Signal Flow Graph

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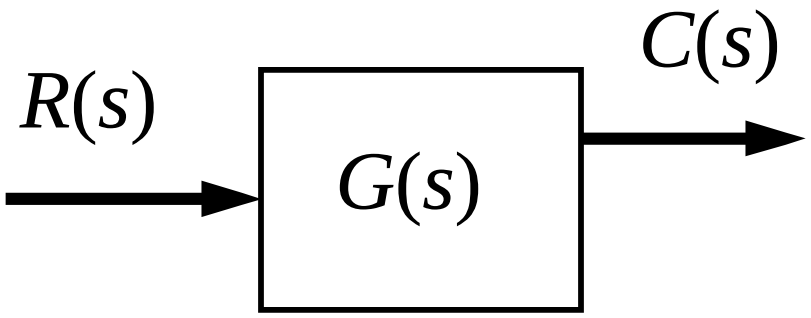
What is Signal Flow Graph?

- SFG is a diagram which represents a set of simultaneous equations.
- This method was developed by S.J.Mason. This method does n't require any reduction technique.
- It consists of nodes and these nodes are connected by a directed line called branches.
- Every branch has an arrow which represents the flow of signal.
- For complicated systems, when Block Diagram (BD) reduction method becomes tedious and time consuming then SFG
  is a good choice.

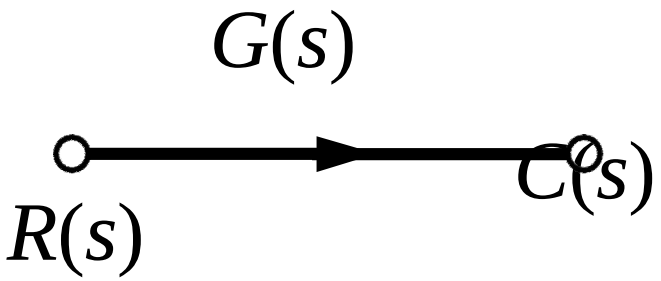
Comparison of BD and SFG

block diagram:



In this case at each step block diagram is to be redrawn. That's why it is tedious method. So wastage of time and space.

signal flow graph:



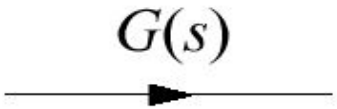
Only one time SFG is to be drawn and then Mason's gain formula is to be evaluated.
So time and space is saved.

SFG

Alternative to block diagram;

Consists only **branches** (systems), and **nodes** (signals)

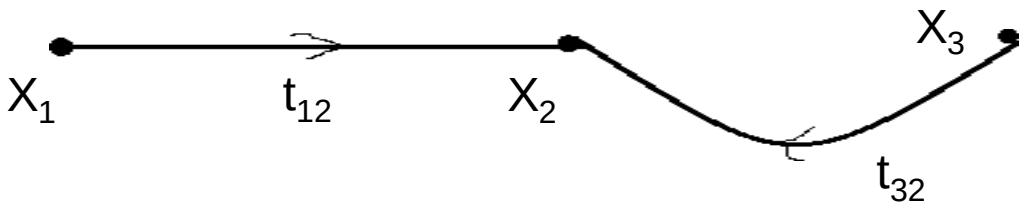
Branches



Nodes

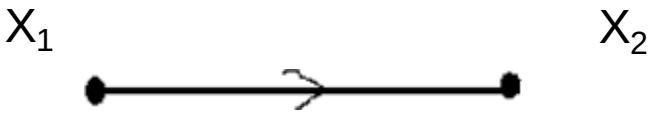
Definition of terms required in SFG

Node: It is a point representing a variable. $x_2 = t_{12} x_1 + t_{32} x_3$



In this SFG there are 3 nodes.

Branch : A line joining two nodes.

