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Lecture for M. Sc. Physics II Semester students

Paper-II: Classical Mechanics

Unit-2 Coriolis Force

Coriolis force

Coriolis force is a fictitious force which appears to be acting on a particle moving with a certain linear velocity with respect to the observer connected to the rotating frame.

Let the angular velocity of the rotating system be constant. Then $\dot{\vec{\omega}} = 0$, let further the origin of the fixed and the moving system coincide. Then $\vec{R} = 0$ and we have $\vec{r'} = \vec{r}$. Hence we can write equation of motion for the rotating system as follows –

$$m\left(\frac{d^2\vec{r}}{dt^2}\right)_{rot} = m\left(\frac{d^2\vec{r}}{dt^2}\right)_{Fix} - 2m\vec{\omega} \times \left(\frac{d\vec{r}}{dt}\right)_{rot} - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) \qquad \text{--------(1)}$$

$$= \vec{F}_{eff}$$

Thus, the forces acting on the particle in the rotating frame are-

1) The real force $\vec{F} = m\left(\frac{d^2\vec{r}}{dt^2}\right)_{Fix}$.
2) The centrifugal force $-m\vec{\omega} \times (\vec{\omega} \times \vec{r})$ arising as a result of the rotation of coordinate axes.

3) The Coriolis force $-2m\vec{\omega} \times \left(\frac{d\vec{r}}{dt}\right)_{rot}$ arising as a result of the motion of the particle in the rotating system.

Astronomical & terrestrial applications of Coriolis force

Motion on the earth :- Let us fix two coordinate systems one at the Centre of the earth but fixed in space and the other at some point in the body of earth but rotating along with the earth with an angular velocity ω. A particle of mass m situated on the surface of the earth will be acted upon by the gravitational force $'mg'$. Some other real force $\vec{F}$ such as an electrostatic or a magnetic force may also act on it. Then, the equation of motion of the particle in the fixed system is,

$$m\left(\frac{d^2\vec{r}}{dt^2}\right)_{Fix} = \vec{F} + mg \quad \text{---------(1)}$$

The equation of motion of the some particle in the rotating system is

$$m\left(\frac{d^2\vec{r}}{dt^2}\right)_{rot} = \vec{F} + mg - 2m\omega\left(\frac{d\vec{r}}{dt}\right)_{rot} - m\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$m\left(\frac{d^2\vec{r}}{dt^2}\right)_{rot} = \vec{F} + m[g - \vec{\omega} \times (\vec{\omega} \times \vec{r})] - 2m\vec{\omega}\left(\frac{d\vec{r}}{dt}\right)_{rot} \quad \text{----(2)}$$

The second term on the R.H.S. of equation (2) represents the effective gravitational acceleration.

$$g_e = g - \vec{\omega} \times (\vec{\omega} \times \vec{r}) \quad \text{---------(3)}$$

The gravitational acceleration measured at any point will be less than the acceleration due to the earth if it were not rotating. The term $-\vec{\omega} \times (\vec{\omega} \times \vec{r})$ is called the centrifugal acceleration. It always points radially outwards. It will be found to be zero at the poles and has the maximum value at the equator.

The third term on the R.H.S. of equation (2) is the Coriolis force that acts on a particle which is moving with a velocity $\vec{v}_r = \left(\frac{d\vec{r}}{dt}\right)_{rot}$ on the earth. The direction of the Coriolis force will be at right angles to the plane formed by $\vec{v}_r$ and $\vec{\omega}$.

Consider now the effect of the Coriolis force on a particle situated at a point P and moving with a velocity $\vec{v}_r$, in a plane perpendicular to the axis of the rotation of the earth and having a latitude θ.

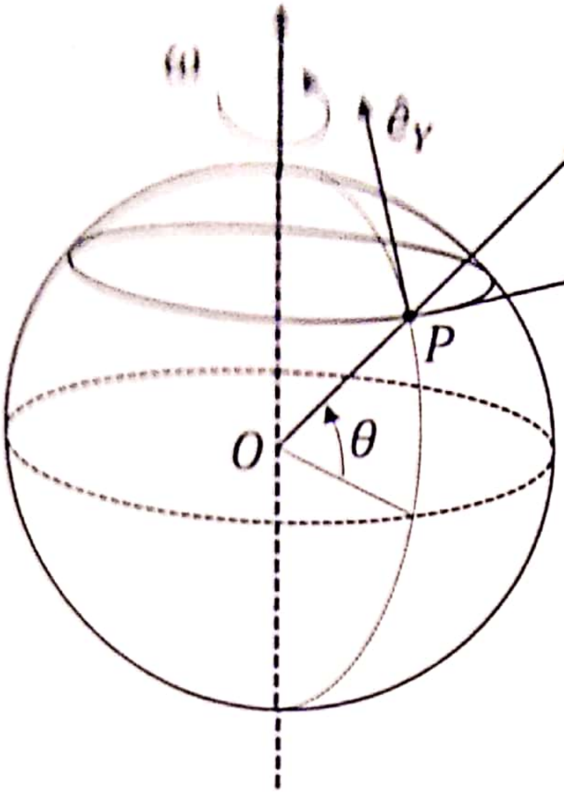

figure (a)

