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Uniformly Rotating Frames: →

We know that the earth itself rotates about its axis in 24 hours. Therefore, any frame fixed with the earth will also rotate with it and so it will be a noninertial frame.

Suppose that a frame $s'(X_r, Y_r, Z_r)$ is rotating with an angular velocity ω relative to an inertial frame S (X_i, Y_i, Z_i). For simplicity, we assume that both of the frames have common origin O and common Z-axis. In case of the earth, the common origin O may be considered as the centre of the earth, Z-axes as coinciding with its rotational axis and the frame s' as rotating with earth relative to the non rotating frame S.

The position vector of a particle P in both frames will be the same, i.e. $R_i = R_r = R$, because the origins are

Coincident. Now, if the particle P is stationary in the frame S, the observer in the Rotating frame S' will see that the particle is moving oppositely with linear velocity $-\omega \times R$. Thus, if the velocity of the particle in the frame S is $\left(\frac{dR}{dt}\right)_i$, then the velocity $\left(\frac{dR}{dt}\right)_r$ in the rotating frame will be given by

$$\left(\frac{dR}{dt}\right)_r = \left(\frac{dR}{dt}\right)_i - \omega \times R$$

or

$$\left(\frac{dR}{dt}\right)_i = \left(\frac{dR}{dt}\right)_r + \omega \times R \qquad \text{—— (1)}$$

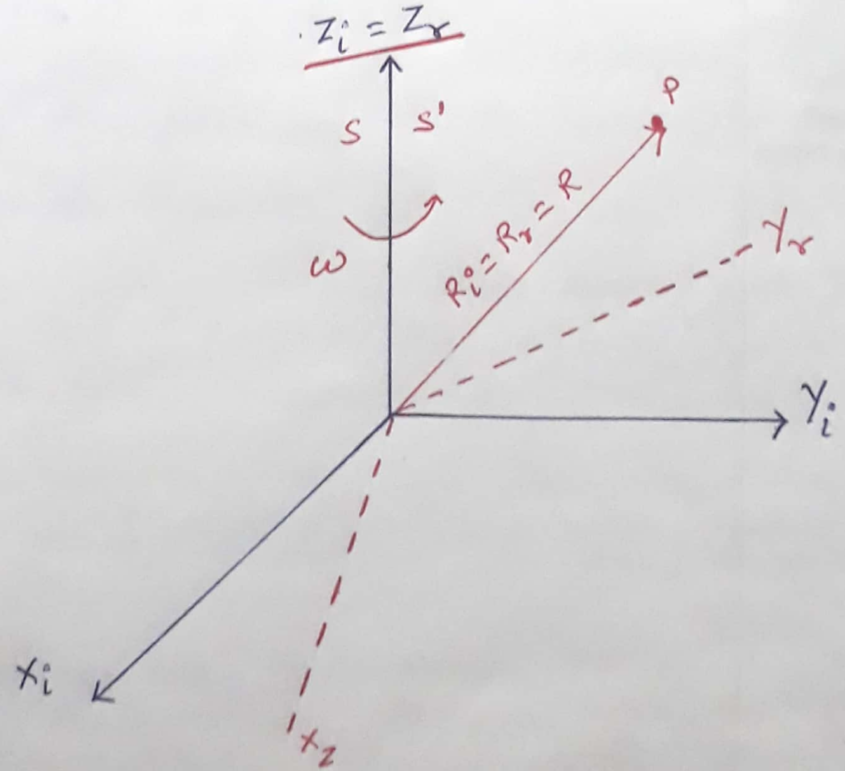

Figure: Uniformaly Rotating Frame

In fact this equation holds for all vectors and relates the time derivatives of a vectors in the frame S and S'. Therefore, relation (1) may be written in the form of operator eqⁿ

$$\left(\frac{d}{dt}\right)_i = \left(\frac{d}{dt}\right)_r + \omega \times R \quad \text{—— (2)}$$

writing $\frac{dR}{dt} = v$ for the velocity of the particle, we have

$$v_i = v_r + \omega \times R \quad \text{—— (3)}$$

Now, if we operate eqⁿ (2) on velocity vector v_i, we have

$$\left(\frac{dv_i}{dt}\right)_i = \left(\frac{dv_i}{dt}\right)_r + \omega \times v_i$$

Substituting the value of v_i in the right hand side of this relation from eqⁿ (3), we obtain

$$\left(\frac{dv_i}{dt}\right)_i = \left[\frac{d}{dt}(v_r + \omega \times R)\right]_r + \omega \times (v_r + \omega \times R)$$

$$= \left(\frac{dv_r}{dt}\right)_r + \frac{d\omega}{dt} \times R + \omega \times \left(\frac{dR}{dt}\right)_r + \omega \times v_r + \omega \times (\omega \times R)$$

if we write the acceleration

$\frac{dv}{dt} = a$ and $\left(\frac{dR}{dt}\right)_r = v_r$, then

$$a_i = a_r + 2\omega \times v_r + \omega \times (\omega \times R) + \frac{d\omega}{dt} \times R$$

For earth, ω is constant so $\frac{d\omega}{dt} = 0$, then

$$a_i = a_r + 2\omega \times v_r + \omega \times (\omega \times R) \quad \text{——— (4)}$$

If m is the mass of the particle then the force in the rotating frame is

$$ma_r = ma_i - 2m\omega \times v_r - m\omega(\omega \times R)$$

But $ma_r = F_i + F_0$, where fictitious force F_0 is given by

$$F_0 = -2m\omega \times v_r - m\omega \times (\omega \times R) \quad \text{——— (5)}$$

Where $-2m\omega \times v_r$ is the Coriolis force and $-m\omega \times (\omega \times R)$, the Centrifugal force

The Centrifugal force is the only fictitious force, acting on a particle which is rest ($v_r = 0$) in the rotating frame. The Centrifugal force can be written as

$$-m\omega \times (\omega \times R) = m\omega^2 r \quad \text{——— (6)}$$

The Coriolis and Centrifugal forces are often called fictitious forces because the bodies don't actually feel them. They are due only to the observer's motion.