Relation between voids ratio (e) and porosity (n)

Voids Ratio $(e) = \frac{V_v}{V_s}$

If $V_v = e$

$\Rightarrow e = \frac{V_v}{V_s} = \frac{e}{V_s}$

$\Rightarrow V_s = 1$

then, $V = 1+e$

Porosity $(n) = \frac{V_v}{V}$

If $V_v = n$

$\Rightarrow n = \frac{V_v}{V} = \frac{n}{V}$

$\Rightarrow V = 1$

then, $V_s = (1-n)$

using phase diagram for voids ratio and porosity

$$e = \frac{V_v}{V_s} = \frac{n}{1-n}$$

and, $n = \frac{V_v}{V} = \frac{e}{1+e}$

Hence,

$$\boxed{e = \frac{n}{1-n}}$$

$$\boxed{n = \frac{e}{1+e}}$$

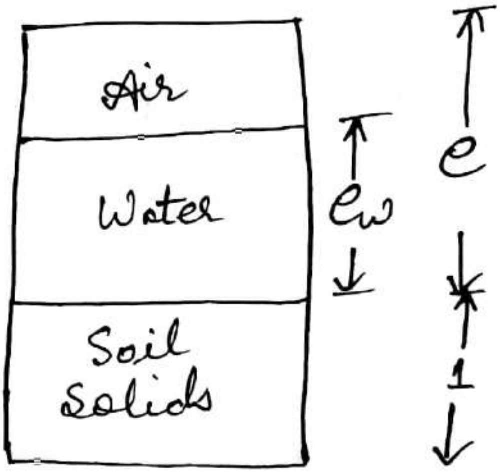
Fundamental Relationships :-

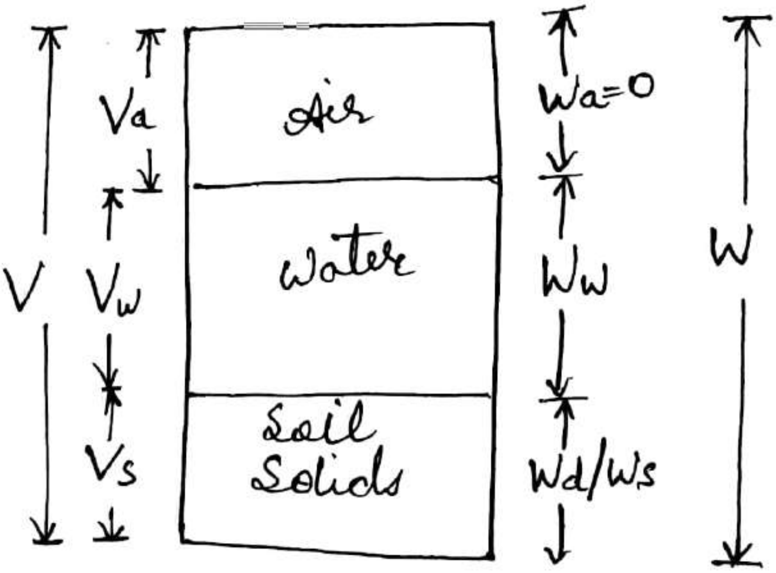
(i) Relation between e, G, w and S

Degree of Saturation (S)

$$S = \frac{V_w}{V_v}$$

using 3- phase diagram in terms of voids Ratio.





we get,

$$S = \frac{V_w}{V_v} = \frac{e_w}{e}$$

$$\Rightarrow e_w = S \cdot e \quad \rightarrow (1)$$

where, e_w = water voids ratio

for saturated sample, $S = 1$

so, eqⁿ (1) becomes

$$e_w = e$$

water content (w)

$$w = \frac{W_w}{W_d}$$

$$\Rightarrow w = \frac{\gamma_w . V_w}{\gamma_s . V_s}$$

[∵ $\gamma = \frac{W}{V}$

then, $\gamma_w = \frac{W_w}{V_w}$

$\Rightarrow W_w = \gamma_w . V_w$

also, $\gamma_s = \frac{W_d}{V_s}$

$\Rightarrow W_d = \gamma_s . V_s$]

