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Paper – III Advanced Quantum Mechanics – II

Unit- II: Born approximation

## Criterion (Condition) for Validity of Born Approximation →

The Born Approximation will be hold good only when the wave function $\psi$ is not much different from the incident wave function $e^{ikr}$. It implies that it will hold good when the scattered wave $\psi_s(r)$ is small compared to $e^{ikr}$ in the region in which $V(r)$ is large.
In most of the cases, both $V(r)$ & $\psi_s(r)$ have been found to largest near the origin so that a rough criterion for the validity of Born approximation will be as follows:

$$|\psi_s(r)|^2 <<< 1 \quad \text{for small values of } r \tag{1}$$

In cases in which $\psi_s(r)$ is small for small values of 'r' but large for intermediate values of 'r' such that $V(r)$ is still appreciable, this criterion has to be applied carefully. further, it is possible that Born approximation may hold good even when the criterion does not get satisfied. Having $\psi_s(r)$ small everywhere gives a sufficient condition for the validity of the approximation but it is not necessary condition.

We know that a change in potential acts like a change in refractive index in optics. On this basis it is possible to derive another criterion for the validity of Born approximation. Therefore, the change of potential will make a change in the phase of the wave function. The total wave function will not be much different from the initially wave function provided the phase of the incident wave does not change when it is passing through the region in which it gets influenced by the perturbing potential. Near the centre of force the magnitude of wave vector is

$$k = \frac{\sqrt{2m(E-V)}}{\hbar}$$

whereas at great distance it is

$$k = \frac{\sqrt{2mE}}{\hbar}$$

Then, the change of phase due to the

potential is put as follows:-

$$\Delta\phi = \int_0^\infty \sqrt{\frac{2m}{\hbar^2}} \left[\sqrt{(E-V)} - \sqrt{E}\right] dr$$
——(2)

When this difference is small in comparison with unity, it implies that the wave function will not be much different from that in the absence of the potential. It follows that the I order Born approximation will hold good when

$$|\Delta\phi| = \left| \sqrt{\left(\frac{2m}{\hbar^2}\right)} \int_0^\infty \left[\sqrt{(E-V)} - \sqrt{E}\right] dr \right| << 1$$
——(3)

When $V << E$, it is possible to simplify the criterion by expressing eqn (3) as a function of the ratio $V/E$ & then expanding the square root. Hence the criterion may take the following form:

$$\sqrt{\frac{2mE}{\hbar^2}} \left| \int_0^\infty \left\{\left(1-\frac{V}{E}\right)^{1/2} - 1\right\} dr \right| << 1$$
——(4)

$$\sqrt{\frac{2mE}{\hbar^2}}\left|\int_0^\infty\left\{\left(1-\frac{V}{2E}+\cdots\right)-1\right\}dr\right| << 1$$

$$\sqrt{\frac{2mE}{\hbar^2}}\left|\int_0^\infty \frac{V}{2E}\,dr\right| << 1 \quad \text{since } \frac{V}{E} << 1$$

$$\sqrt{\frac{m}{2\hbar^2 E}}\left|\int_0^\infty V\,dr\right| << 1$$

$$\sqrt{\frac{m}{2\hbar^2 E}}\;\bar{V}\,\bar{r} << 1 \quad \text{—— (5)}$$

where $\bar{V}$ represents the avg. potential & $\bar{r}$ the mean range.
If the high energy particles are scattered by the spherical potential well of radius 'a' & depth $V_0$, it means that $V_0 << E$. This condition yields the following result.

$$\sqrt{\frac{m}{2\hbar^2 E}}\;V_0\,a << 1 \quad \text{—— (6a)}$$

$$\boxed{E >> \frac{m}{2}\left(\frac{V_0 a}{\hbar}\right)^2}$$