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Lecture for M.Sc. Physics II Semester students

Paper – III Quantum Mechanics – II

Unit- III: Identical Particle

Spin Angular Momentum :-

∵ The orbital angular momentum $\boxed{L = r \times p}$ is expressed in terms of the co-ordinates & Momentum. The quantity L^2 is not in general a Constant of Motion. thus the quantum number l need not be well definite. It is possible to have an angular Momentum S that satisfy the fundamental communication relation $\boxed{J \times J = i\hbar J}$ & such that S^2 commutes with all the dynamical Variable. A necessary condition is that S cannot be expressed in terms of r & p. Then S^2 is strictly of constant of Motion & can be replaced by number $S(S+1)\hbar^2$ where S is an integer or half and odd Integer.

It is found experimentally that electron, protons, Neutrons, Neutrinos & New Meson's can have $S = \frac{1}{2}$ & ~~Protons~~, photons Phonons have $S = 1$ & 2 Pi-mesons have $S = 0$.

Aggregates of particles that are sufficiently lightly bound can be regarded as particles & can be characterized by definite magnitudes of their total internal angular Momentum as their internal motion & their relative spin Orientations of their component particles are not sufficiently affected by the interaction between aggregates.

The addition of angular ~~xx~~ Momenta shows the possible magnitudes of the total angular Momentum of any aggregate of fundamental particle called the spin of the aggregate.

If it consist of n particles, each of which has $s = \frac{1}{2}$ & any no. of particle with $s = 0$ & if the internal orbital angular momentum of these particle is ignored, the total S can be any integer from 0 to $\frac{1}{2n}$, If n is even or vary from $\frac{1}{2}$ to $\frac{1}{2n}$. If n is odd the total orbital angular Momentum is an integer or zero.

The electron possess an internal angular momentum called the spin which can assume only the values $+\hbar/2, -\hbar/2$ in same arbitrary chosen direction. Infact, all elementary particles have a spin degree of freedom. fermions possess 1/2 integral spin ~~by~~ while Bosons have Integral spin (Including zero).

Let the spin Operator be $\hat{S} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$
If n is a unit vector pointing in some arbitrary direction then the Klebsch - Gorden experiment, n operator has only two values i.e.;

$$\boxed{\hat{S}, n|n, \pm\rangle = \pm \frac{\hbar}{2}|n, \pm\rangle} \quad \text{---} (1)$$

we can choose n point in the z-direction. then $\hat{S}_n = \hat{S}_z$. Then, the eigen Value equation takes the form;

$$\left.\boxed{\begin{aligned} \hat{S}_z|+\rangle &= \frac{\hbar}{2}|+\rangle \\ \hat{S}_z|-\rangle &= -\frac{\hbar}{2}|-\rangle \end{aligned}}\right\} (2)$$

Where $|+\rangle$ corresponds to the spin operator pointing in the positive z direction, i.e. a spin up state & $|-\rangle$ corresponds to the spin operator pointing in the negative z-axis direction. i.e. a spin-down state.
Since spin is a physical Observable, S_z is a Hermitian & state belonging to the distinct eigen Values are orthogonal i.e.

$$\boxed{\langle +|-\rangle = 0} \quad \text{---} (3)$$

further Normalizing them to unity get

$$\boxed{\langle + | + \rangle = \langle - | - \rangle = 1} \quad \text{——— (4)}$$

The spin aperator satisfy the commutation Relation;

$$\left.\boxed{\begin{aligned} [\hat{S}_i , \hat{S}_j] &= i\hbar \,\varepsilon_{ijk} \,\hat{S}_k \\ [\hat{S}_z , \hat{S}_\pm] &= \pm \hbar \hat{S}_\pm \\ [\hat{S}_+ , \hat{S}_-] &= 2\hbar S_z \end{aligned}}\right\} \quad (5)$$

where $S_\pm = \hat{S}_x \pm i\hat{S}_y$

$$\hat{S}_z = \frac{1}{2}\left(\hat{S}_+ + \hat{S}_-\right)$$

$$\hat{S}_y = -i \cdot \frac{1}{2}\left(\hat{S}_+ + \hat{S}_-\right)$$

for spin $S = \frac{1}{2}$, S has Eigen Value $\frac{3}{4}\hbar^2$

$$\boxed{\begin{aligned} \hat{S}^2 | + \rangle &= \frac{3}{4}\hbar^2 | + \rangle \\ \hat{S}^2 | - \rangle &= -\frac{3}{4}\hbar^2 | - \rangle \end{aligned}}$$