Using 3-phase relationship diagram, we get;

$$w = \frac{\gamma_w . V_w}{\gamma_s . V_s}$$

$$\Rightarrow w = \frac{e_w . \gamma_w}{\gamma_s . 1} \rightarrow (2)$$

Sp. Gravity (G)

$$G = \frac{\gamma_s}{\gamma_w}$$

$$\Rightarrow \gamma_s = G . \gamma_w \rightarrow (3)$$

Substitute value of eqⁿ (3) in eqⁿ (2) we get;

$$\omega = \frac{e_w \cdot \gamma_w}{\gamma_s \cdot 1}$$

$$\Rightarrow \omega = \frac{e_w \cdot \cancel{\gamma_w}}{G \cdot \cancel{\gamma_w} \cdot 1}$$

$$\Rightarrow \omega = \frac{e_w}{G}$$

$$\Rightarrow e_w = \omega \cdot G \rightarrow (4)$$

equating equations (1) & (4), we get;

$$\boxed{S \cdot e = \omega \cdot G}$$

(ii) Relation between n_a, a_c and n

Air content, $a_c = \frac{V_a}{V_v}$

and, $n = \frac{V_v}{V}$

Percentage air voids, $n_a = \frac{V_a}{V} \rightarrow (1)$

$$a_c \cdot n = \frac{V_a}{\cancel{V_v}} \times \frac{\cancel{V_v}}{V} = \frac{V_a}{V} \rightarrow (2)$$

from eq^n (1) & (2); we get

$$\boxed{n_a = n \cdot a_c}$$

(iii) Relation between γ_d, G and e (or n)

$$\gamma_d = \frac{W_d}{V}$$

$$\Rightarrow \gamma_d = \frac{\gamma_s \cdot V_s}{V} \quad \left[\because \gamma_s = \frac{W_d}{V_s} \Rightarrow W_d = \gamma_s \cdot V_s \right]$$

From 3-phase relationships for voids ratio

$V_s = 1$ and $V = (1+e)$

$$\Rightarrow \gamma_d = \frac{\gamma_s \cdot V_s}{V} = \frac{1 \cdot \gamma_s}{1+e} \rightarrow (1)$$

Sp. Gravity, $G = \frac{\gamma_s}{\gamma_w}$

$$\Rightarrow \gamma_s = G \cdot \gamma_w \rightarrow (2)$$

Put value of eq^n (2) in eq^n (1):

$$\boxed{\gamma_d = \frac{G \cdot \gamma_w}{1+e}}$$

Similarly, from 3-phase relationship diagram for porosity

$V_s = (1-n)$ and $V = 1$

$$\gamma_d = \frac{\gamma_s \cdot V_s}{V} = \frac{(1-n) \cdot \gamma_s}{1} \rightarrow (3)$$

Sp. gravity, $G = \frac{\gamma_s}{\gamma_w}$

$$\Rightarrow \gamma_s = G \cdot \gamma_w \rightarrow (4)$$

Put value of eq^n (4) in eq^n (3):

$$\boxed{\gamma_d = (1-n) G \cdot \gamma_w}$$

(iv) Relation between e, S and n_a

Percentage air voids (n_a)

$$n_a = \frac{V_a}{V}$$

$V = V_v + V_s = 1+e$ [Using 3-phase diagram we have this]

$V_a = V_v - V_w = e - e_w$

$$n_a = \frac{V_a}{V}$$

$$\Rightarrow n_a = \frac{V_v - V_w}{V_v + V_s} = \frac{e - e_w}{1+e}$$

Since, $e_w = e \cdot S$ (Derived from previous relation)

$$n_a = \frac{e - e \cdot S}{1+e}$$

$$\Rightarrow n_a = \frac{e(1-S)}{1+e}$$

$$\boxed{n_a = \frac{e(1-S)}{1+e}}$$

(v) Relation between γ, G, e and S

Bulk unit wt. density (γ)

$$\gamma = \frac{W}{V}$$

$$\Rightarrow \gamma = \frac{(W_w + W_d)}{V}$$

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$$\Rightarrow \gamma = \frac{\gamma_s . V_s + \gamma_w . V_w}{V}$$ $\left[\because \gamma_s = \frac{W_d}{V_s} \Rightarrow W_d = \gamma_s . V_s.\right.$ also, $\gamma_w = \frac{W_w}{V_w} \Rightarrow \left. W_w = \gamma_w . V_w \right]$

