Uncertanity relation of x and P :- (by operator method) :-

~~Completeness of Eigen function:-~~

The wave particle duality is contained in the uncertanity principle of Heisenberg. In other words the uncertinity principle protects wave particle duality.

According to this principle it is impossible to measure simoultenously the position of a particle in a particullar direction and also its momentum P_x in this direction. If Δx is the uncertanity in position and ΔP_x is the uncertanity in momentum.

$$\Delta x \cdot \Delta P_x \geq \hbar/2$$

$$\Delta y \cdot \Delta P_y \geq \hbar/2$$

$$\Delta z \cdot \Delta P_z \geq \hbar/2$$

Proof → let Δx is the uncertanity in position & ΔP_x is the uncertanity in momentum, then we know that

$$\rightarrow \quad \Delta x \cdot \Delta P_x \geq \hbar/2 \quad \text{——— (1)}$$

by using opertor method for P_x and x, we may calculate the average value $\langle P_x \rangle$ and $\langle x \rangle$, then mean square deviation of position from the average $\langle x \rangle$ may be obtained by

$$\rightarrow \quad (\Delta x)^2 = \int_{-\infty}^{+\infty} \psi^* [x - \langle x \rangle]^2 \psi \, dx \quad \text{— (2)}$$

Similarly the mean square deviation of momentum from the average $\langle P_x \rangle$ is

$$\rightarrow \quad (\Delta P_x)^2 = \int_{-\infty}^{+\infty} \psi^* [P_x - \langle P_x \rangle]^2 \psi \, dx \quad \text{— (3)}$$

Now, we define two new operators as

$$\rightarrow \quad \alpha = [x - \langle x \rangle] \quad \text{and} \quad \beta = [P_x - \langle P_x \rangle]$$

then from eqn (2) & (3) we have

$$\rightarrow \quad (\Delta x)^2 . (\Delta P_x)^2 = \int_{-\infty}^{+\infty} \psi^* \alpha^2 \psi \, dx \int_{-\infty}^{+\infty} \psi^* \beta^2 \psi \, dx$$

$$= \int_{-\infty}^{+\infty} \psi^* \alpha . \alpha \psi \, dx . \int_{-\infty}^{+\infty} \psi^* \beta . \beta \psi . dx \quad \text{— (4)}$$

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Now we know that if the operator α is Hermition

$$\rightarrow \int_{-\infty}^{+\infty} g^* (\alpha\ fun.)\, dx = \left[\int_{-\infty}^{+\infty} f^* (\alpha g)\, dx = \int_{-\infty}^{+\infty} (\alpha g)^* f\, dx \right. \quad \text{--- (5)}$$

Putting this property in eq^n (4), then we have

$$\rightarrow (\Delta x)^2 \cdot (\Delta P_x)^2 = \int_{-\infty}^{+\infty} (\alpha \psi)^* (\alpha \psi)\, dx \int_{-\infty}^{+\infty} (\beta \psi)^* (\beta \psi)\, dx \quad \text{--- (6)}$$

Now, we know that Schwartz's inequality

$$\rightarrow \int_{-\infty}^{+\infty} a^* a\, dx \int_{-\infty}^{+\infty} b \cdot b^*\, dx \geq \left| \int_{-\infty}^{+\infty} a^* b\, dx \right|^2 \quad \text{--- (7)}$$

~~were~~ where, a & b are two continous function of x

Applying this property in eq^n (6) we get

$$\rightarrow (\Delta x)^2 (\Delta P_x)^2 \geq \left| \int_{-\infty}^{+\infty} (\alpha \psi)^* (\beta \psi)\, dx \right|^2$$

$$\geq \left| \int_{-\infty}^{+\infty} \psi^* \alpha \beta \psi\, dx \right|^2$$

{ by Harmition properties.

$$\geq \left| \int_{-\infty}^{+\infty} \psi^* \frac{1}{2} (\alpha\beta - \beta\alpha) \psi\, dx + \int_{-\infty}^{+\infty} \psi^* \frac{1}{2} (\alpha\beta + \beta\alpha) \psi\, dx \right|^2$$

$$\left\{ \because \alpha\beta = \frac{1}{2}(\alpha\beta - \beta\alpha) + \frac{1}{2}(\alpha\beta + \beta\alpha) \right)$$

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again

$$\rightarrow (\Delta x)^2 . (\Delta P_x)^2 \geq \frac{1}{4} \left| \int_{-\infty}^{+\infty} \psi^* (\alpha\beta - \beta\alpha) \psi \, dx \right|^2 + \frac{1}{4} \left| \int_{-\infty}^{+\infty} \psi^* (\alpha\beta + \beta\alpha) \psi \, dx \right|^2 \quad \text{——— (8)}$$

the factor having cross term on the right hand side is omitted because it vanishes

then eq^n (8) becomes

$$\rightarrow (\Delta x)^2 (\Delta P_x)^2 \geq \frac{1}{4} \left| \int_{-\infty}^{+\infty} \psi^* (\alpha\beta - \beta\alpha) \psi \, dx \right|^2$$

$$\geq \frac{1}{4} \left| \int_{-\infty}^{+\infty} \psi^* (x P_x - P_x x) \psi \, dx \right|^2 \quad \text{——— (9)}$$

Now the Commutation relation between P_x & x

i.e. $$P_x . x - x . P_x = \frac{\hbar}{i}$$

$$\therefore (\Delta x)^2 . (\Delta P_x)^2 \geq \frac{1}{4} \hbar^2 \text{ (using above relation}$$

$$\boxed{(\Delta x)(\Delta P_x) \geq \frac{\hbar}{2}} \quad \text{——— (10)}$$

or

$$\boxed{\begin{aligned} \Delta t . \Delta E &\geq \hbar/2 \\ \Delta \phi . \Delta L &\geq \hbar/2 \end{aligned}} \quad \text{——— (11)}$$

Thus Heisenburg uncertanity relation proved.

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