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Paper – II Unit – I

Symmetry properties of space and time with Conservation law

When we considered the motion of a free particle or a closed system in an intertial frame, the space is assumed to be homogenous and isotropic and the time to be also homogeneous.

The space ~~to be~~ is said to be homogeneous, if the physical properties of a closed system are not affected by an arbitrary displacement of the origin of the frame of reference.

The space is said to be isotropic, if the physical properties of closed system are not changed for arbitrary rotation about the origin of the frame of reference.

The time is said to be homogeneous, if the physical properties of a closed system are not affected by an arbitrary displacement of the origin of time.

The homogeneity and isotropy of space and homogeneity of time imply the invariance of the physical properties of a closed system under certain operations

Known as symmetry operations. These operations leave the configuration and states of motion unchanged. The homogeneity of space correspond to an arbitrary tran--slation (symmetry operation), isotropy of space to an arbitrary rotation and homogeneity of time to an arbitrary shifting of the time or time-translation.

We can describe a closed system by its Lagrangian. This Lagrangian must be invariant under the operations of translation and rotation in space and time shifting. These symmetry operations on the Lagrangian have very important consequences. Thus every symmetry in the Lagrangian corresponds to a conservation law. Homogeneity of space resuts in the conservation law of linear momentum, isotropy of space in the conservation law of angular momentum and homogeneity of time in the conservation law of energy. These conservation laws have been obtained in the following discussion.

(1) Homogeneity of space and Conservation of Linear Momentum :-

The homogeneity of space implies that the Lagrangian of a closed system is not changed by an arbitrary translation of all the particles of the system. In Carterian Coordinates a small arbitrary translation of the coordinates of the i^{th} particle can be written as

$$r_i \longrightarrow r_i + \delta r_i \quad \text{or} \quad r_i \rightarrow r_i + \varepsilon$$

where $\delta r_i = \varepsilon$ is constant small translation for each particle.

Now Corresponding to this change in coordinate, the change δL in L is

$$\delta L = \sum_i \frac{\partial L}{\partial r_i} \cdot \varepsilon = -\varepsilon \cdot \sum_i \frac{\partial L}{\partial r_i} \qquad \text{———} (1)$$

However for any arbitrary translation ε, $\delta L = 0$. This means

$$\sum_i \frac{\partial L}{\partial r_i} = 0 \qquad \text{———} (2)$$

We know that from Langrange eqn is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}_i}\right) - \frac{\partial L}{\partial r_i} = 0$$

Hence for all particles of the system

$$\sum_i \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}_i}\right) - \sum_i \frac{\partial L}{\partial r_i} = 0$$

using eqn (2)

$$\sum_i \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}_i}\right) = 0 \quad \text{or} \quad \frac{d}{dt}\sum_i \left(\frac{\partial L}{\partial \dot{r}_i}\right) = 0 \quad \text{—— (3)}$$

But

$$L = T - V = \sum_i \frac{1}{2} m_i \dot{r}_i^2 - V(r_1 \dots r_N)$$

$$\therefore \quad \frac{\partial L}{\partial \dot{r}_i} = m_i \dot{r}_i = P_i \text{ linear momentum of } i^{th} \text{ particle} \quad \text{—— (4)}$$

Hence from eqn (3)

$$\frac{d}{dt}\left(\sum_i P_i\right) = 0 \quad \sum_i P_i = \text{Constant} \quad \text{—— (5)}$$

Where $\sum_i P_i = P$ is the total momentum of the system.

Thus, the total linear momentum of a closed system is conserved due to be homogeneity of the space.

To be cont.