Using 3- phase diagram

$V_s = 1$, $V = (1+e)$ and $V_w = e_w$

$$\gamma = \frac{\gamma_s . 1 + \gamma_w . e_w}{1+e}$$

$$\Rightarrow \gamma = \frac{G . \gamma_w + \gamma_w . e_w}{1+e}$$ $\left[\because G = \frac{\gamma_s}{\gamma_w} \Rightarrow \gamma_s = G . \gamma_w\right]$

$$\Rightarrow \gamma = \frac{G . \gamma_w + \gamma_w . S . e}{1+e}$$ $\left[\because e_w = S . e\right]$

$$\Rightarrow \gamma = \frac{G . \gamma_w + \gamma_w . S . e}{1+e}$$

$$\Rightarrow \gamma = \frac{(G + e . S)\gamma_w}{1+e}$$

$$\boxed{\gamma = \frac{(G + eS)\gamma_w}{1+e}}$$

(vi) Relation between γ_{sat}, G and e (or n)

$$\gamma_{sat} = \frac{W_{sat}}{V}$$

$$\Rightarrow \gamma_{sat} = \frac{W_d + W_w}{V}$$

$$\Rightarrow \gamma_{sat} = \frac{\gamma_s . V_s + \gamma_w . V_w}{V}$$

$\left[\because \gamma_s = \frac{W_d}{V_s} \Rightarrow W_d = \gamma_s . V_s \right.$

also,

$\left. \gamma_w = \frac{W_w}{V_w} \Rightarrow W_w = \gamma_w . V_w \right]$

Using 3-phase relationship diagram:-

$$V = (1+e)\ ,\ V_s = 1,\ V_w = e$$

$$\gamma_{sat} = \frac{\gamma_s . V_s + \gamma_w . V_w}{V}$$

$$\Rightarrow \gamma_{sat} = \frac{\gamma_s . 1 + \gamma_w . e}{1+e}$$

$$\Rightarrow \gamma_{sat} = \frac{G . \gamma_w + \gamma_w . e}{1+e}$$

$\left[\because G = \frac{\gamma_s}{\gamma_w} \Rightarrow \gamma_s = G . \gamma_w \right]$

$$\Rightarrow \gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}$$

$$\boxed{\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}}$$

Similarly, 3-phase relationship for porosity

$$V = 1\ ,\ V_s = (1-n) \text{ and } V_w = n$$

$$\gamma_{sat} = \frac{\gamma_s . V_s + \gamma_w . V_w}{V}$$

$$\Rightarrow \gamma_{sat} = \frac{\gamma_s . (1-n) + \gamma_w . n}{1}$$

$$\Rightarrow \gamma_{sat} = G . \gamma_w (1-n) + \gamma_w . n$$

$$\left[\because G = \frac{\gamma_s}{\gamma_w} \Rightarrow \gamma_s = G \cdot \gamma_w \right]$$

$$\boxed{\gamma_{sat} = G . \gamma_w (1-n) + \gamma_w . n}$$

(vii) Relation between γ_d, γ and w water content (w)

$$w = \frac{W_w}{W_d}$$

Adding 1 on both the sides;

$$w + 1 = \frac{W_w}{W_d} + 1$$

$$\Rightarrow w + 1 = \frac{W_w + W_d}{W_d}$$

$$\Rightarrow 1 + w = \frac{W}{W_d}$$

$$\Rightarrow W_d = \frac{W}{1+w} \rightarrow (1)$$

Dry unit wt. Density, $\gamma_d = \frac{W_d}{V} \rightarrow (2)$

$$\gamma_d = \frac{W_d}{V}$$

putting value of eq^n (1) in eq^n (2);

$$\gamma_d = \frac{W}{(1+w)V}$$

$$\Rightarrow \gamma_d = \frac{\gamma}{1+w} \quad \left[\because \gamma = \frac{W}{V}\right]$$

$$\boxed{\gamma_d = \frac{\gamma}{1+w}}$$

(viii) Relation between γ_{sat}, γ, γ_d and S.

$$\gamma = \frac{(G+eS)\gamma_w}{1+e}$$

$$\Rightarrow \gamma = \frac{G\gamma_w}{1+e} + S.\frac{e\gamma_w}{1+e}$$

$$\Rightarrow \gamma = \frac{G.\gamma_w}{1+e} + S\left[\frac{(G+e)\gamma_w}{1+e} - \frac{G\gamma_w}{1+e}\right]$$

