Reciprocal Lattice:-

Every crystal structure has two lattice associated with it. The direct lattice and reciprocal lattice.

The crystal lattice is in position space, while reciprocal lattice is a lattice in K space or momentum space.

we know that in crystal lattice there are large number of groups of parallel lattice planes, the distance b/w them ie. and orientations are different. If we draw normals on the common origin of this parallel lattice planes then the end point of this normal expressed a lattice which is called reciprocal lattice.

Thus, Reciprocal lattice is a lattice obtained from the end point of normal drawn on different parallel lattice planes from common origin.

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the primitive vector of a direct lattice. Then the primitive vectors of a reciprocal lattice are assumed to be A, B & C which can be defined in terms of $\vec{a}, \vec{b}, \vec{c}$ as.

$$\left.\begin{aligned}\vec{A} &= 2\pi . \frac{\vec{b}\times\vec{c}}{\vec{a}.(\vec{b}\times\vec{c})} \\ \vec{B} &= 2\pi . \frac{\vec{c}\times\vec{a}}{\vec{a}.(\vec{b}\times\vec{c})} \\ \vec{C} &= 2\pi . \frac{\vec{a}\times\vec{b}}{\vec{a}.(\vec{b}\times\vec{c})}\end{aligned}\right\} \text{———} (1)$$

$$V = \vec{a}.(\vec{b}\times\vec{c})$$

The primitive vectors direct lattice i.e, $\vec{a}, \vec{b}$ and $\vec{c}$ has dimensions of [Length]. while primitive vectors of reciprocal lattice i.e. $\vec{A}, \vec{B}, \& \vec{C}$ have the dimension of $[\text{Length}]^{-1}$ or [1/Length]. A microscopic image is the image of direct lattice, while the x-ray diffraction pattern is a mapping of reciprocal lattice of a crystal.

From above relation (1), it is clear that $\vec{A}$ is perpendicular to $\vec{b}$ and $\vec{c}$, $\vec{B}$ is perpendicular to $\vec{c}$ and $\vec{a}$, and $\vec{C}$ is perpendicular to $\vec{a}$ and $\vec{b}$, Also from (1) we have

$$\vec{A}\cdot\vec{a} = 2\pi \;,\quad \vec{B}\cdot\vec{a} = 0 \;,\quad \vec{C}\cdot\vec{a} = 0$$

$$\left.\begin{array}{lll} \vec{A}\cdot\vec{b} = 0 & \vec{B}\cdot\vec{b} = 2\pi \;, & \vec{C}\cdot\vec{b} = 0 \\ \vec{A}\cdot\vec{c} = 0 & \vec{B}\cdot\vec{c} = 0 & \vec{C}\cdot\vec{c} = 2\pi \end{array}\right\} \quad (2)$$

Above equation (2) relates the primitive vectors of ~~different~~ direct lattice to the primitive vectors of reciprocal lattice.

Hence, $\vec{a}\cdot(\vec{b}\times\vec{c})$ is the volume of unit cell of direct lattice of crystal W.K.T. as position Vector of a lattice point in direct Lattice is expressed by translational vector T.

$$T = n_1\vec{a} + n_2\vec{b} + n_3\vec{c}$$

similarly, the position of lattice point in reciprocal lattice is represented by reciprocal lattice vector $\vec{G}$ and expressed as

$$\vec{G} = h\vec{A} + k\vec{B} + l\vec{C}$$

where n_1, n_2, n_3, h, K and l all are the integers, Hence

$$\vec{G}\cdot\vec{T} = 2\pi N$$