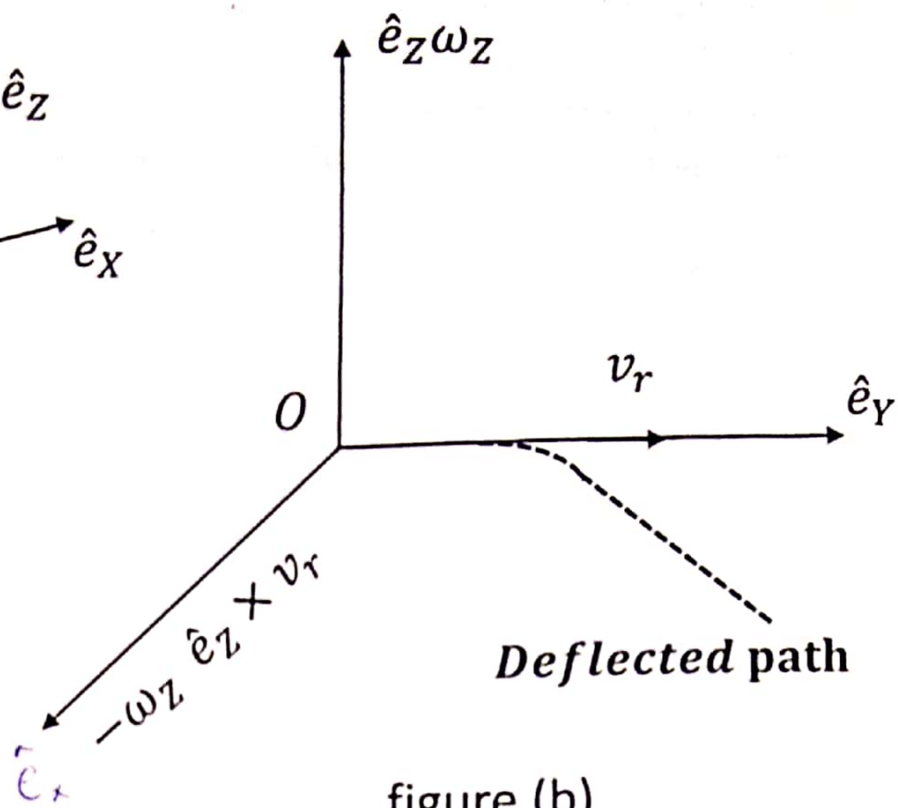

figure (b)

Particle P has latitude θ

Deflection of the path of a particle due to Coriolis force

From figure (a), it is clear that $\hat{e}_X$ and $\hat{e}_Y$ axes form a 'horizontal' plane at the point P under consideration while the $\hat{e}_Z$ axis is vertical. The component of ω along the vertical direction is $\omega_Z\hat{e}_Z = \hat{e}_Z\omega \sin\theta$ and is directed upward in the northern hemisphere and downward in the southern hemisphere. Hence, the path of the particle will be deflected towards the right in the northern hemisphere and towards the left in the southern hemisphere due to the Coriolis acceleration.

Effect of Coriolis force on a freely falling particle

Consider a particle which is falling freely towards the earth. Let the height h through which the particle falls be so small that the variation in g can be neglected. The acceleration of the particle is given by

$$\vec{a} = \vec{g} - 2\vec{\omega} \times \vec{v} \qquad \text{--------(1)}$$

And where $\vec{a}$ and $\vec{v}$ are measured with respect to the earth, i.e. in the rotating frame. From figure (a) in the northern hemisphere, we have

$\omega_x = 0, \omega_y = \omega\cos\theta \ \ and \ \ \omega_z = \omega\sin\theta$

The deflection produced by the Coriolis force is quite small and hence for a particle moving along $-\hat{e}_Z$ direction, the force will have negligibly small components along the directions $\hat{e}_X$ and $\hat{e}_Y$. Thus,

$$\dot{x} \cong 0, \dot{y} \cong 0 \ and \ \dot{z} \cong -gt$$

Hence, $\omega \times v = -\omega gt \cos\theta \, \hat{e}_X$ and lies along the X-direction. As g is directed along the $-\hat{e}_Z$ direction. The components of the acceleration of the particle are,

$$a_x = \ddot{x} = 2\omega gt \cos\theta \quad \text{---------(2)}$$

$$a_y = \ddot{y} = 0 \quad \text{---------(3)}$$

$$a_z = \ddot{z} = -g \quad \text{---------(4)}$$

Thus, from equation (2) the acceleration along the $\hat{e}_X$ direction is seen to be due to the Coriolis force.

On integrating equation (2) & (4) we get the solution,

$$x = \frac{1}{3}\omega gt^3 \cos\theta \quad \text{---------(5)}$$

$$z = z_0 - \frac{1}{2}gt^2 \quad \text{---------(6)}$$

Where the initial position is $z(0) = z_0$ and $x(0) = 0$ and the initial velocity is

$$\dot{x}(0) = \dot{z}(0) = 0$$

Now, from equation (6), the time of fall from height h is

$$0 = h - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}} \quad \text{---------(7)}$$

Hence the deflection of a particle towards the east when it is dropped from rest is given by, $\frac{1}{3}\omega \cos\theta \left(\frac{8h^3}{g}\right)^{1/2}$.