$$\Rightarrow \gamma = \gamma_d + S\left[\gamma_{sat} - \gamma_d\right]$$

$$\boxed{\gamma = \gamma_d + S\left[\gamma_{sat} - \gamma_d\right]}$$

(ix) Relation between γ', γ_d and n

Since, $\gamma' = \gamma_{sat} - \gamma_w$

Also, $\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}$

Then,

$$\gamma' = \frac{(G+e)\gamma_w}{1+e} - \gamma_w$$

$$\Rightarrow \gamma' = \frac{G\gamma_w}{1+e} + \frac{e\gamma_w}{1+e} - \gamma_w$$

$$\Rightarrow \gamma' = \frac{G\gamma_w}{1+e} + \frac{e\gamma_w - \gamma_w(1+e)}{1+e}$$

$$\Rightarrow \gamma' = \frac{G\gamma_w}{1+e} + \frac{e\gamma_w - \gamma_w - e\gamma_w}{1+e}$$

$$\Rightarrow \gamma' = \frac{G\gamma_w}{1+e} - \frac{\gamma_w}{1+e}$$

[Since, $\gamma_d = \frac{G\gamma_w}{1+e}$]

$$\Rightarrow \gamma' = \gamma_d - \frac{\gamma_w}{1+e}$$

[Since, $e = \frac{n}{1-n}$]

$$\Rightarrow \gamma' = \gamma_d - \frac{\gamma_w}{1+\left(\frac{n}{1-n}\right)}$$

$$\Rightarrow \gamma' = \gamma_d - (1-n)\gamma_w$$

$$\boxed{\gamma' = \gamma_d - (1-n)\gamma_w}$$

(x) Relation between γ_d, G, w and n_a

$$V = V_a + V_w + V_s$$

$$V = V_a + \frac{W_w}{\gamma_w} + \frac{W_d}{\gamma_s} \quad \left[\because \gamma_w = \frac{W_w}{V_w} \Rightarrow W_w/\gamma_w = V_w \right.$$

also,

$$\left. \gamma_s = \frac{W_d}{V_s} \Rightarrow V_s = W_d/\gamma_s \right]$$

$$\Rightarrow 1 = \frac{V_a}{V} + \frac{W_w}{\gamma_w \cdot V} + \frac{W_d}{\gamma_s \cdot V}$$

$$\Rightarrow 1 = \frac{V_a}{V} + \frac{w \cdot W_d}{\gamma_w \cdot V} + \frac{W_d}{\gamma_s \cdot V} \quad \left[\because w = \frac{W_w}{W_d} \Rightarrow W_w = w \cdot W_d \right]$$

also,

$$\left[\because \gamma_d = \frac{W_d}{V} \right]$$

$$\Rightarrow 1 = \frac{V_a}{V} + \frac{w \cdot \gamma_d}{\gamma_w} + \frac{\gamma_d}{\gamma_s}$$

$$\Rightarrow \left(1 - \frac{V_a}{V}\right) = \frac{w \cdot \gamma_d}{\gamma_w} + \frac{\gamma_d}{\gamma_s}$$

$$\Rightarrow \left(1 - \frac{V_a}{V}\right) = \frac{w \cdot \gamma_d}{\gamma_w} + \frac{\gamma_d}{G \cdot \gamma_w} \quad \left[\because G = \frac{\gamma_s}{\gamma_w} \Rightarrow \gamma_s = G \cdot \gamma_w \right]$$

$$\Rightarrow \left(1 - \frac{V_a}{V}\right) = \frac{\gamma_d}{\gamma_w}\left(w + \frac{1}{G}\right)$$

$$\Rightarrow (1 - n_a) = \frac{\gamma_d}{\gamma_w}\left(w + \frac{1}{G}\right)$$

$$\Rightarrow \gamma_d = \frac{(1-n_a)\gamma_w}{\left(w+\frac{1}{G}\right)}$$

$$\Rightarrow \gamma_d = \frac{(1-n_a)\,G\cdot\gamma_w}{1+wG}$$

$$\boxed{\gamma_d = \frac{(1-n_a)\,G\cdot\gamma_w}{1+wG}}$$

(xii) Relation between γ_d, G, w and S

$$\gamma_d = \frac{G\gamma_w}{1+e}$$

Since, $S\cdot e = wG$

$$\Rightarrow e = \frac{wG}{S}$$

So,

$$\gamma_d = \frac{G\gamma_w}{1+\frac{wG}{S}}$$

for $S=1$, $w = w_{sat}$

$$\Rightarrow \gamma_d = \frac{G\gamma_w}{1+wG}$$

$$\boxed{\gamma_d = \frac{G\gamma_w}{1+w_{sat}\cdot G}}$$