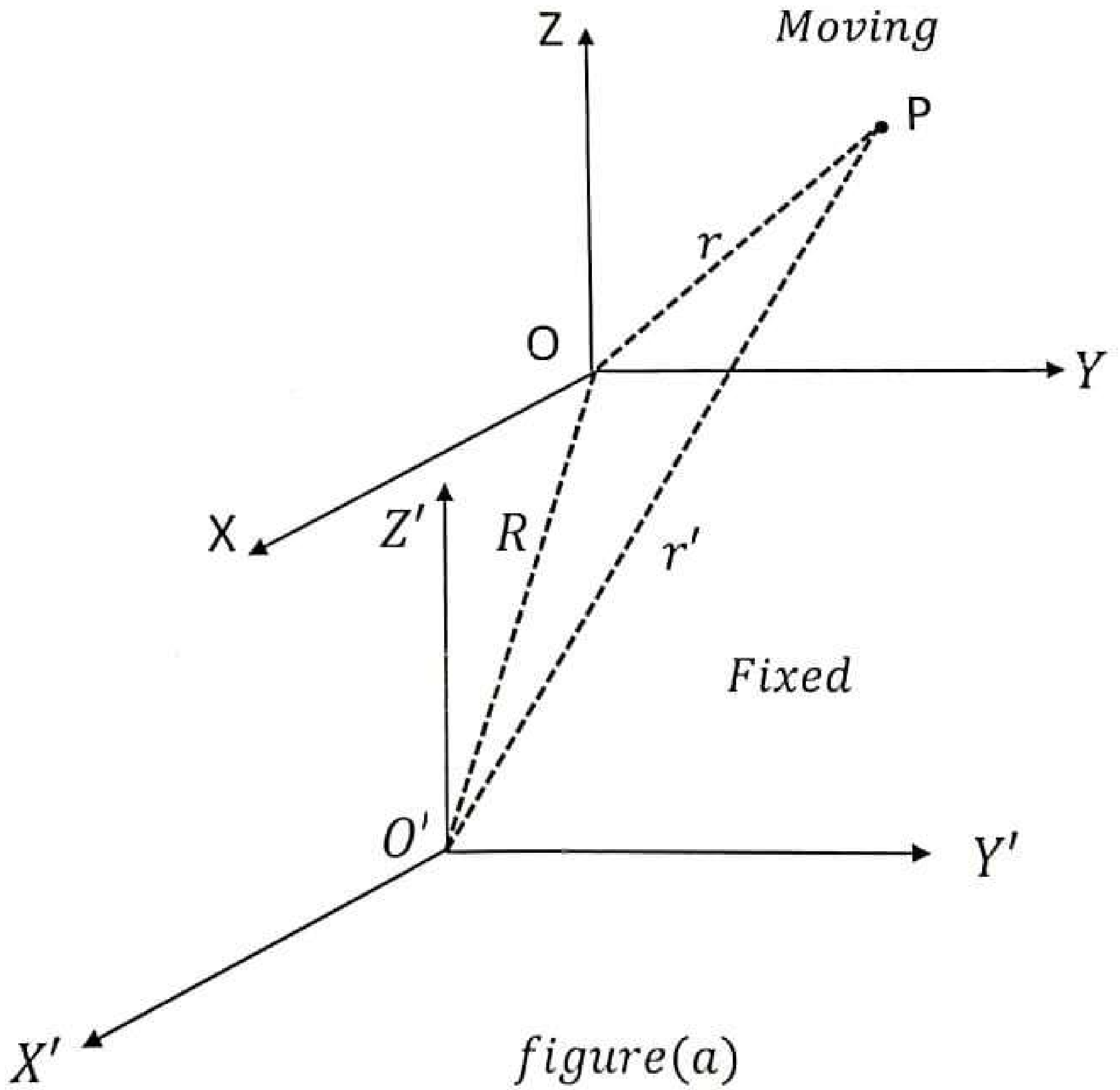
Paper-II: Classical Mechanics

Unit-2 Inertial Force

Introduction of Inertial force

All bodies have a tendency to remain in a state of rest or of uniform rectilinear motion, this tendency to resist the change in motion is called Inertia. To resist the change in motion, a body exerts a force which is equal and opposite to the force applied on the body to change its motion, this force exerted by the body is called ***Inertial force***.

Consider two coordinate system $O'(X',Y',Z')$ fixed in space and $O(X,Y,Z)$ fixed to a certain body that is moving with translational velocity with respect to the first system or fix frame.



figure(a)

Let $\vec{R}$ be the position vector of the point O′ with respect to O at a certain instant t.

Let P be any point whose position vector with respect to O and O′ are $\vec{r}$ and $\vec{r'}$ thus from figure (a).

$$\vec{r'} = \vec{r} + \vec{R} \qquad \text{--------(1)}$$

Differentiating equation (1) with respect to time

$$\frac{d\vec{r'}}{dt} = \frac{d\vec{r}}{dt} + \frac{d\vec{R}}{dt} \qquad \text{---------(2)}$$

Thus the velocity $\frac{d\vec{r'}}{dt}$ of a particle at the point P measured in the fixed system is the vector addition of the velocity $\frac{d\vec{R}}{dt}$ of moving frame and the velocity $\frac{d\vec{r}}{dt}$ of the particle at the point P in the moving frame.

