Dr. Priya Dubey (Guest Lecturer)

School of Studies in Physics, Vikram University, Ujjain

Lecture for M.Sc. Physics II Semester students

Paper – III Quantum Mechanics – II

Unit- III: Identical Particle

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IDENTICAL PARTICLES :-

Identical particles are those in a system for which the system remains unalter by interchanging the particles. As each particle is described quantum Mechanically by a wave packet. These particles can be distinguished from one another only if wave packet do not overlap. Similarly, If particles have spin which are alligned in different position in the process at any Interaction. They can be identified from one another as a result of such interaction.

Thus, the word Identicle in quantum Mechanics is to describe, the particles that can be substituted for each other under the most general possible circumstances with no change in physical situation of the System.

For Example: The components of spin along some particular axis is supposed not to change during elastic collision. It means the particles can be distinguished, if they have different spin components. By Identical particles, we mean the particles like electron which can't be distinguished by means of any Inherent property. Since, otherwise they would not be identical in all respects.

$(1, 2, 3 --- n, t) \rightarrow$ co-ordinates of particles

Thus there are two general catagories of particles :-

a) classical particles which are identical but indistinguishable.

b) Quantum particles which are identical but indistinguishable.

Symmetric & Anti-Symmetric Wave Functions :-

We can write the Schrödinger wave equation for n-particles as follows :-

$$\boxed{H(1,2,3 -- n)\,\Psi(1,2,3 --- n, t) = i\hbar \frac{\partial \Psi(1,2,3 -- n, t)}{\partial t} \qquad (1)}$$

Where the number 1 to n represents the co-ordinates of each particles, in this case by co-ordinates we mean x, y, z-axis & the spins. The Hamiltonian is symmetrical as by changing the particles ~~along~~ among themselves. The Hamiltonian remains the same as the particles are identical. for a particular interest there are also two types of solution of wave function Ψ as given by eq.(1).

Symmetric wave function (ψ_S):
A wave function is said to be symmetric, if the interchange of any pair of particle have the wave functions unchanged.

Anti-Symmetric wave function (ψ_A):
A wave function is said to be anti-symmetric, if an exchange of co-ordinate of a pair of particles, change the sign of original wave function.

It is Noted that, the symmetric character of wave function doesn't changes with time 't'. Thus, if ψ_S is Symmetric at time t, then $H\psi_S$ is also symmetric at time t. & By using eq. (1) $\frac{d\psi_S}{dt}$ is also symmetric at time t.

As ψ_S & $\frac{d\psi_S}{dt}$ are symmetric at time t, ψ_S at infinitesimaly later time $t + dt$ given by $\psi_S + \frac{d\psi_S}{dt} dt$ is also symmetric.

In this way, a step-by-step Integration of the wave function can be carried out for large time intervals & ψ_S is found to be symmetric always.

Similarly, if ψ_A is anti-symmetric wave function at any time t, $H\psi_A$ & Hence $\frac{\delta}{\delta t}\psi_A$ are also anti symmetric.

Then the Integration of wave function ψ_A shows that ψ_A is always anti-symmetric.

If P is an exchange Operator. Then, we must have;

$$\boxed{\psi_S = \psi(1,2) + \psi(2,1)}$$

$$\Rightarrow P\psi_S = P\psi(1,2) + P\psi(2,1)$$

$$= \psi(2,1) + \psi(1,2)$$

$$\Rightarrow \boxed{P\psi_S = \psi_S}$$

& if $\boxed{\psi_A = \psi(1,2) - \psi(2,1)}$

$$\Rightarrow P\psi_A = P\psi(1,2) - P\psi(2,1)$$

$$= \psi(2,1) - \psi(1,2)$$

$$= -\left[\psi(1,2) - \psi(2,1)\right]$$

$$\Rightarrow \boxed{P\psi_A = -\psi_A}$$