- **Introduction**
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Antenna Array: Introduction

- **Array is an assembly of antenna elements arranged in an orderly fashion. The elements are usually identical.**
- **Why array? When high gain and/or narrow beam are required:**
  - Single element -> Wide beam (low directivity)
  - Increasing size -> difficult to build and expensive
  - Useful especially when the element gain is low.

Antenna Array: Introduction (2)

- **Advantages**
  - Higher directivity
  - Narrower beam
  - Lower sidelobes
  - Electronic steerable beam
- **Types**
  - Fix direction
  - Steerable : Mechanical or Electronic (phased arrays)

Antenna Array: Introduction (3)

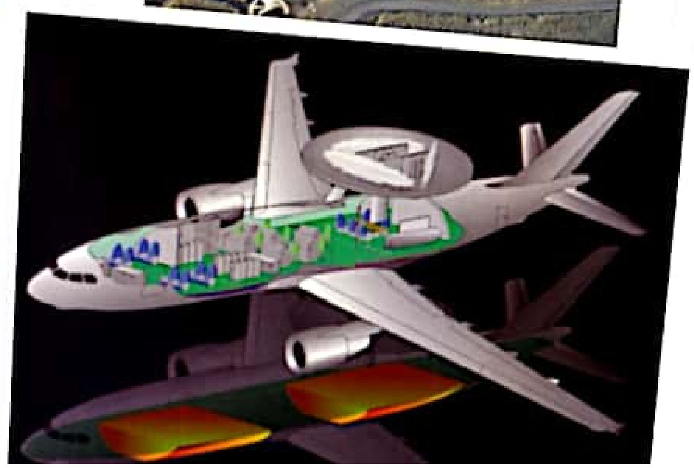
In an array of identical elements, there are in general five controls that can be used to shape the overall pattern of the antenna:

1. Geometrical configuration (linear, circular, etc.)
2. Relative displacement between elements
3. Excitation amplitude of individual elements
4. Excitation phase of individual elements
5. Relative pattern of individual elements

Examples

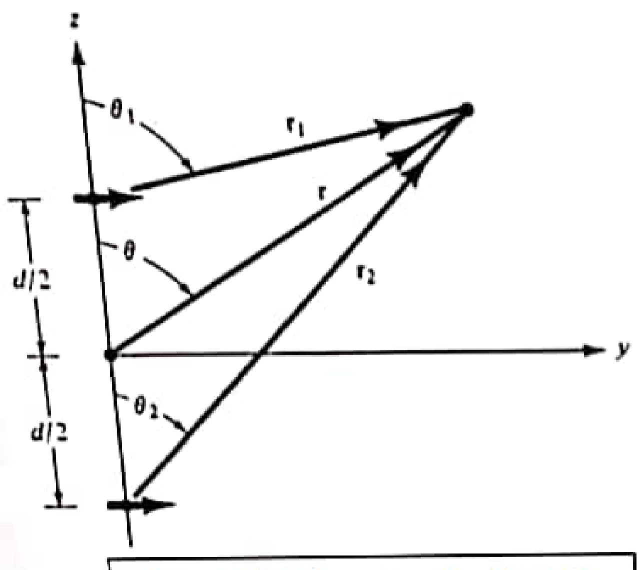
Very Large Antenna (VLA)

Airborne Warning and Control System (AWACS)

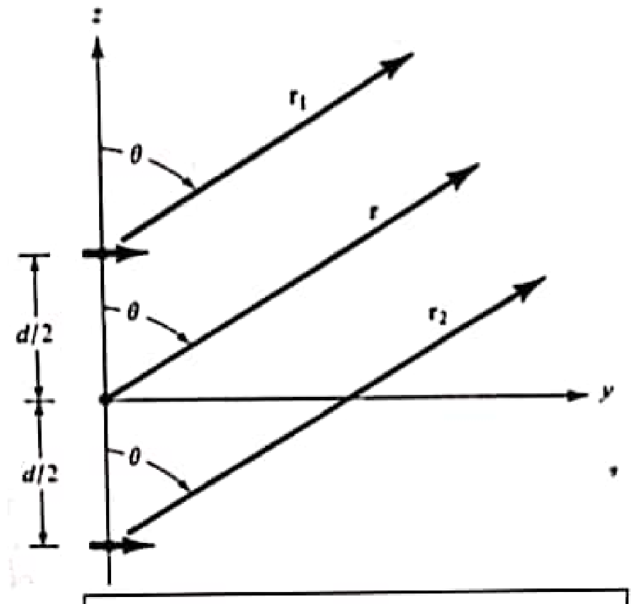


Two-element Array

Consider two-element array of horizontal infinitesimal dipoles (assume no coupling between elements)



Two infinitesimal dipoles



Far-field observation

Two-element Array (2)