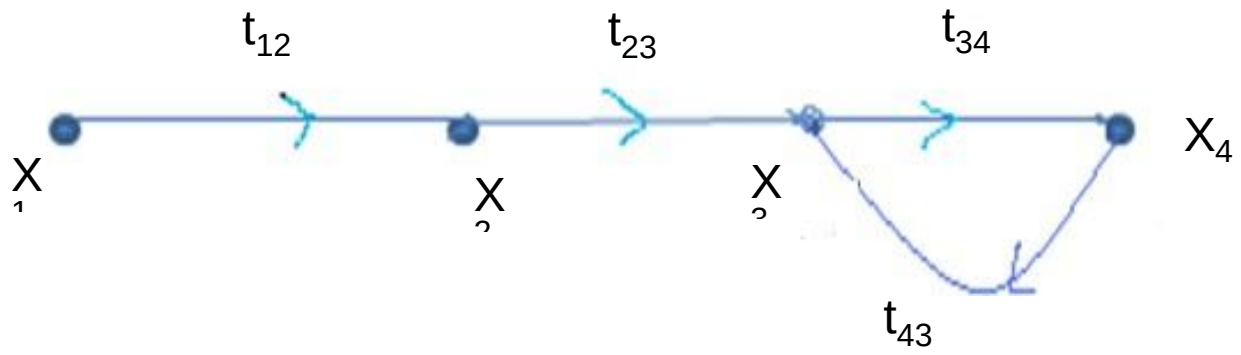


Input Node : Node which has only outgoing branches.
X_1 is input node.

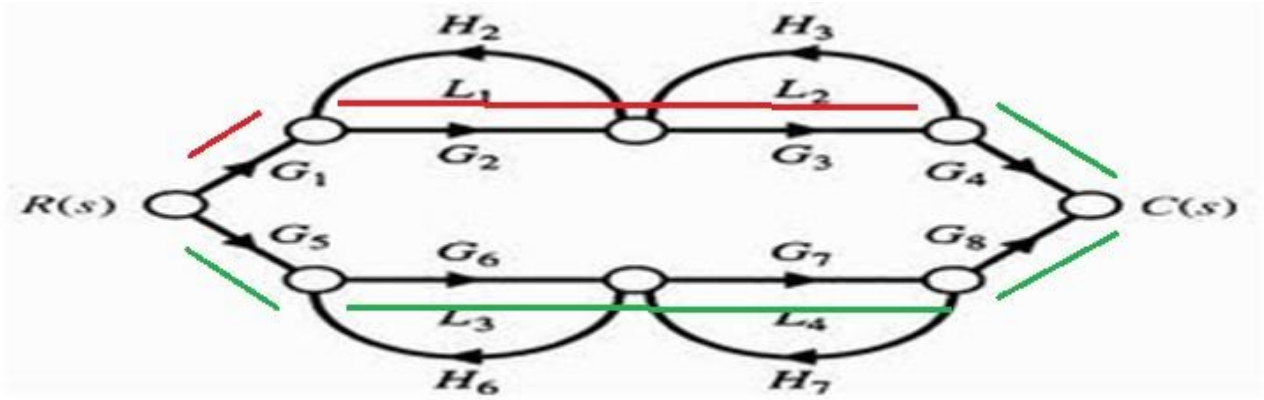
Output node/ sink node: Only incoming branches.

Mixed nodes: Has both incoming and outgoing branches.

Transmittance : It is the gain between two nodes. It is generally written on the branch near the arrow.



- **Path** : It is the traversal of connected branches in the direction of branch arrows, such that no node is traversed more than once.
- **Forward path** : A path which originates from the input node and terminates at the output node and along which no node is traversed more than once.
- **Forward Path gain** : It is the product of branch transmittances of a forward path.

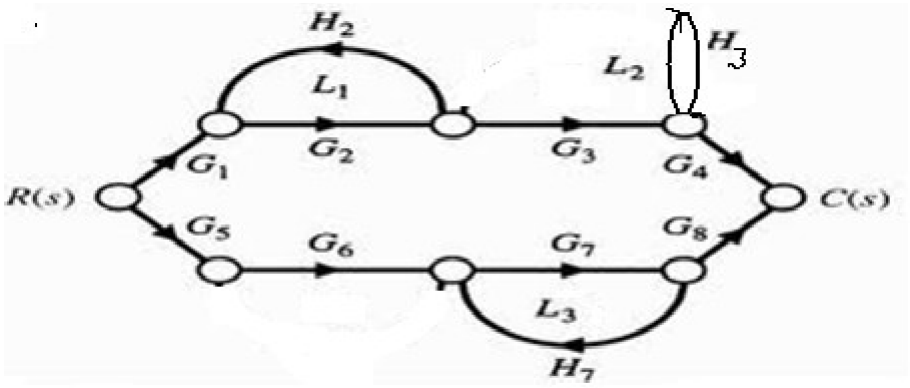


$P_1 = G_1 G_2 G_3 G_4$, $P_2 = G_5 G_6 G_7 G_8$

Loop : Path that originates and terminates at the same node and along which no other node is traversed more than once.

Self loop: Path that originates and terminates at the same node.

Loop gain: it is the product of branch transmittances of a loop. **Non-touching loops**: Loops that don't have any common node or branch.

