Zero-Energy Scattering - The Scattering Length

The necessary condition for Partial wave is that the energy be small & the potential be of short-range. That is, the method of partial waves is suited for low-energy scattering.

If the energy is so low that only particles with $l=0$ are scattered, then in this case, only δ_0 (the s-wave phase shift) would be different from zero. Then we have

$$f_k(\theta) = f_k^{(0)}(\theta) = \frac{1}{k} e^{i\delta_0} \sin\delta_0 \quad \text{——} \quad (1)$$

which is independent of the scattering angle. The limiting value of the energy for which eq. (1) holds good, is called 'zero-energy'. Thus the angular distribution of the scattered particles at zero energy, is independent of the scattering angle. In other words scattering is isotropic.

The negative of the scattering amplitude in the zero energy limit is called the Scattering Length. & is denoted by 'a'. Thus

$$a = \lim_{E\to 0}\left[-f_k(\theta)\right] = -\frac{1}{k} e^{i\delta_0} \sin\delta_0 \quad \text{——} \quad (2)$$

In terms of 'a' the zero-energy total scattering cross-sect? is given by

$$\sigma = 4\pi |f_k(\theta)|^2 = \cancel{4a}$$

$$\boxed{\sigma = 4\pi a^2} \quad \text{——} \quad (3)$$

Geometrical Interpretation of Scattering Length -

The radial Schrodinger eq. for $l=0$, reduces to

$$\left[\frac{d^2}{dr^2} + k^2 - U(r)\right] u_0(r) = 0 \qquad (4)$$

In the zero-energy limit & for $r >> r_0$, this becomes

$$\frac{d^2 u_0}{dr^2} \simeq 0 \qquad (5)$$

Thus, the asymptotic value of $u_0(r)$ is given by

$$\{u_0(r)\}_{r>>r_0} = br + c \qquad (6)$$

Where, b & c are constants.

But,

$$u_0(r) = r R_0(r) \underset{r>>r_0}{\simeq} r e^{ik.r} + f_k(\theta) e^{ikr}$$

$$\underset{k\to 0}{\simeq} (r-a) = u_0(r) \qquad (7)$$

In eq. (7), a normalizat? const. is arbitrary so that we may write,

$$u_0(r) = \alpha(r-a) \qquad (8)$$

Further choosing the normalization, $|\alpha| = 1$, we have

$$u_0(r) = \pm(r-a), \quad r >> r_0 \qquad (9)$$

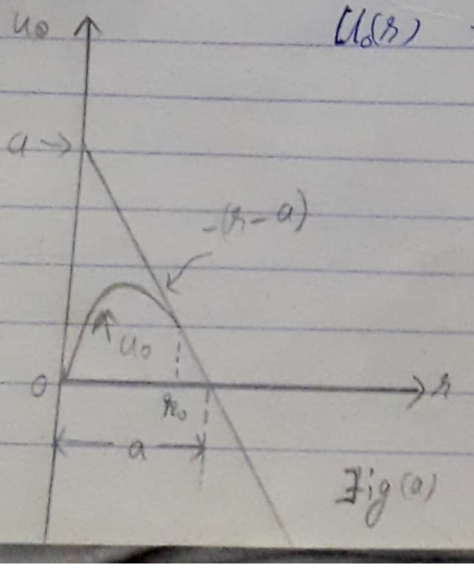

Fig (a)

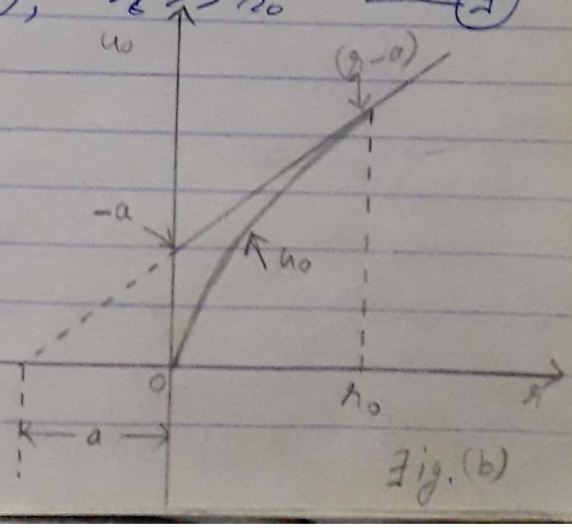

Fig. (b)

Now, eq. 6 is the eq. of a straight line with slope 'b' & intercept on the u_0 – axis equal to 'c'. Comparing eq. 6 & 9 we see that '±a' is the intercept of the straight line $u_0(s)$ on the u_0-axis when the slope of the straight line is negative as in fig. (a). & '–a' is the intercept of the straight line $u_0(s)$ on u_0 - axis when the slope of the st. line is positive as in fig. (b). In either case, 'a' is equal to the intercept of the St. line on the s-axis as is seen by putting $u_0(s) = 0$ in eq. 9.

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