Recall the far-zone electric field of horizontal infinitesimal dipole in the y-z plane

$$\mathbf{E} = \hat{\theta} jk\eta I_0 l \frac{e^{-jkr}}{4\pi r} \cos\theta$$

Thus the total electric field becomes:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \hat{\theta} jk\eta l \left(I_1 \frac{e^{-jkr_1}}{4\pi r_1} \cos\theta_1 + I_2 \frac{e^{-jkr_2}}{4\pi r_2} \cos\theta_2 \right)$$

Two-Element Array (3)

Using the far-field approximation

$$r_1 \cong r_2 \cong r \quad r_1 \cong r - \frac{d}{2}\cos\theta \quad r_2 \cong r + \frac{d}{2}\cos\theta$$

The total field becomes:

$$\mathbf{E} = \hat{\theta} jk\eta l \cos\theta \frac{e^{-jkr}}{4\pi r}\left(I_1 e^{jkd\cos\theta/2} + I_2 e^{-jkd\cos\theta/2}\right)$$

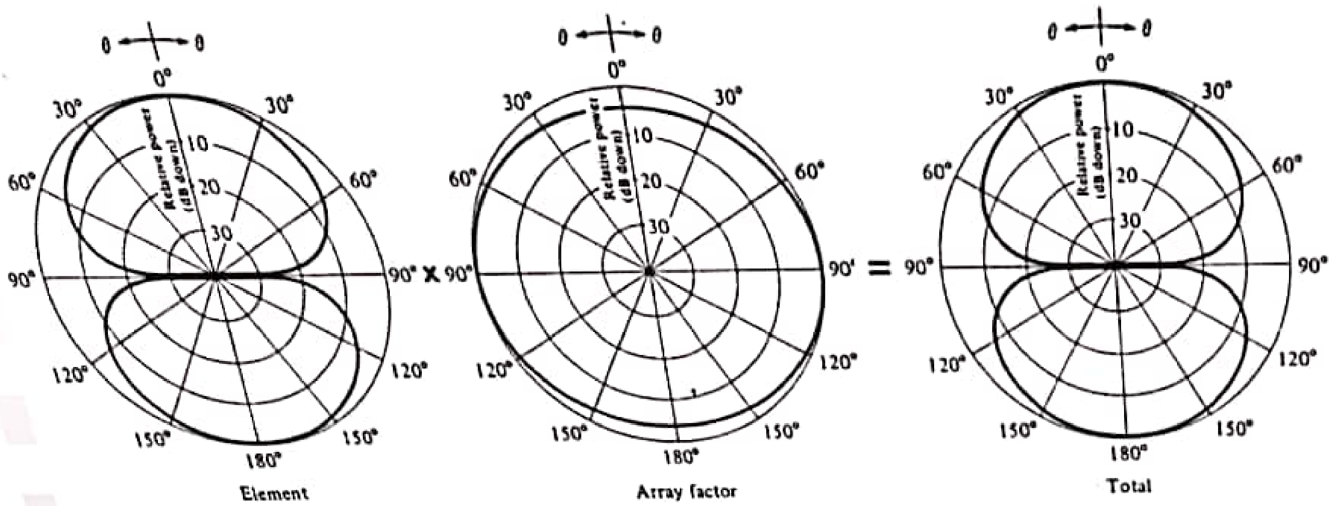
If $I_1 = I_0 e^{j\beta/2}; I_2 = I_0 e^{-j\beta/2}$, β: phase difference

$$\mathbf{E} = \hat{\theta} jk\eta I_0 l \cos\theta \frac{e^{-jkr}}{4\pi r} 2\cos(k\frac{d}{2}\cos\theta + \frac{\beta}{2})$$

$$\boxed{\text{total field} = (\text{element factor}) \times (\text{array factor(AF)})}$$