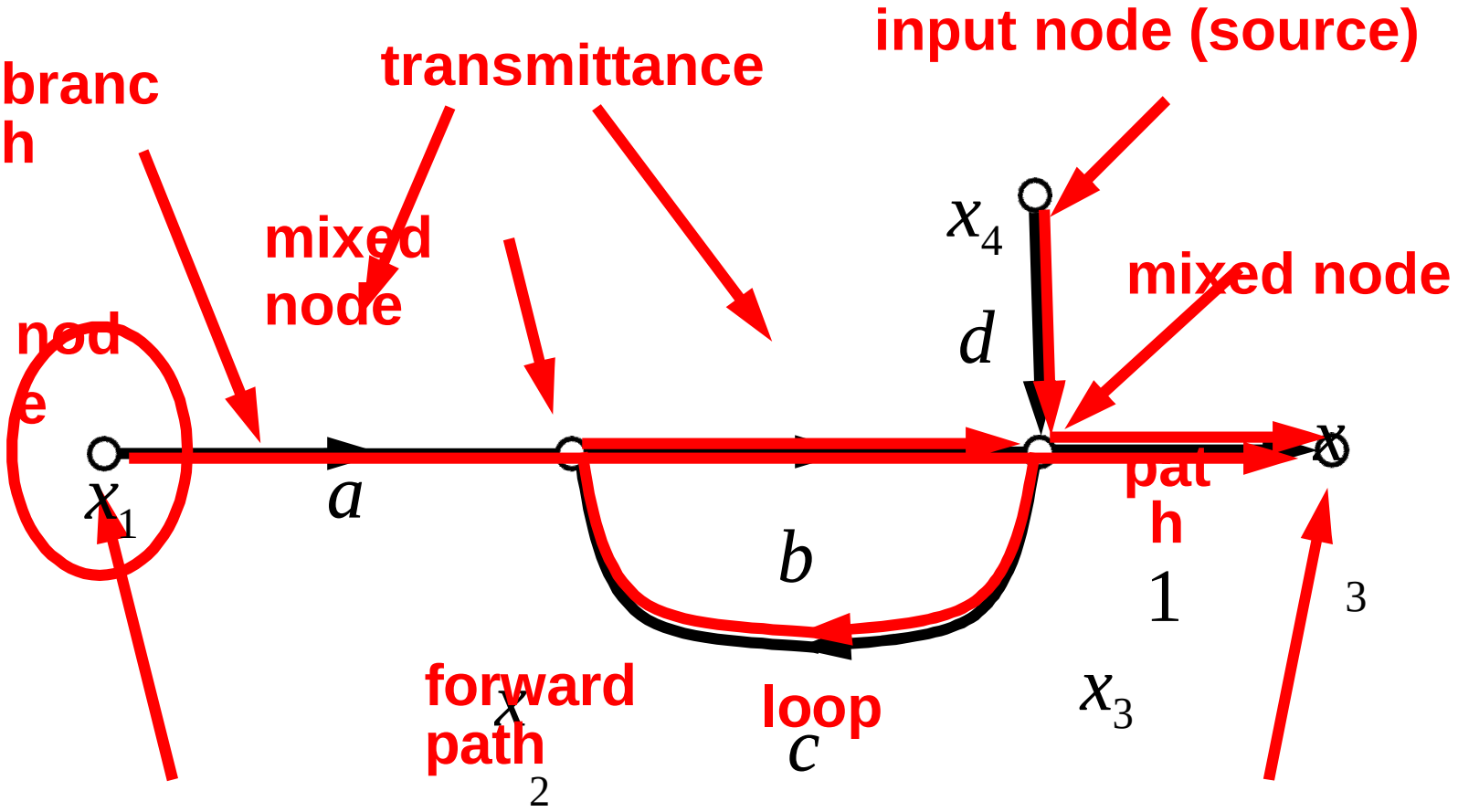


$L_1 = G_2 H_2$

$L_2 = H_3$, $L_3 = G_7 H_7$

Non-touching loops are L1 & L2, L1 & L3, L2 &L3

SFG terms representation



Rules for drawing of SFG from Block diagram

- All variables, summing points and take off points are represented by nodes.
- If a summing point is placed before a take off point in the direction of signal flow, in such a case the summing point and take off point shall be represented by a single node.
- If a summing point is placed after a take off point in the direction of signal flow, in such a case the summing point and take off point shall be represented by separate nodes connected by a branch having transmittance unity.

