Subject – Thermal Engineering and Gas Dynamics

Unit-4
Air Compressors

Lecture Notes

AIR COMPRESSORS

4.1 Introduction

Compression of air and vapour plays an important role in engineering fields. Compression of air is mostly used since it is easy to transmit air compared with vapour.

Uses of compressed air:

The applications of compressed air are listed below:

1) In gas turbines and propulsion units.
2) In pneumatic tools for concrete breaking, clay or rock drilling, caulking, riveting etc.
3) It is used in rotary type pneumatic tools for drilling, grinding, hammering etc.
4) Pneumatic lifts and elevators work by compressed air.
5) For Cleaning
6) In paint spray and spray guns as atomizer
7) Pile drivers, extractors, concrete vibrators require compressed air.
8) Air-operated brakes are used in railways and heavy vehicles such as buses and lorries.
9) Sand blasting operation for cleaning of iron castings needs compressed air.
10) It is used for blast furnaces and air-operated chucks.
11) Compressed air is used for starting I.C.engines and also super charging them.

4.2 Working Principle of Air Compressor

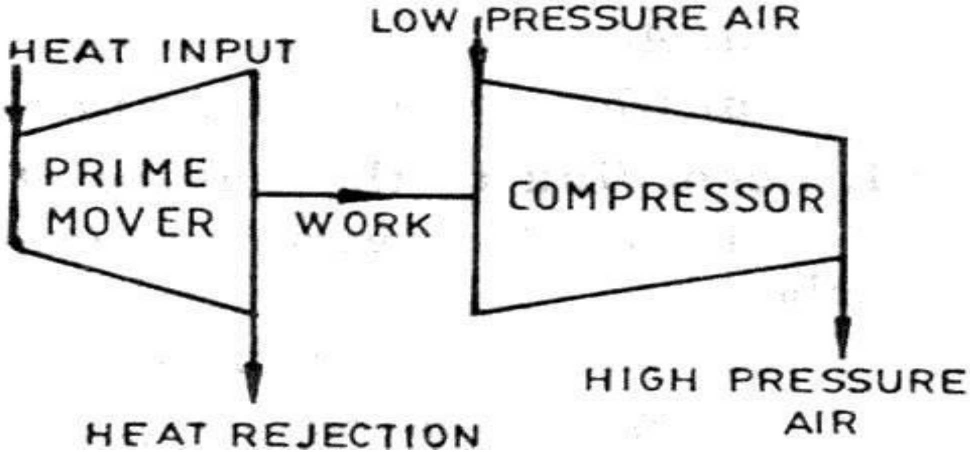


Fig.1.1Air Compressor

A line diagram of a compressor unit is shown in fig 4.1. The compression process requires work input. Hence a compressor is driven by a prime mover. Generally, an electric motor is used as prime mover. Air from atmosphere enters into the compressor It is compressed to a high pressure. Then, this high pressure air is delivered to a storage vessel (reservoir). From the reservoir, it can be conveyed to the desired place through pipe lines.

Some of the energy supplied by the prime mover is absorbed in work done against friction. Some portion of energy is lost due to radiation and coolant. The rest of the energy is maintained within the high pressure air delivered.