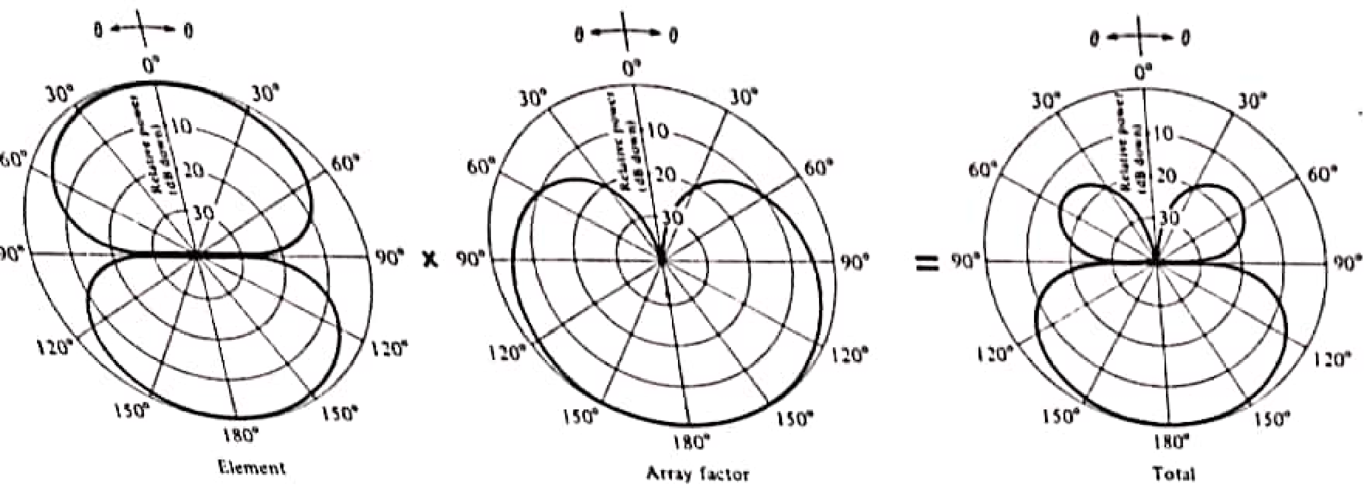
Electric Field Pattern

$\beta = 0, d = \lambda / 4$



Electric Field Pattern (2)

$\beta = \pi / 2, d = \lambda / 4$



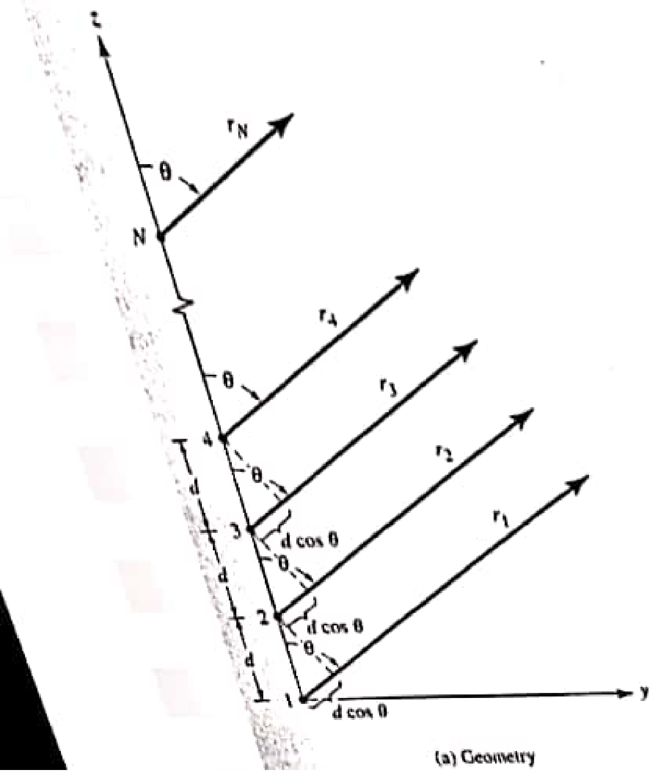
N-element Linear Array: Uniform amplitude & spacing

If the amplitude and spacing are both uniform, the array factor becomes

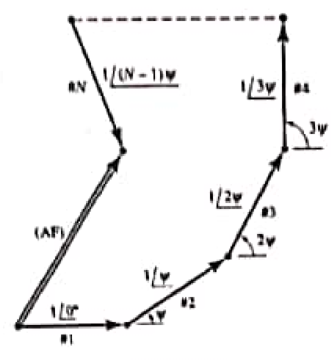
$$AF = 1 + e^{j(kd\cos\theta+\beta)} + e^{j2(kd\cos\theta+\beta)} + \cdots + e^{j(N-1)(kd\cos\theta+\beta)}$$

$$= \sum_{n=1}^{N} e^{j(n-1)\psi}$$

where $\psi = kd\cos\theta + \beta$



(a) Geometry



(b) Phasor diagram

thus

$$\mathrm{AF} = \frac{e^{jN\psi} - 1}{e^{j\psi} - 1} = e^{j\frac{N-1}{2}\psi} \frac{e^{j\frac{N}{2}\psi} - e^{-j\frac{N}{2}\psi}}{e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}}}$$

$$= e^{j\frac{N-1}{2}\psi} \frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\frac{\psi}{2}}$$

If the reference point is the physical center of the array

$$\mathrm{AF} = \frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\frac{\psi}{2}} \underset{\psi:\text{small}}{\cong} \frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{\psi}{2}}$$

Normalized AF

$$(\mathrm{AF})_n = \frac{\sin\left(\frac{N}{2}\psi\right)}{N\sin\frac{\psi}{2}} \underset{\psi:\text{small}}{\cong} \frac{\sin\left(\frac{N}{2}\psi\right)}{N\frac{\psi}{2}} = \operatorname{sinc}\left(\frac{N}{2}\psi\right)$$

Nulls

$$\sin\left(\frac{N}{2}\psi\right) = 0 \Rightarrow \frac{N}{2}\psi\Big|_{\theta=\theta_n} = \pm n\pi$$

$$\Rightarrow \theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]$$

$$n = 1,2,3,\ldots;\, n \neq N, 2N, 3N, \ldots$$

Maxima

$$\frac{\psi}{2}\Big|_{\theta=\theta_m} = \pm m\pi \Rightarrow \theta_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right]$$

$$m = 0,1,2,\ldots$$