Mason's GainFormula

- A technique to reduce a signal-flow graph to a single transfer function requires the application of one formula.
- The transfer function, C(s)/R(s), of a system represented by a

  signal-flow graph is

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k P_k \Delta_k}{\Delta}$$

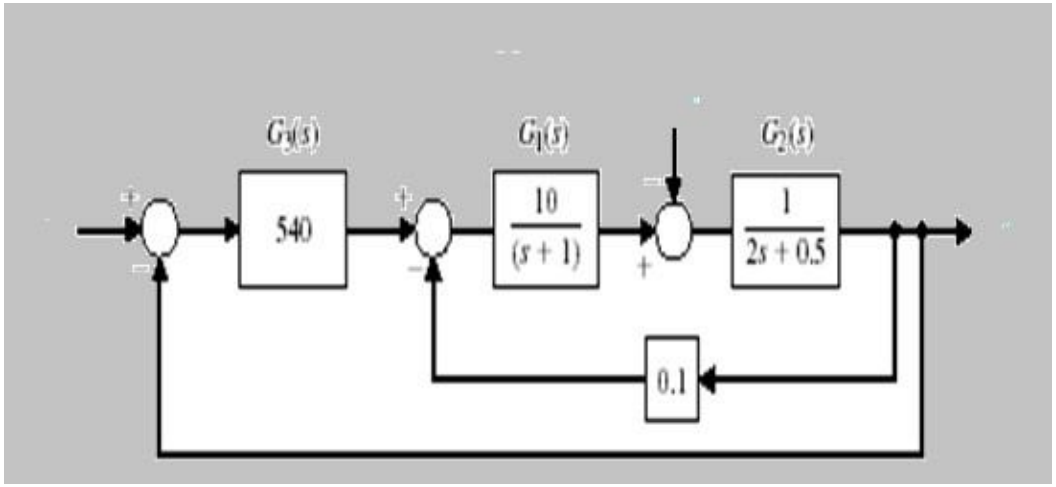
k = number of forwardpath P_k = the kth forward path

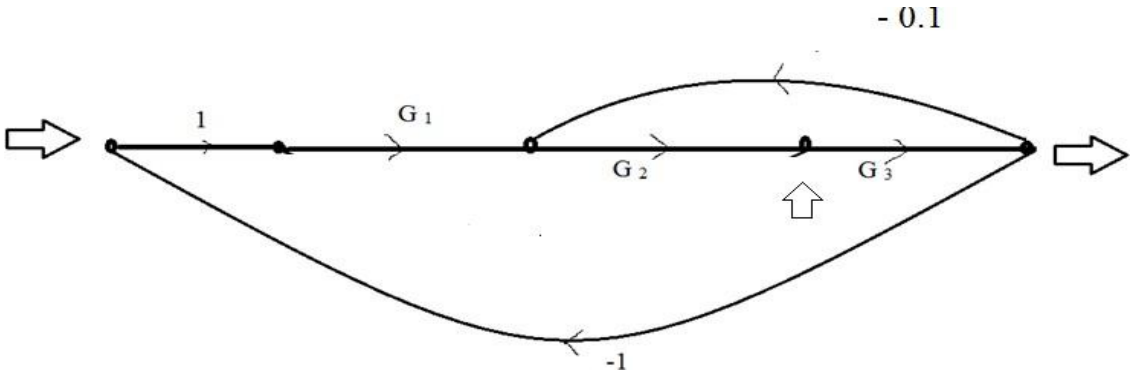
gain

$\Delta = 1 - (\Sigma$ loop gains$) + (\Sigma$ non-touching loop gains taken two at a time$) - (\Sigma$ non-touching loop gains taken three at a time$)+$ so on .

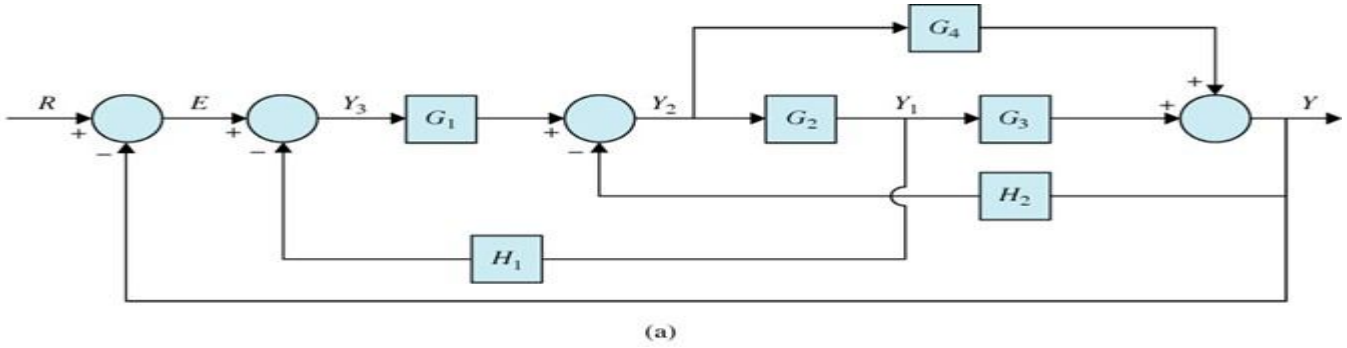
Δ_k = *1 – (loop-gain which does not touch the forward path)*