4.3 Classification of compressors:

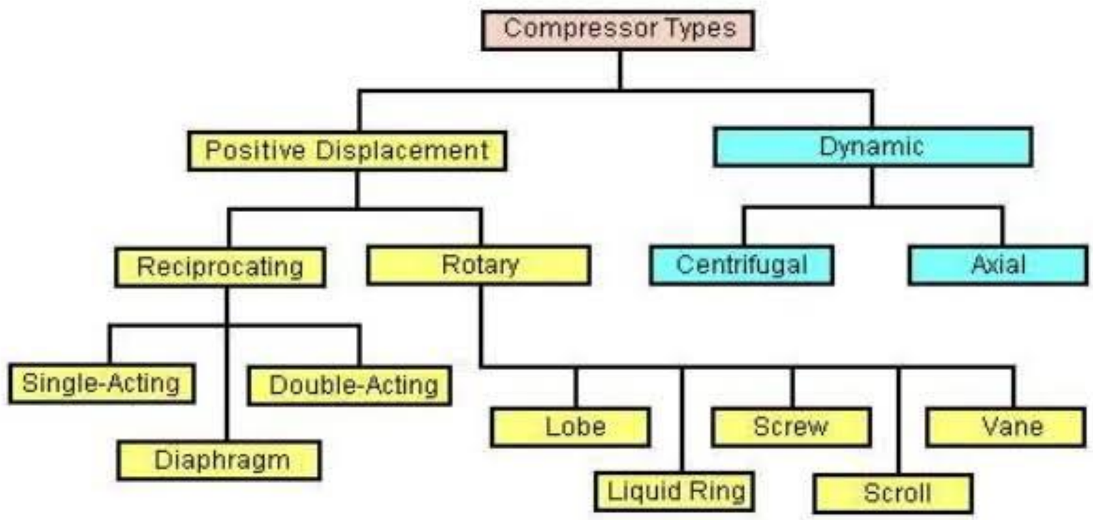


Fig.4.2Classification of Air Compressor

Air compressors may be classified as follows:

- **According to design and principle of operation:**
  (a) Reciprocating compressors in which a piston reciprocates inside the cylinder.
  (b) Rotary compressors in which a rotor is rotated.

- **According to number of stages:**
  (a) Single stage compressors in which compression of air takes place in one cylinder only.
  (b)Multi stage compressors in which compression of air takes place in more than one cylinder.

- **According to pressure limit:**
  (a) Low pressure compressors (delivery pressure is less than 10 bar)
  (b) Medium pressure compressor (delivery pressure is 10 bar to 80 bar)
  (c) High pressure compressors (delivery pressure is 80 to 100 bar)

- **According to capacity:**

(a) Low capacity compressor (delivers 0.15m /s of compressed air),
(b) Medium capacity compressor (delivers 5m/s of compressed air)
(c) High capacity compressor(more than 5m/s of compressed air).

- **According to method of cooling:**

  a)Air cooled compressor
  b) cooled compressor

- **According to the nature of installation:**

(a) Portable compressors
(b) Semi-fixed compressors
(c) Fixed compressors

- **According to applications:**

  (a) Rock drill compressors (used for drilling rocks),
  (b) Quarrying compressors (used in quarries),
  (c) Sandblasting compressors (used for cleaning of cast iron)
  (d) Spray painting compressors (used for spray painting).

- **According to number of air cylinders**

  (a) Simplex - contains one air cylinder
  (b) Duplex - contains two air cylinders
  (c) Triplex - contains three air cylinders

4.4 Reciprocating compressors

(a) Single acting compressors in which suction, compression and delivery of air (or gas) take place on one side of the piston.

(b) Double acting compressors in which suction, compression and delivery of air (or gas) take place on both sides of the piston.

4.5 Single stage reciprocating air compressor:

In a single stage compressor, the compression of air (or gas) takes place in a single cylinder. A schematic diagram of a single stage, single acting compressor is shown in fig.4.3.

Construction: It consists of a piston which reciprocates inside a cylinder. The piston is connected to the crankshaft by means of a connecting rod and a crank. Thus, the rotary movement of the crankshaft is converted into the reciprocating motion of the piston. Inlet and outlet valves (suction and delivery valves) are provided at the top of the cylinder.

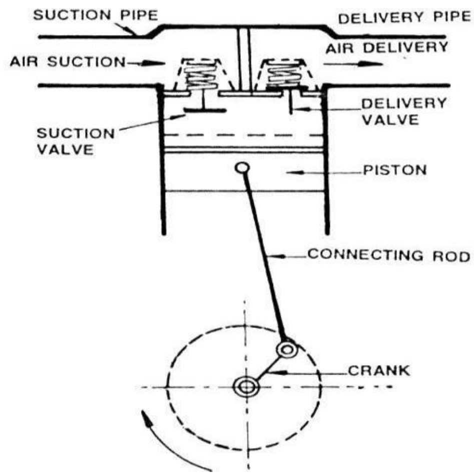


Fig 4.3 Reciprocating Compressor

Working: When the piston moves down, the pressure inside the cylinder is reduced. When the cylinder pressure is reduced below atmospheric pressure, the inlet valve opens. Atmospheric air is drawn into the cylinder till the piston reaches the bottom dead centre. The delivery valve remains closed during this period. When the piston moves up, the pressure inside the cylinder increases. The inlet valve is closed, since the pressure inside the cylinder is above atmospheric. The pressure of air inside the cylinder is increased steadily. The outlet valve is then opened and the high pressure air is delivered through the outlet valve in to the delivery pipe line.

