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Lecture for M.Sc. Physics IV Semester students

Paper – III Advanced Quantum Mechanics – II

Unit- II: Born approximation

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Scattering of Electrons By atoms
or
Rutherford's scattering formula from Born approximation
or
Application of Born approximation in screening Coulomb potential -

When $V(r)$ is the screened Coulomb potential, it is possible to find the differential scattering cross-section which is as follows

$$V(r') = \frac{Z Ze^2}{4\pi\varepsilon_0 r} e^{-r/r_0} \quad (1)$$

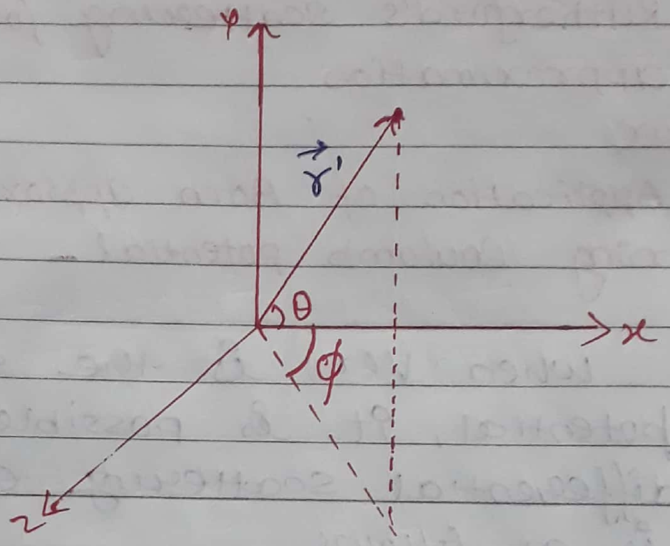
where Z & Ze represents the charge of target & scattered particle & r_0 is screening radius (shielding distance).

From Born approximation, the expression for scattering amplitude may be obtained which is as follows:-

$$f(\theta, \phi) = -\frac{m}{2\pi\hbar^2} \int V(r') e^{i\vec{k}.\vec{r}} \, d\vec{r}' \quad (2)$$

here

$$\vec{K} \cdot \vec{r}' = K r' \cos\theta$$



and

$$d\vec{r}' = -2\pi r'^2 \, dr' \, d(\cos\theta) \, d\theta$$

(for spherical symmetric potential)

Thus eqn (2) reduces to

$$f(\theta, \phi) = \int_{r'=0}^{\infty} V(r') \int_{\cos\theta=-1}^{+1} \exp^{ikr'\cos\theta} \, r'^2 \, d(\cos\theta) \, d\phi \, dr' \qquad (3)$$

But,

$$\int_{-1}^{+1} e^{ikr'\cos\theta} \, d(\cos\theta) = \left[\frac{e^{ikr'\cos\theta}}{ikr'} \right]_{\cos\theta=-1}^{+1}$$

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$$= \frac{e^{ikr'} - e^{-ikr'}}{ikr'}$$

$$= \frac{2\sin(kr')}{kr'}$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

so, eqn (3) becomes

$$f(\theta,\phi) = \int_{r'=0}^{\infty} V(r')\, r'^2 \cdot \frac{2\sin(kr')}{kr'}\, dr' \left(\frac{-m}{\hbar^2}\right)$$

$$f(\theta,\phi) = \frac{-2m}{\hbar^2} \cdot \frac{1}{k}\left[\int_0^{\infty} V(r')\, r' \sin(kr')\, dr'\right] \quad \text{—— (4)}$$

using eqn (1), we get

$$f(\theta,\phi) = \frac{-2m}{\hbar^2} \cdot \frac{1}{k}\left[\int_0^{\infty} \frac{Z Ze^2}{r'} \cdot \frac{e^{-r'/r_0}}{4\pi\varepsilon_0} \cdot \sin(kr') \cdot r'\, dr'\right]$$

$$f(\theta,\phi) = \frac{-2m Z Ze^2}{\hbar^2 k\, 4\pi\varepsilon_0}\left[\int_{r'=0}^{\infty} e^{-r/r_0} \sin(kr')\, dr'\right] \quad \text{—— (5)}$$

Solving integral we get

$$\int_{r'=0}^{\infty} e^{-r'/r_0} [\sin(kr')] dr' = \frac{k r_0^2}{k^2 r_0^2 + 1} \quad \text{——(6)}$$

Using eqn (6) in (5) we get

$$f(\theta, \phi) = \frac{-2mZze^2}{4\pi\varepsilon_0 k\hbar^2} \cdot \frac{k r_0^2}{k^2 r_0^2 + 1}$$

$$= \frac{-2mZze^2}{4\pi\varepsilon_0 \hbar^2} \left[\frac{1}{k^2 + \frac{1}{r_0^2}} \right]$$

But $k = 2K \sin\theta/2$

$\therefore$

$$f(\theta, \phi) = \frac{-2mZze^2}{4\pi\varepsilon_0 \hbar^2} \cdot \frac{1}{4K^2 \sin^2\frac{\theta}{2} + \frac{1}{r_0^2}} \quad \text{——(7)}$$

When $r_0 \to \infty$, $V(r)$ becomes the ordinary Coulomb potential

$$V(r) = \frac{1}{4\pi\varepsilon_0} \frac{Zze^2}{r}$$

Hence eqn (7) becomes

$$f(\theta,\phi) = \frac{-2mzZe^2}{4\pi\epsilon_0\hbar^2} \frac{1}{4k^2\sin^2\frac{\theta}{2}} \quad \text{---} (8)$$

Now, scattering cross-section is obtained by squaring eqn (8)

$$\sigma(\theta,\phi) = |f(\theta,\phi)|^2 =$$

$$= \left(\frac{2mzZe^2}{4\pi\epsilon_0}\right)^2 \frac{1}{\left(4\hbar^2 k^2 \sin^2\frac{\theta}{2}\right)^2}$$

But $\cancel{\hbar/k} \approx p = \hbar k$

$$\therefore \sigma(\theta,\phi) = \frac{4m^2}{4^2p^4}\left(\frac{zZe^2}{4\pi\epsilon_0}\right)^2 \operatorname{cosec}^4\frac{\theta}{2}$$

But $p^2 = 2mE$

$$\therefore \sigma(\theta,\phi) = \frac{4m^2}{16\times 4m^2E^2}\left(\frac{zZe^2}{4\pi\epsilon_0}\right) \operatorname{cosec}^4\frac{\theta}{2}$$

$$\sigma(\theta,\phi) = \frac{1}{16E^2}\left(\frac{zZe^2}{4\pi\epsilon_0}\right) \operatorname{cosec}^4\frac{\theta}{2}$$

The obtained ~~the~~ expression for scattering cross section is found to be same as Rutherford Crosssection.