1.6 BINDING ENERGY

It might at first be supposed that the mass of an atom should be the sum of the masses of its constituent particles, *i.e.*, electrons, protons and neutrons. But the mass of any permanently stable atom is found to be less than the sum of the masses of its constituents. Aston called the decrease in the mass or loss of mass upon coalescence of the elementary constituents the *mass defect*, now generally called *mass deficiency*. This can be explained as under :

The nucleons that hold the stable nucleus are held together by strong attractive forces, and therefore work must be done in separating them from each other until they are very large distance apart. In other words *energy must be supplied to the nucleus to separate it into the individual constituents*, and the total energy of the constituents when at a very large distance apart is greater than when they form the nucleus. For convenience the masses of atoms rather than the masses of nuclei are used in all calculations. This causes no difficulty, except that the binding energy of the atomic electrons should also be considered. For simplicity, we usually omit it. The principle of equivalence of mass and energy confirms the idea that the amount of energy required to break up the atom into its constituents is equivalent to the *mass deficiency*. It can also be defined as the amount of mass which would be converted into energy if a particular atom is to be assembled from its constituents. The energy equivalent of mass difference or mass *deficiency* is a measure of binding energy of the atom. *It is analogous to the heat of formation of a chemical compound.*

The constituents of the atom are consequently Z protons and Z electrons, which are equivalent in mass to Z hydrogen atoms, and $(A - Z)$ neutrons and their total mass is $ZM_H + (A - Z)\, M_n$, where M_H and M_n represent the masses of the hydrogen atom and of the neutron respectively. If ${}^A_Z M$ is isotropic atomic mass of an atom, then we can write, *mass difference or mass deficiency*

$$\Delta M = ZM_H + (A - Z)M_n - {}^A_Z M. \qquad ...(67)$$

As the binding energy is a measure of the energy equivalent of mass defect, hence we can write *binding energy*

$$E_B = \Delta Mc^2 = [ZM_H + (A - Z)M_n - {}^A_Z M]c^2. \qquad ...(68)$$

Since the state of infinite separation of nucleons is taken to be zero level of energy hence the total energy of the nucleus is just $-E_B$, or simply written as $-B$.

The binding energy may either be expressed in u or converted into MeV as *one atomic mass unit.*

$$u = \frac{mass\,of\ {}^{12}C\,atom}{12} = \frac{1kg}{N_A} = \frac{1}{6.0221367(36) \times 10^{26}}$$

$$= 1.6605402(10) \times 10^{-27}\ \text{kg}$$

It is equivalent to 931.49432(28) MeV.

Aston. in 1927 expressed the deviations of the masses from mass numbers in the form of a packing fraction for each nuclide, given by

$$\textit{Packing fraction} \quad f = ({}^A_Z M - A)/A.$$

$$\therefore \quad {}^A_Z M/A = 1 + f. \qquad ...(69)$$

To minimise confusion with mass defect, $({}^A_Z M - A)$ can be called the *mass excess.*

The binding energy per nucleon in terms of mass unit u

$$\bar{B} = \frac{B}{A} = \frac{Z}{A}M_H + \left(1 - \frac{Z}{A}\right)M_n - \frac{{}^A_Z M}{A}$$

$$= \frac{Z}{A}[M_H - M_n] + M_n - (1 + f) = -\frac{Z}{A}[0.0008397] + 0.0086649 - f. \qquad ...(70)$$

Thus the binding energy per nucleon is maximum when f is minimum. In the intermediate region of atomic weights f is negative, hence the $\bar{B}$ is greater and the nuclei are more stable. For the nuclides from ${}^{40}Ca$ to ${}^{120}Sn$, Z/A varies between 0.50 and 0.42 with the mean value 0.46. In this region the average value of $f = = -6 \times 10^{-4}$ u/nucleon, hence the value of $\bar{B} = 0.009\ u = 8.5\ MeV$/nucleon.

The work necessary to separate a proton, neutron, deuteron or alpha particle from a nucleus is called the *separation energy S*. Conversely, this energy is released when the nucleus captures such a particle. The separation energy is expressed in terms of binding energies of these nuclei, such as

$$S_n = [M(A-1, Z) + M_n - M(A, Z)c^2 = B(A, Z) - B(A-1, Z)$$

$$S_\alpha = B(A, Z) - B(A-4, Z-2) - B(4, 2).$$

The separation energy of nucleus is analogous to the heat of vaporization of solids or liquids. The average binding energy per molecule, called the latent heat of vaporisation is of the order of eV, while the average nuclear binding energy is of the order of MeV.