At the top dead centre of the piston, a small volume of high pressure air is left in the clearance space. When the piston moves down again, this air is expanded and pressure reduces, Again the inlet valve opens and thus the cycle is repeated.

Disadvantages

i. High pressure air results in leakage through the piston.
ii. Cooling of the gas *is not effective.*
iii. Requires a stronger cylinder to withstand high delivery pressure.

Applications: It is used in places where the required pressure ratio is small.

4.6 Compression processes:

The air may be compressed by the following processes.

(a) Isentropic or adiabatic compression,
(b) Polytropic compression and
(c) Isothermal compression

(a)Isentropic(or)adiabatic compression*:*

In internal combustion engines, the air (or air fuel mixture) is compressed isentropically.

By isentropic compression, maximum available energy in the gas is obtained.

(b)Polytropic compression:

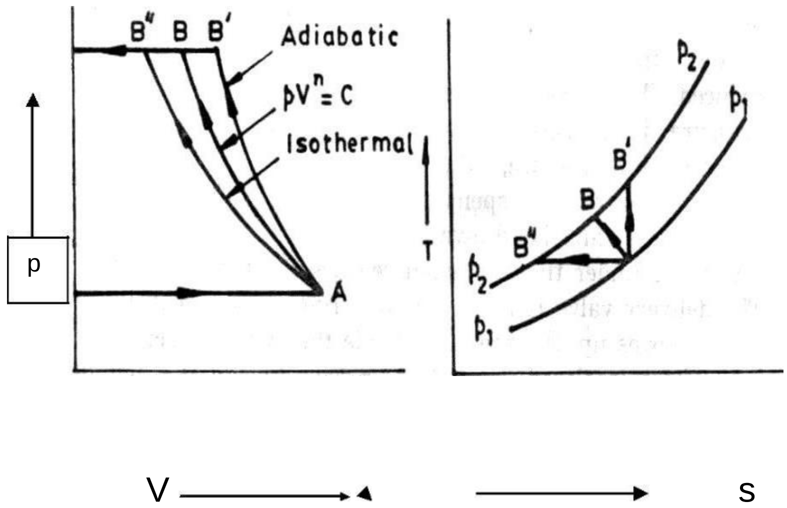


Fig: 4.4 Compression processes
A-B": Isothermal; A-B: Polytropic; A-B': Isentropic

The compression follows the law pV^n = Constant. This type of compression may be used in Bell-Coleman cycle of refrigeration.

(c)Isothermal compression:

When compressed air (or gas) is stored in a tank, it loses its heat to the surroundings. It attains the temperature of surroundings after some time. Hence, the overall effect of this compression process is to increase the pressure of the gas keeping the temperature constant. Thus isothermal compression is suitable if the compressed air (or gas) is to be stored.

4.7 Power required for driving the compressor:

The following assumptions are made in deriving the power required to drive the compressor.

1. There is no pressure drop through suction and delivery valves.
2. Complete compression process takes place in one cylinder.
3. There is no clearance volume in the compressor cylinder.
4. Pressure in the suction line remains constant. Similarly, pressure in the delivery line remains constant.
5. The working fluid behaves as a perfect gas.
6. There is no frictional losses.

The cycle can be analysed for the three different case of compression. Work required can be obtained from the p - V diagram.

Let,

P_1 = Pressure of the air (kN/m), before compression
V_1 = Volume of the air (m), before compression
T_1=Temperature of the air *(K),* before compression

P_2, V_2 and T_2 be the corresponding values after compression.

m - Mass of air induced or delivered by the cycle (kg). *N* - Speed in RPM.