Ex: SFG from BD

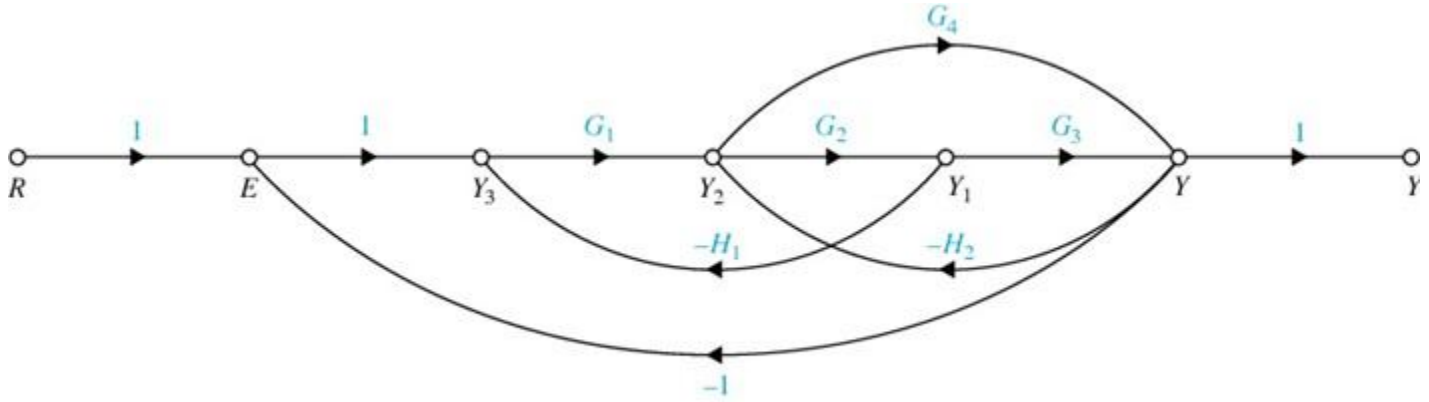




EX: To find T/F of the given block diagram

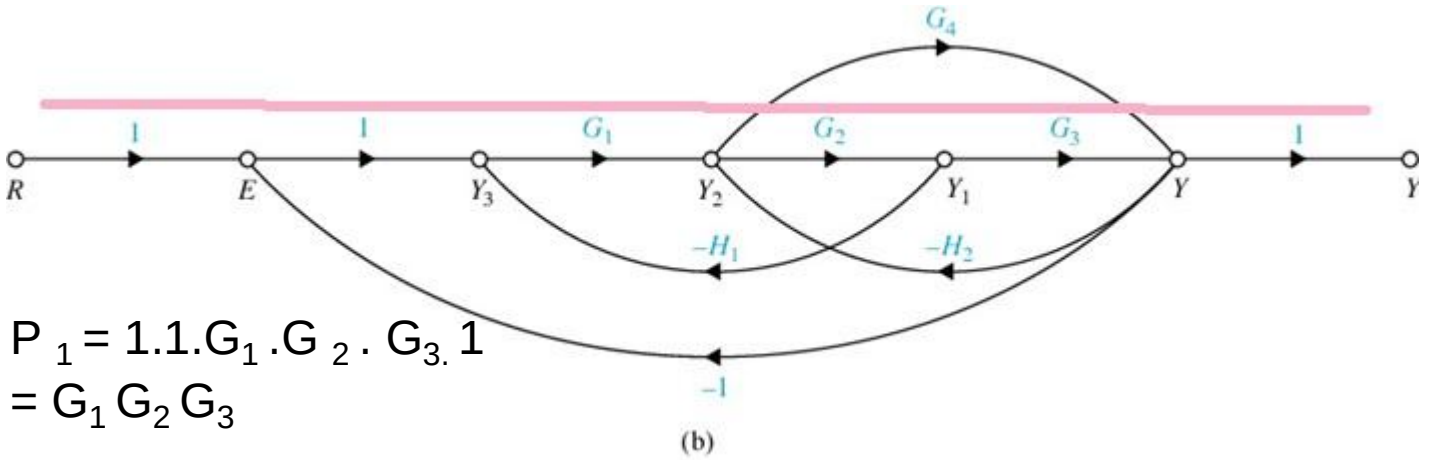


(a)



