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Classical mechanics

Central force; Definition and characteristics →

The properties of a particle of mass m moving in a particular type of force field, a central force field. Central forces are very important in physics and engineering. For example, the gravitational force of attraction between two point masses in a central force. The Coulomb force of attraction and repulsion between charged particles is a central force. Because of their importance they deserve special consideration. We begin a precise definition of central force, or central force field.

A force acting on a particles of mass m has the properties that:

* The force is always directed from m toward or away, from a fixed point O

★ The magnitude of the force only depends on the distance r from O.

Forces having these properties are called central forces. The particle is said to move in a central force field. The point O is referred to as the ~~central~~ centre of force.

If $f(r) < 0$ the force is said to be attractive towards O. If $f(r) > 0$ the force is said to be repulsive from O.

Properties of a particle moving under the influence of a Central force → If a particle moves in a centre force field then the following properties hold:

1. The path of the ~~curve~~ particle must be a plane curve, i.e. it must lie in a plane.
2. The angular momentum of the particle is conserved, i.e. it is constant in time.
3. The particle moves in such a way that the position vector (from the point O) sweeps out equal

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areas in equal times. In other words, the time rate of change in area is constant. This is referred to as the Law of Areas.

Reduction to the Equivalent one-body problem in terms of Lagrangian:-

Consider a monogenic system of two mass points, m_1 and m_2 (fig. 1), where the only forces are those due to an interaction potential U. Firstly we assume that U is any function of the vector between the two particles, $r_2 - r_1$. or of their relative velocity $\dot{r}_2 - \dot{r}_1$, or of any higher derivatives of $r_2 - r_1$. such a system has six degrees of freedom and hence six independent generalized coordinates. If $\vec{R}$ be the radius vector to the centre of mass, the Lagrangian will have the form;

$$L = T(\dot{\vec{R}}, \dot{\vec{r}}) - U(\vec{r}, \dot{\vec{r}} \ldots) \quad \text{——} (1)$$

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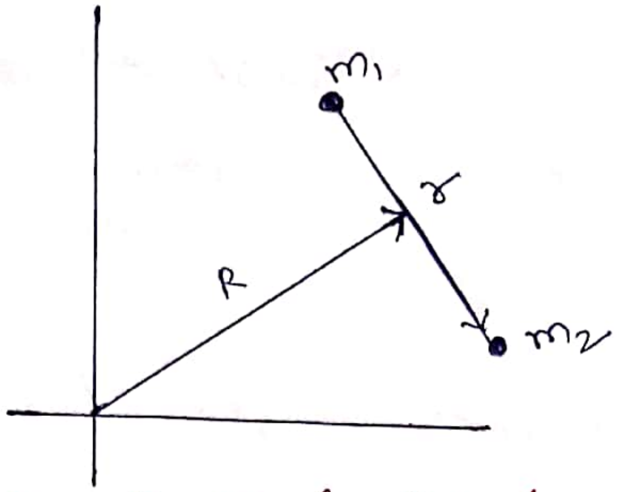

Fig. 1 Coordinates for the two-body problem.

Now the kinetic energy T can be written as the sum of the kinetic energy of the motion of the centre of mass, and the kinetic energy of motion about the centre of mass T':

$$T = \frac{1}{2}(m_1 + m_2)\dot{\vec{R}}^2 + T'$$

with

$$T' = \frac{1}{2} m_1 \dot{\vec{r}}_1'^2 + \frac{1}{2} m_2 \dot{\vec{r}}_2'^2 .$$

Here $\vec{r}_1'$ and $\vec{r}_2'$ are the radii vectors of the two particles relative to the centre of mass and are related to $\vec{r}$ by

$$\vec{r}_1' = -\frac{m_2}{m_1 + m_2}\vec{r}$$

$$\vec{r}_2' = \frac{m_1}{m_1 + m_2}\vec{r} \qquad \text{———} (2)$$

Substituting the value of $\vec{r}_1'$ & $\vec{r}_2'$, T' we get

$$T' = \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}\dot{\vec{r}}^2$$

and the total Lagrangian eqn (1) is

$$L = \frac{m_1+m_2}{2}\dot{\vec{R}}^2 + \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}\dot{\vec{r}}^2 - U(\vec{r}, \dot{\vec{r}}, --) \quad \text{———(3)}$$

It is seen that the three coordinates $\vec{R}$ are cyclic, so that the centre of mass is either at rest or moving uniformly.

The rest of the Langrangian is exactly expected to be a fixed centre of force with a single particle at a distance r from it, having a mass

$$\mu = \frac{m_1 m_2}{m_1+m_2} \quad \text{———(4)}$$

where μ is known as the reduced mass. Eqn (4) is written in the form

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} \quad \text{———(5)}$$

Thus the central force motion of two bodies about their centre of mass can always be reduced to an equivalent one-body problem.