4.7.1 Polytropic Compression

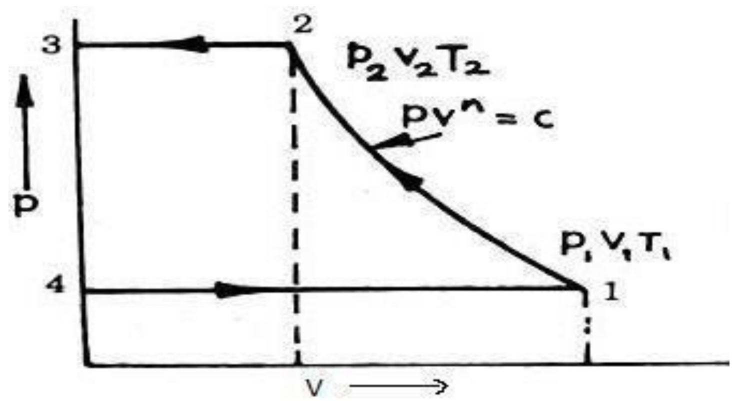


Fig:4.5 Polytropic compression

Let n= Index of polytropic compression

Net work done on air/cycle is given by

W = Area 1-2-3-4-1

= Work done during compression (1-2) + Work done during air delivery (2-3) - Work done during suction (4-1).

$$W = \frac{p_2v_2 - p_1v_1}{n-1} + p_2v_2 - p_1v_1$$

$$W = \frac{p_2v_2 - p_1 + (n-1)p_2v_2 - (n-1)p_1v_1}{n-1}$$

$$= \frac{np_2v_2 - np_1v_1}{n-1} = \left(\frac{n}{n-1}\right) p_2v_2 - p_1v_1$$

We know that, $p_1V_1 = m\,RT_1$ & $p_2V_2 = m\,RT_2$

Therefore, $\mathbf{W = \frac{n}{n-1}\,m\,R\,(T_2 - T_1)}$

$$\mathbf{W = \frac{n}{n-1}\,m\,R\,T_1 \left[\frac{T_2}{T_1} - 1\right]}$$

For polytropic process, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$

Therefore, $W = \frac{n}{n-1}\,m\,R\,T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$ kJ/cycle

$$W = \frac{n}{n-1}\,p_1\,V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \text{ kJ/cycle}$$

Indicated power (or) Power required,

P = W x N , kW for single acting reciprocating compressor;

P= W x 2N, kW for double acting reciprocating compressor.

4.7.2 Isentropic compression

Compression follows, pV^{γ}= Constant

Let γ = Index of isentropic compression

Net work done on air/cycle is given by

W = Area 1-2-3-4-1

= Work done during compression (1-2) + Work done during air delivery (2-3) - Wc done during suction (4-1).

$$W = \frac{p_2v_2 - p_1v_1}{\gamma - 1} + p_2v_2 - p_1v_1$$

$$W = \frac{p_2v_2 - p_1 + (\gamma-1)p_2v_2 - (\gamma-1)p_1v_1}{\gamma - 1}$$

$$= \frac{\gamma p_2v_2 - \gamma p_1v_1}{\gamma - 1} = \left(\frac{\gamma}{\gamma-1}\right) p_2v_2 - p_1v_1$$

We know that, $p_1V_1 = m\,RT_1$ & $p_2V_2 = m\,RT_2$

$$\mathbf{W = \frac{\gamma}{\gamma-1}\, m\, R\, (T_2 - T_1)}$$

$$\mathbf{W = \frac{\gamma}{\gamma-1}\, m\, R\, T_1 \left[\frac{T_2}{T_1} - 1\right]}$$

For isentropic process, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$

Therefore, $W = \frac{\gamma}{\gamma-1}\, m\, R\, T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$ kJ/cycle

$$W = \frac{\gamma}{\gamma-1}\, p_1\, V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \text{ kJ/cycle}$$