(b)

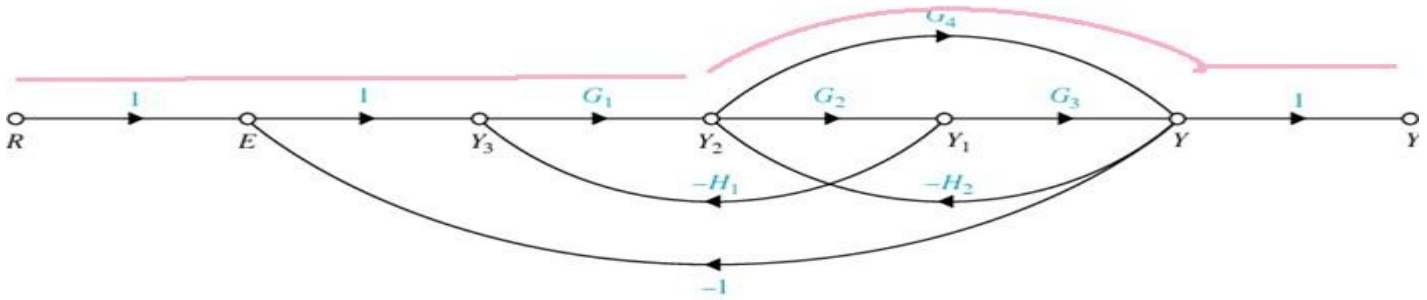
Identification of Forward Paths



(b)

$P_1 = 1.1.G_1 .G_2 . G_3 . 1$

$= G_1 G_2 G_3$

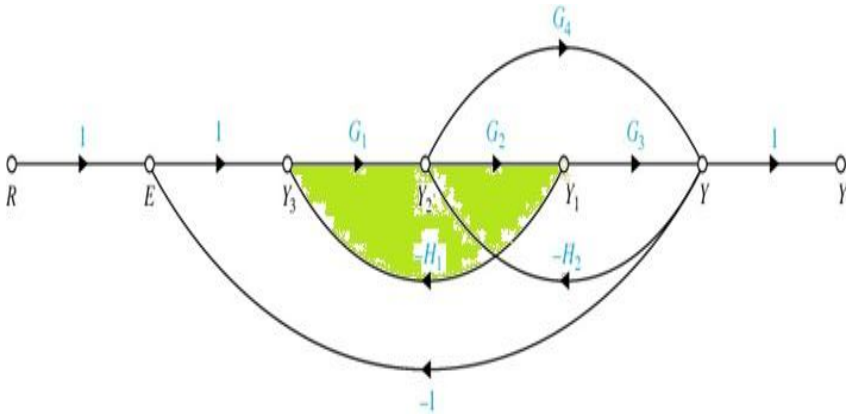


(b)