The corresponding acceleration is given by

$$\frac{d^2\vec{r'}}{dt^2} = \frac{d^2\vec{r}}{dt^2} + \frac{d^2\vec{R}}{dt^2} \qquad \text{---------(3)}$$

The equation of motion of the particle at point P in the fixed frame is

$$m\frac{d^2\vec{r'}}{dt^2} = \vec{F}$$

Where $\vec{F}$ is total force acting on the particle P. The equation of motion of particle at point P in the moving frame of reference is:

$$m\frac{d^2\vec{r}}{dt^2} = m\frac{d^2\vec{r'}}{dt^2} - m\frac{d^2\vec{R}}{dt^2}$$

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F} - m\frac{d^2\vec{R}}{dt^2} \quad \text{And} \quad m\frac{d^2\vec{r}}{dt^2} = \vec{F}_{eff} \qquad \text{--------(4)}$$

Thus, if the moving frame of reference has an acceleration $\ddot{\vec{R}}$. The effective force acting on a particle is smaller than actual force by an amount $m\frac{d^2\vec{R}}{dt^2}$.

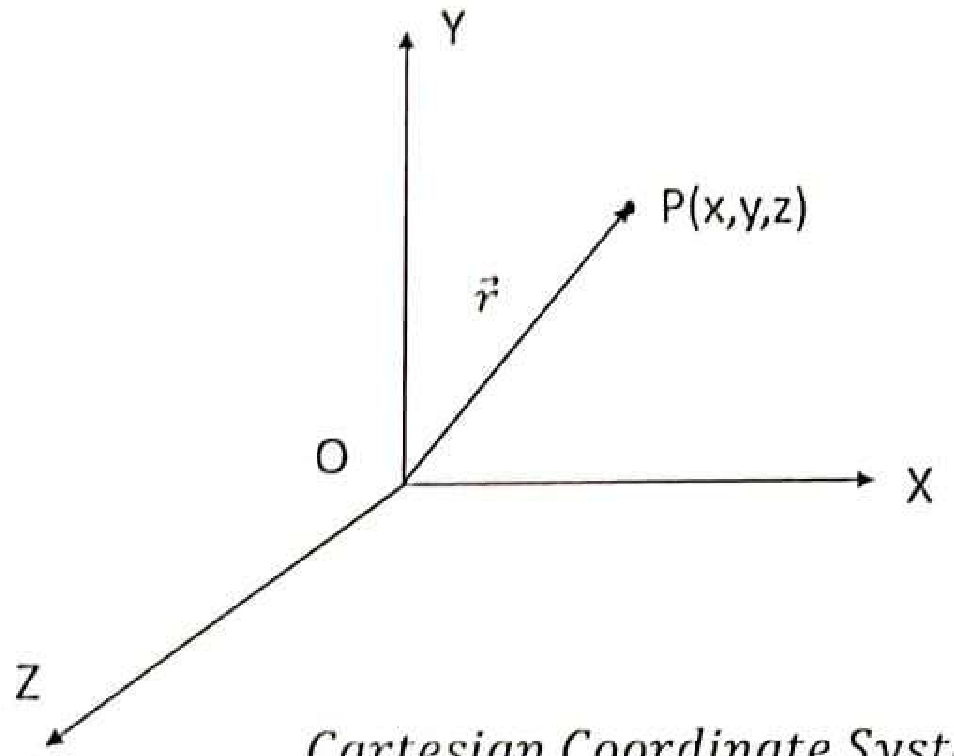
This additional term in R.H.S. of equation (4) doesn't represent any real force. It is called a Pseudo, Fictitious or non-inertial force. This reduction in force arises due to the acceleration $\frac{d^2\vec{R}}{dt^2}$ of the moving frame. When $\ddot{\vec{R}}$ equal to zero the equation of motion is identical in the two system. In other words Newton's law of motion are valid in the two systems moving with a uniform velocity. This is known as principle Newtonian relativity and Galilean invariance.

Frame of Reference & Its types

The frame relative to which the position or the motion of a body is specified, is called the ***Frame of reference.*** The position of other objects is specified relative to the frame of reference being assumed to be connected with a rigid body. For this, a convenient point is chosen as the origin of the frame of reference and the position of the body is specified by the position vector of the body from the origin.

The motion of a body appears to be different in different frames of reference. The simplest frame of reference is *Cartesian coordinate system.* In this system, the position of a particle at any point of its path is given by the position vector, expressed in terms of three coordinates (X, Y, Z) as

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$



Cartesian Coordinate System

Mainly types of frame of reference

1) Inertial frame of reference
2) Non-inertial frame of reference

Inertial frame of reference: - The frame of reference in which the Newton's law holds, are called inertial frames.

i.e. if a particle, subjected to no external force, is found to move in a straight line with constant velocity (or to remain at rest), then the coordinate system used for this purpose is called inertial frame. Thus in an inertial frame, a body not experiencing any force ($\vec{F} = 0$) appears unaccelerated ($\vec{a} = 0$) because from Newton's second law.

$$\vec{F} = m\vec{a} = 0$$

$$\text{Or} \quad \vec{a} = \frac{d^2\vec{r}}{dt^2} = 0$$

The law of inertia was first stated by Galileo, therefore the inertial frames are also called Galilean frames of reference.

Thus all those frames of reference which are either stationary relative to each other or are in uniform motion, are called the inertial frames provided that one of these frames is inertial. Inertial frames are necessarily the unaccelerated frames.

Non-inertial frame of reference :- The frames of reference in which Newton's law are not valid, are called the non-inertial frames because in such a frame a force –free particle will seem to have an acceleration. If we do not consider the acceleration of the frame but apply Newton's laws to the motion of the force-free particle, then it will appear that a force is acting on it. This means that in the accelerated frames, Newton's law of inertia is not valid. Thus a non-inertial frame of reference is defined as a frame of reference in which Newton's first law does not hold true. So all rotating frames are also non-inertial.