4.7.3 Isothermal Compression

Compression follows, pV= Constant

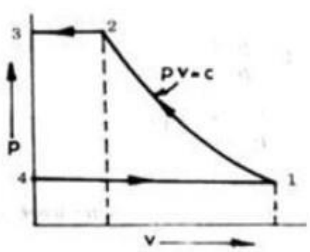


Fig: 4.5 Isothermal Compression

Isothermal Work input, W = Area 1-2-3-4-1 = area under 1-2 + area under 2-3 - area under 4-1

$$W = p_1V_1\, ln\left(\frac{V_1}{V_2}\right) + p_2V_2 - p_1V_1$$

But $p_1V_1 = p_2V_2$

$$W = p_1V_1 \ ln\left(\frac{V_1}{V_2}\right) \qquad and \ \frac{V_1}{V_2} = \frac{p_2}{p_1}$$

Therefore, $W = p_2V_2 \ ln\left(\frac{p_2}{p_1}\right)$ kJ/cycle

4.8 Isothermal efficiency: Isothermal efficiency is defined as the ratio of isothermal work input to the actual work input. This is used for comparing the compressors.

Isothermal efficiency, $\eta_{iso} = \frac{Isothermal\ work\ input}{Actual\ work\ output}$

4.9 Adiabatic efficiency: Adiabatic efficiency is defined as the ratio of adiabatic work input to the actual work input. This is used for comparing the compressors.

Adiabatic efficiency, $\eta_{adia} = \frac{Adiabatic\ work\ input}{Actual\ work\ output}$

4.10 Mechanical efficiency:

The compressor is driven by a prime mover. The power input to the compressor is the shaft power (brake power) of the prime mover. This is also known as brake power of the compressor.

Mechanical efficiency is defined as the ratio of indicated power of the compressor to the power input to the compressor.

$$\eta_m = \frac{\text{Indicated power of compressor}}{\text{Power input}}$$

Indicated Power, IP $= \frac{p_m l\, aNk}{60}$,

where, p_m = mean effective pressure, kN/m^2

l = length of stroke of piston, m

a = area of cross section of cylinder, m^2

N= crank speed in rpm, and

K =number of cylinders

4.11 Clearance and clearance volume:

When the piston reaches top dead centre (TDC) in the cylinder, there is a dead space between piston top and the cylinder head. This space is known as clearance space and the volume occupied by this space is known as clearance volume, Vc.

The clearance volume is expressed as percentage of piston displacement. Its value ranges from 5% - 10% of swept volume or stroke volume (Vs).The p - V diagram for a single stage compressor, considering clearance volume is shown in fig. . At the end of delivery of high pressure air (at point 3), a small amount of high pressure air at p2 remains in the clearance space. This high pressure air which remains at the clearance space when the piston is at TDC is known as remnant air. It is expanded polytropically till atmospheric pressure (p4=p1) is reached. The inlet valve is opened and the fresh air is sucked into the cylinder. The suction of air takes place for the rest of stroke (upto point 1). The volume of air sucked is known as effective suction volume (V1 - V4). At point 1, the air is compressed polytropically till the delivery pressure (p2) is reached. Then the delivery valve is opened and high pressure air is discharged into the receiver. The delivery of air continues till the piston reaches its top dead centre, then the cycle is repeated.

4.11.1 Effect of clearance volume:

The following are the effects of clearance space.

1. Suction volume (volume of air sucked) is reduced.
2. Mass of air is reduced.
3. If clearance volume increases, heavy compression is required.
4. Heavy compression increases mechanical losses

