Completness of Eigen function :- Prove $\sum_n |n\rangle\langle n| = 1$

Consider, the eigen ket $|n\rangle$ of real linear operator then, they will satisfy following conditions :-

(i) They from a complete set, i.e. any $|p\rangle$ can be expressed as a linear combination of eigen ket.

$$|p\rangle = \sum_n a_n |n\rangle \quad \text{——— (1)}$$

(ii) They are orthogonal and normalised. i.e. they satisfy the condition of orthonormality

i.e. $\langle n|m\rangle = \delta_{n,m}$; $\delta_{n,m} = 0$ for $\Rightarrow m \neq n$

$\delta_{n,m} = 1$ for $\Rightarrow m = n$

(iii) they are linearly independent, i.e no eigen ket can be reepresented in terms of the other

thus, if $\sum_n a_n |n\rangle = 0$

then, each term must be zero

and $\therefore$ $a_n = 0$

(IV) $\sum_n |n\rangle\langle n| = 1$ —— (3)

where, 1 is the unit linear operator

i.e $\rightarrow$ $1 . |p\rangle = |p\rangle$

then, for ~~orthogonal~~ orthonormal set of eigen function

$\rightarrow |p\rangle = \sum_n a_n |n\rangle$ by eq^n (1)

multiply both side by $\langle m|$

$\rightarrow \langle m|p\rangle = \sum_n a_n \langle m|n\rangle$

$= \sum_n a_n \delta_{m,n}$ by eq^n (2) $(m=n)$

$= \sum_n a_n$ —— (★)

$= a_n = a_m$

than $\langle m|p\rangle = a_m$
or $\langle n|p\rangle = a_n$ $\Big\}$ —— (4)

$|p\rangle = \sum_n a_n |n\rangle$

$|p\rangle = \sum_n |n\rangle a_n$

i.e. $|p\rangle = \sum_n |n\rangle \langle n|p\rangle$ —— by eq^n (★)

or $\sum_n |n\rangle\langle n| = 1$ $\Big\{$ by unit linear operator eq^n (3)

Hence proved

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