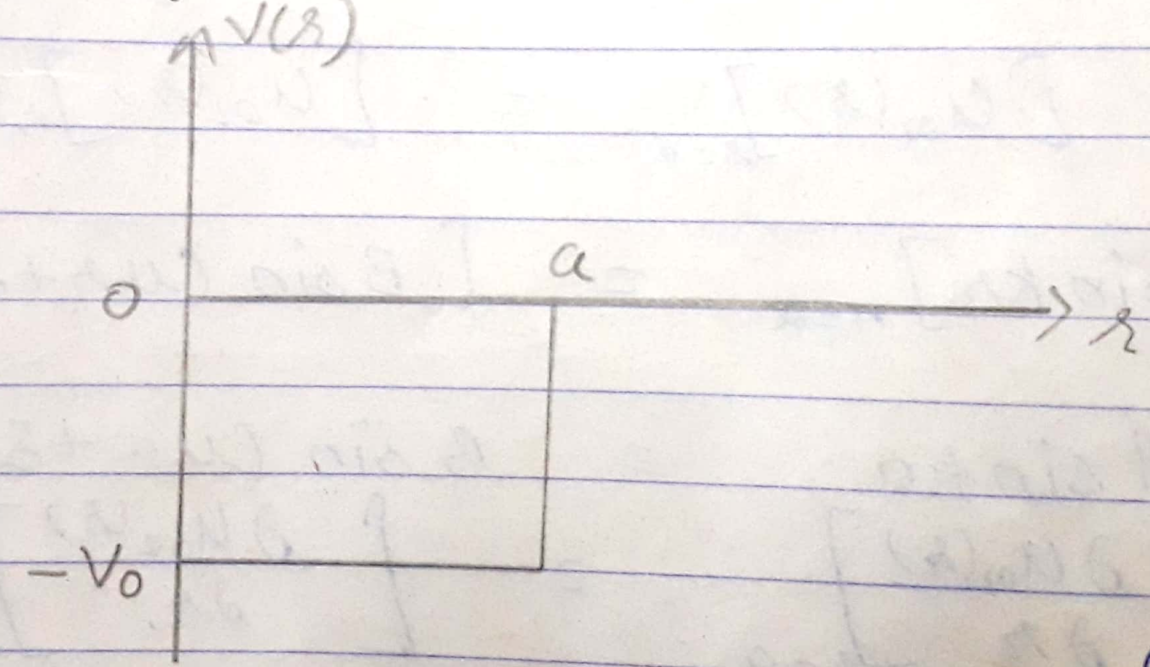
Scattering By a Square Well Potential -



We consider the S-wave scattering by a square well

$$V(r) = -V_0 \quad \text{for } 0 < r < a$$
$$V(r) = 0 \quad \text{for } r > a$$

For $l = 0$, the radial part of Schrodinger eq. would be

Case (1) → $$\frac{d^2 u_{01}}{dr^2} + k^2 u_{01}(r) = 0 \quad \text{—— (1)}$$

for $0 < r < a$

where $k^2 = \frac{2m}{\hbar^2}(E + V_0)$ —— (2)

Case (2) → $$\frac{d^2 u_{02}}{dr^2} + \mu^2 u_{02}(r) = 0 \quad \text{—— (3)}$$

for $r > a$

where $\mu^2 = \frac{2mE}{\hbar^2}$

The general solu. of eq. (1) is

$$u_{01}(r) = A \sin kr + B \cos kr \quad \text{—— (4)}$$

The cosine solu. will make $R_0(r) = u(r)/r$ go to ∞ at $r = 0$

only acceptable solu. is

$$u_{01}(r) = A \sin kr \quad \text{—— (5)}$$

Simillarly the solu. of eq. (3) can be taken as

$$u_{02}(r) = B \sin(\mu r + \delta_0) \quad \text{—— (6)}$$

Because the difference will be only in phase factor. Putting the boundary condit? at $r = a$.

$$[u_{01}(r)]_{r=a} = [u_{02}(r)]_{r=a}$$

$$[A \sin kr]_{r=a} = [B \sin(\mu r + \delta_0)]_{r=a}$$

$$A \sin ka = B \sin(\mu a + \delta_0) \quad \text{—— (7)}$$

& $$\left[\frac{\partial u_{01}(r)}{\partial r}\right]_{r=a} = \left[\frac{\partial u_{02}(r)}{\partial r}\right]_{r=a}$$

$$\left[\frac{\partial}{\partial r} A \sin kr\right]_{r=a} = \left[\frac{\partial}{\partial r} B \sin(\mu r + \delta_0)\right]_{r=a}$$

$$\left[A . k \cos kr\right]_{r=a} = \left[B . \mu \cos(\mu r + \delta_0)\right]_{r=a}$$

$$A k \cos ka = B \mu \cos(\mu a + \delta_0) \quad \text{——} (8)$$

Dividing eq. (7) by (8)

$$\frac{\sin ka}{k \cos ka} = \frac{\sin(\mu a + \delta_0)}{\mu \cos(\mu a + \delta_0)}$$

$$\Rightarrow \tan(\mu a + \delta_0) = \frac{\mu}{k} \tan ka.$$

$$\mu a + \delta_0 = \tan^{-1}\left\{\frac{\mu}{k} \tan ka\right\}$$

$$\delta_0 = \tan^{-1}\left\{\frac{\mu}{k} \tan ka\right\} - \mu a \quad \text{——} (9)$$

$$\because \sigma = \frac{4\pi}{k^2} \sin^2 \delta_0$$

$$\sigma = \frac{4\pi}{k^2}\left[1 + \frac{1}{\tan^2 \delta_0}\right]^{-1} \quad \text{——} (10)$$

$\mu a << 1$

In eq. (9) we notice that $\tan \delta_0$ would vanish if

$$\boxed{\frac{\tan ka}{k} = \frac{\tan \mu a}{\mu}}$$

We are assuming $\mu a << 1$ so that δ_1, δ_2 etc are negligible.
This known as RAMSAUER effect.