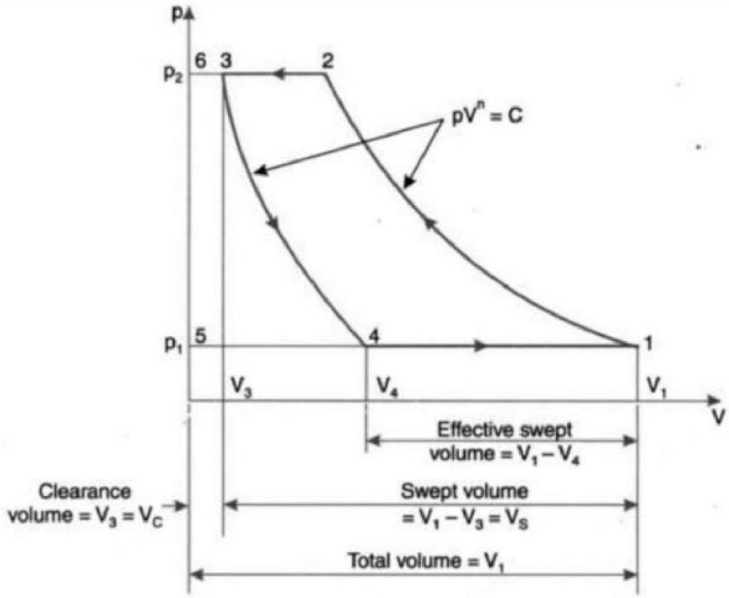


Fig: 4.6 p-V diagram with clearance volume

4.11.3 Work input considering clearance volume:

Assuming the expansion (3-4) and compression (1-2) follow the law $p V^n = C$,
Work input per cycle is given by,
W = Area (1-2-3-6-5-4-1)- Area (3-6-5-4 -3)
W = Workdone during compression - Work done during expansion

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] - \frac{n}{n-1} p_4 V_4 \left[\left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} - 1\right]$$

But, $p_3 = p_2$ and $p_4 = p_1$
therefore

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] - \frac{n}{n-1} p_1 V_4 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W = \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] \text{ kJ/cycle}$$

V_1-V_4 is called as effective suction volume.

4.12 Volumetric efficiency:

The clearance volume in a compressor reduces the intake capacity of the cylinder. This leads to a term called volumetric efficiency.

The volumetric efficiency is denned as the volume of free air sucked into the compressor per cycle to the stroke volume of the cylinder,the volume measured at the intake pressure and temperature or at standard atmospheric conditions,(p_s =101.325 kN/m^2 and T_s = 288K)

$$\text{Volumetric efficiency, } \eta_{vol} = \frac{\text{Volume of free air taken in per cycle}}{\text{Stroke volume of the cylinder}}$$

$$= \frac{\text{Effective suction volume}}{\text{Swept volume}} = \frac{(V1 - V4)}{(V1 - V3)} = \frac{V_1 - V_4}{V_s}$$

Clearance ratio: Clearance ratio is defined as, the ratio of clearance volume to swept volume It is denoted by the letter C.

$$\text{Clearance ratio,C} = \frac{\textit{Clearance volume}}{\textit{Swept volume}} = \frac{V_c}{V_s} = \frac{V_c}{V_{1-V_3}}$$

$$\text{Pressure ratio, } R_p = \frac{\textit{Delivery pressure}}{\textit{Suction pressure}} = \frac{p_2}{p_1} = \frac{p_3}{p_4}$$

4.12.1 Expression for Volumetric efficiency

Let the compression and expansion follows the law, pV^n =Constant.

$$\text{Clearance ratio, C} = \frac{\textit{Clearance volume}}{\textit{Swept volume}} = \frac{V_c}{V_s} = \frac{V_3}{V_{1-V_3}}$$

$V_1 - V_3 = \frac{V_3}{C}$ -------------(1)

$V_1 = \frac{V_3}{C} + V_3$

$V_1 = V_3\left(\frac{1}{C} + 1\right)$ --------------------- (2)

We know that, Pressure ratio, $R_p = \frac{Delivery\ pressure}{Suction\ pressure} = \frac{p_2}{p_1} = \frac{p_3}{p_4}$

By polytropic expansion process 3-4:

$$\frac{p_3}{p_4} = \left(\frac{V_4}{V_3}\right)^n$$

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{1/n} = (R_p)^{\frac{1}{n}}$$

Therefore, $V_4 = V_3(R_p)^{\frac{1}{n}}$ ----------------- (3)

Volumetric efficiency, $\eta_{vol} = \frac{\text{Effective suction volume}}{\text{Swept volume}} = \frac{(V_1 - V_4)}{(V_1 - V_3)}$ ------------ (4)