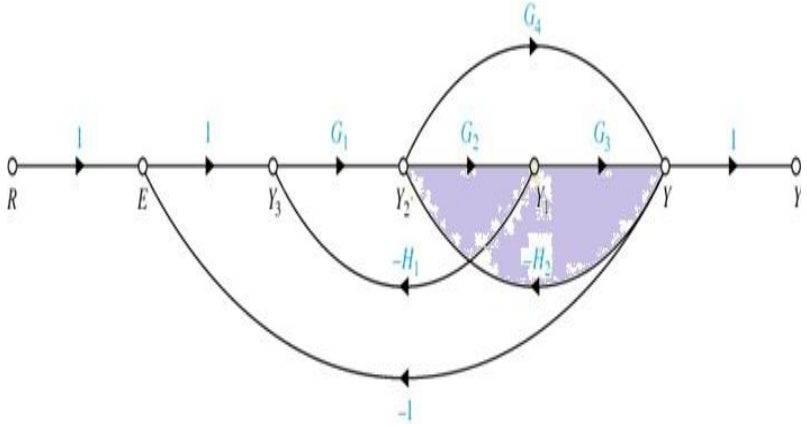
$P_2 = 1.1.G_2 . G_3 . 1$

$= G_2 G_3$

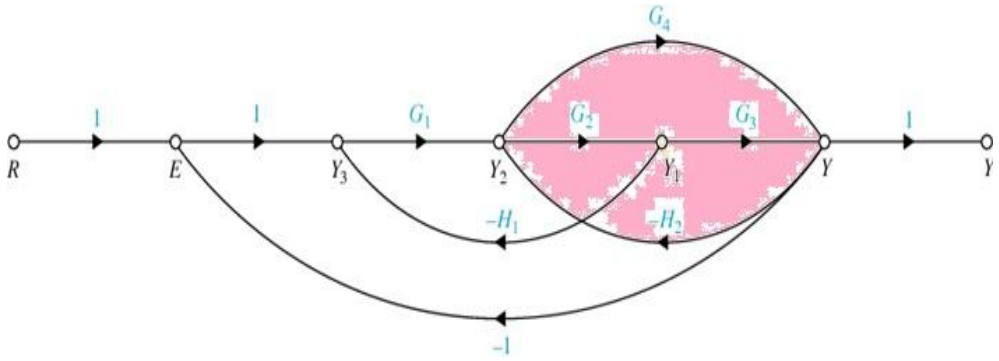
Individual Loops



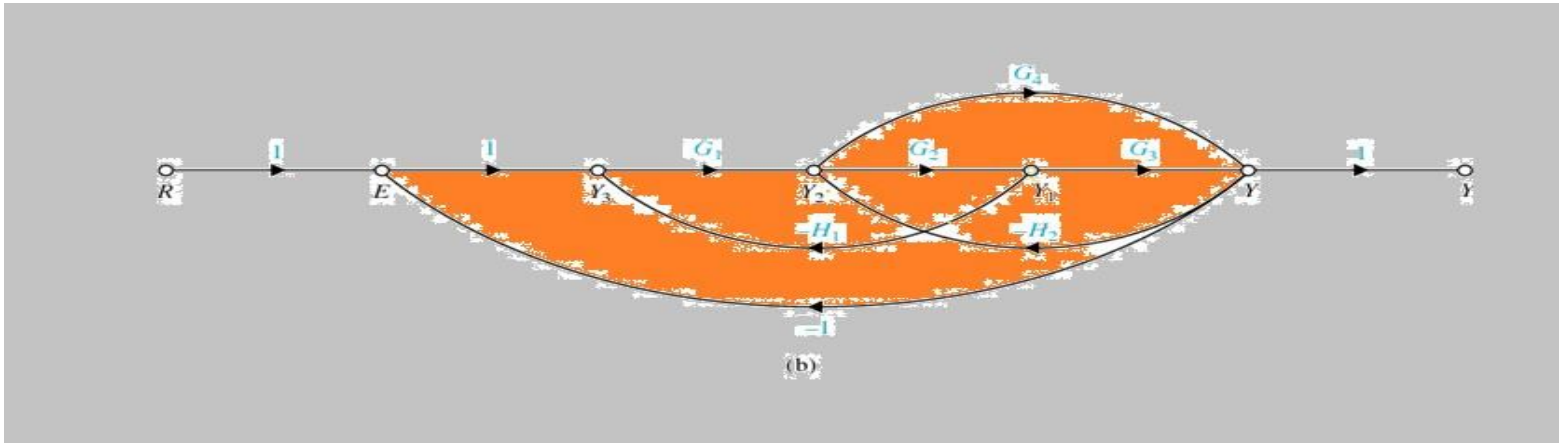
$L_1 = G_1 G_2 H_1$



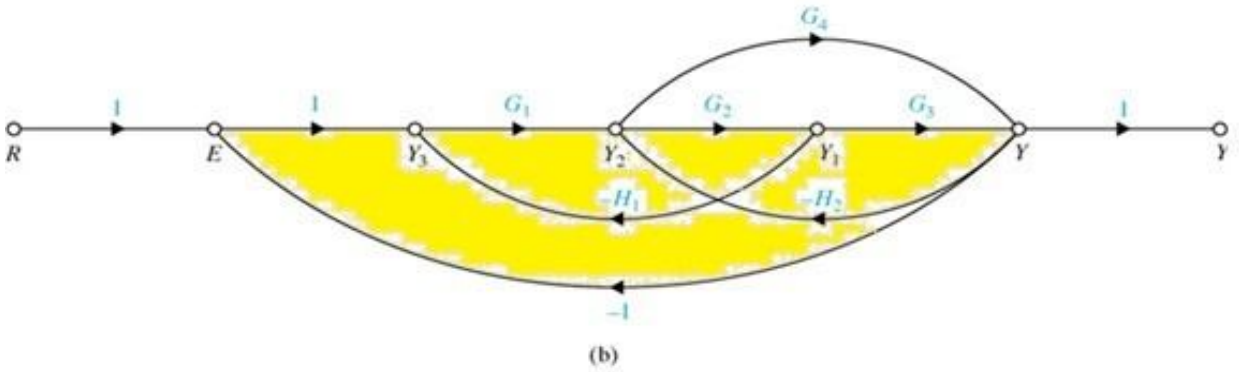
$L_2 = - G_2 G_3 H_2$



$L_3 = - G_4 H_2$



$L_4 = - G_1 G_4$



$L_5 = - G_1 G_2 G_3$

Construction of SFG from simultaneous equations

$$y_2 = t_{21}y_1 + t_{23}y_3$$

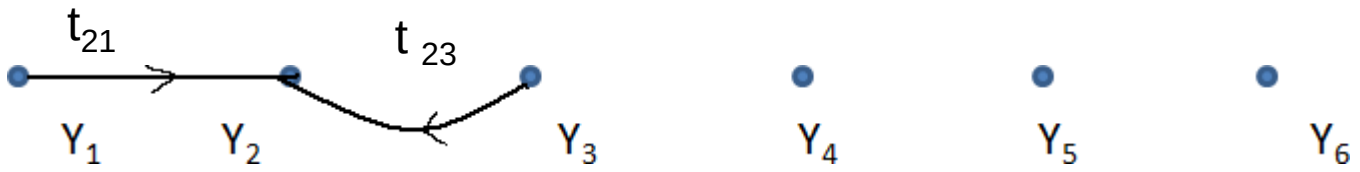
