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ARTIFICIAL SATELLITES →

A body which revolves constantly round a comparatively much larger body is said to be Satellite, like the Motion of a planet and its orbit around the sun. We know that the earth and other planets revolve round the sun in their specified orbits.

The moon revolves round the earth and the Planets Jupiter and Saturn have six and nine moons respectively revolving around them. All these are the examples of natural satellites.

Scientists have also been able to placed man-made statellites, revolving round the earth or sun. They are called artificial satellites. The theory disussed above for the orbits and planetary motion is valid for the discussion of satellites.

An artificial satellite of the earth is a body, place in a stable orbit around the earth with the help of multistage rocket. In order to ~~launch~~ launch a satellite in a stable orbit, first it is necessary to take the satellite to the altitude h, where at the point P by some mechanism, it is given the necessary orbiting velocity, called the insertion velocity v_i (in fig.).

The total Energy of the satellite at P relative to the earth is given by

$$E = \frac{1}{2} m v_i^2 - \frac{GMm}{R+h} \qquad (1)$$

where m is the mass of the satellite and M that of the earth having radius R.

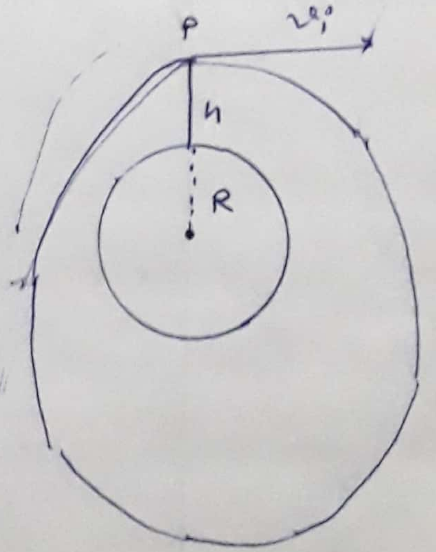

Fig. 1 Elliptical path of a body projected horizontally from a height h above the Earth surface

The orbit will be an ellipse, a parabola, depending on whether E is negative, zero or positive. In each case the centre of the earth is at one focus of the path. Therefore, the satellite will moving in an elliptical orbit if,

$$v_i^2 < \frac{2GM}{R+h} \quad \text{——— (2)}$$

The total energy E determines the size or semi-major axis of the orbit. However the shape or eccentricity e of the orbit is determined by both total Energy E and angular momentum J by the relation:

$$e = \sqrt{1+\frac{2EJ^2}{mk^2}} \quad \text{——— (3)}$$

with $k = GMm$. For elliptical orbits, layer the angular momentum, the less elongated is the orbit (Fig. 2)

For Circular ~~path~~ orbit, the ~~insertation~~

For Circular orbit, the insertion velocity is found by equating the centripetal force mv^2/r to the gravitational force GMm/r^2 i.e.

$$\frac{mv_i^2}{r} = \frac{GMm}{r^2} \quad \text{or} \quad v_i^2 = \frac{GM}{r} = \frac{GM}{R+h} \qquad (4)$$

where $r = R+h$

Remeber that for circular orbit $e=0$, so that

$$1 + \frac{2EJ^2}{mk^2} = 0 \qquad \left\{ \begin{array}{l} J = mva \text{ ; Angular momentum} \\ \& \ E = -\frac{k}{2a} \text{ ; from eq}^n \ (10,c) \end{array} \right.$$

$$1 + 2\times\left(-\frac{k}{2a}\right)\times\frac{m^2 v_i^2 a^2}{mk^2} \quad \text{or} \quad v_i^2 = \frac{GM}{R+h}$$

where $r = a = R+h$, $k = GMm$ and $J = mv(R+h)$

For the circular orbit at the height h above the Earth's surface, the period of revolution is

$$T = \frac{2\pi r}{v_i} = \frac{2\pi(R+h)}{v_i} = \frac{2\pi(R+h)^{3/2}}{\sqrt{GM}} \qquad (5)$$

Geosynchronous orbit: if the period of revolution is equal to the period of earth's diuronal (one day) rotation, the orbit is said to be geosychronous orbit.

For a geosynchronous orbit, the eccentricity can have any value and the orbit can have any orientation with respect to the equator of the earth.

Geostationary orbit: If the height of an artificial satellite at equator above the earth's surface is such that its period of revolution is exactly equal to the period of rotation of the earth, then the satellite would appear stationary over a point on earth's equator. Such a satellite is called geostationary satellite and its orbit is called geostationary orbit. Therefore for a geostationary satellite we must have the orbit

(1) to be geosynchronous

(2) to be circular

(3) to stay over the geographical equator of the each

Uses of Artificial satellites :–

Artificial satellites are used in the following:

(1) Distance transmission of radio and TV signals.

(2) To study upper regions of the atmosphere.

(3) High altitude satellites for astronomical observations (as the effects of atmosphere are not present)

(4) weather forecasting.

(5) Eearth measurments (gravitation and magnetic field)

Q.1 An artificial satellite is revolving round the earth at a distance of 620 km. Calculate the orbital velocity and the period of revolution. Radius of earth = 6380 km & $g = 9.8 m/s^2$

Sol. Radius of earth's Satellite orbit $r = R+h = 7\times10^6$ m/sec

period of revolution $T = \frac{2\pi r}{R}\sqrt{\frac{r}{g}} = 5775$ sec.

& orbital Velocity $v = R\sqrt{\frac{g}{r}} = 7.55\times10^3$ m/sec.

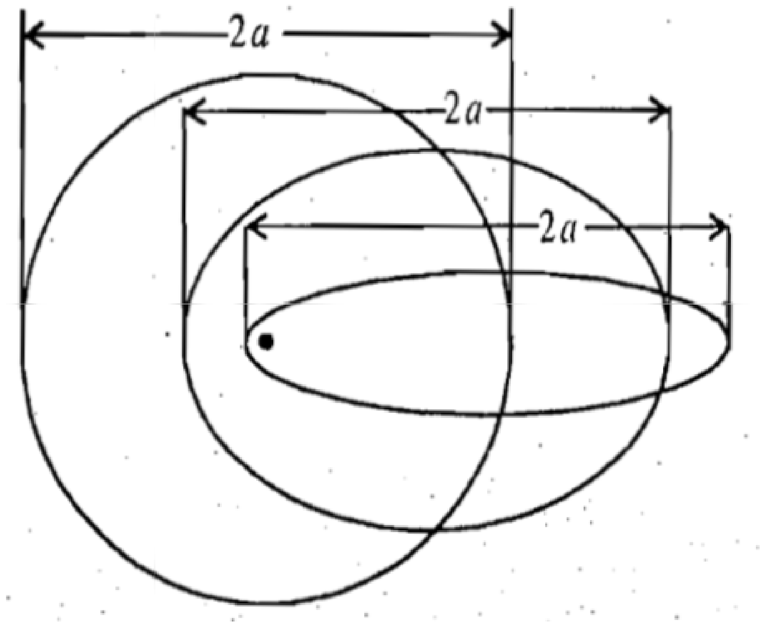

Figure 2 Elliptical orbits fir different values of the angular momentum J with same energy E, various orbits have the same focus and semi-major axis but differing in eccentricity.