$$y_3 = t_{32}y_2 + t_{33}y_3 + t_{31}y_1$$

$$y_4 = t_{43}y_3 + t_{42}y_2$$

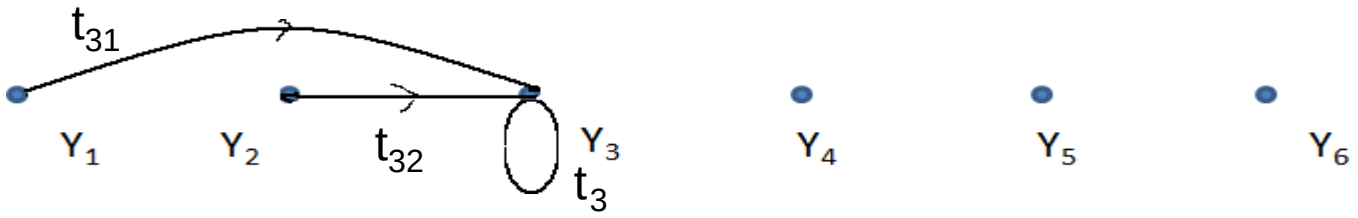
$$y_5 = t_{54}y_4$$

$$y_6 = t_{65}y_{51} + t_{64}y_4$$

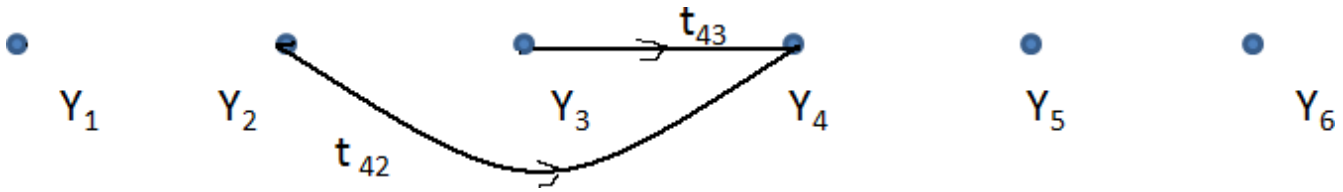
$$y_2 = t_{21}y_1 + t_{23}y_3$$



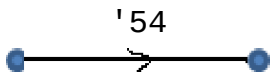
$$y_3 = t_{32}y_2 + t_{33}y_3 + t_{31}y_1$$



$$y_4 = t_{43}y_3 + t_{42}y_2$$

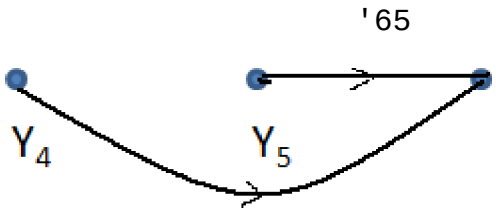


$$y_5 = t_{54} y_4$$



Y1 ’2 3 4 5 6

$$y_6 = t_{65} y_{51} + t_{64} y_4$$



Y1 2 3 ’6