Using equations 1,2 and 3 in 4,

$$\eta_{vol} = \frac{V_3\left(\frac{1}{C}+1\right) - V_3[R_p]^{1/n}}{\frac{V_3}{C}} = \frac{V_3\left\{\left(\frac{1}{C}+1\right) - [R_p]^{1/n}\right\}}{V_3\left(\frac{1}{C}\right)} = \frac{\left\{\left(\frac{1}{C}+1\right) - [R_p]^{1/n}\right\}}{\left(\frac{1}{C}\right)} = C\left[\left(\frac{1}{C} + 1\right) - [R_p]^{1/n}\right]$$

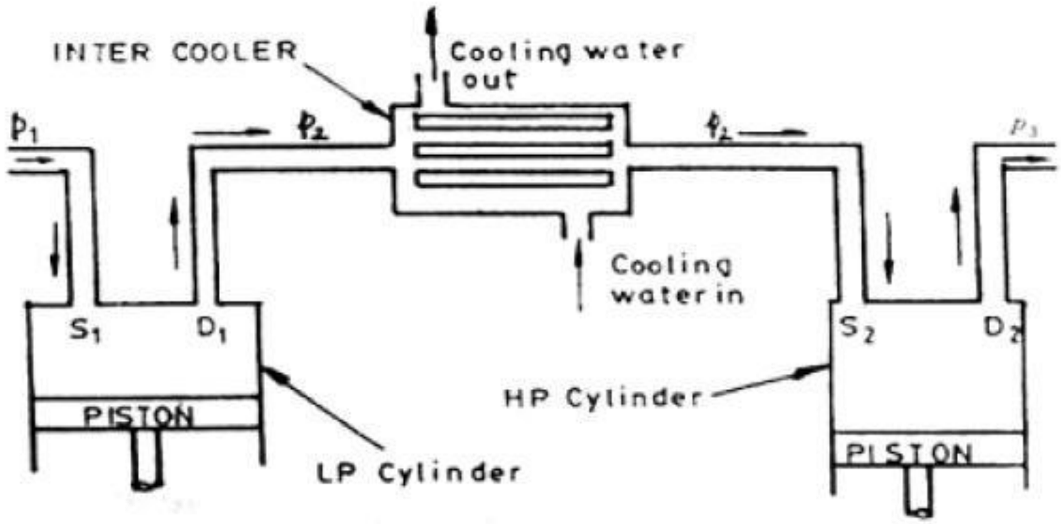
$$\eta_{vol} = 1 + C - C[R_p]^{1/n} = 1 + C - C\left[\frac{p_2}{p_1}\right]^{1/n}$$

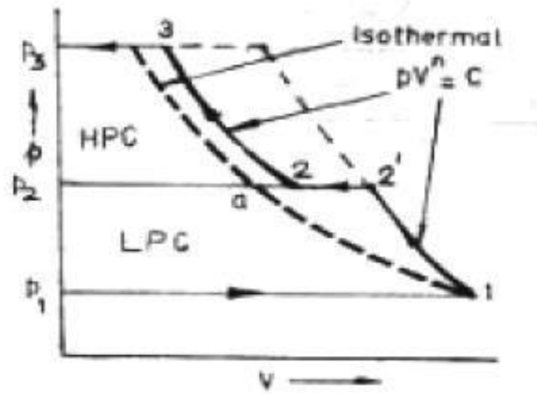
4.13 Multi-stage air compressor:

In a multi stage air compressor, compression of air takes place in more than one cylinder. Multi stage air compressor is used in places where high pressure air is required. Fig. shows the general arrangement of a two-stage air compressor. It consists of a low pressure (L.P) cylinder, an intercooler and a high pressure (H.P) cylinder. Both the pistons (in L.P and H.P cylinders) are driven b y a single prime mover through a common shaft.

Atmospheric air at pressure p1 taken into the low pressure cylinder is compressed to a high pressure (p2). This pressure is intermediate between intake pressure (p1) and delivery pressure p 3). Hence this is known as intermediate pressure.

The air from low pressure cylinder is then passed into an intercooler. In the intercooler, the air is cooled at constant pressure by circulating cold water. The cooled air from the intercooler is then taken into the high pressure cylinder. In the high pressure cylinder, air is further compressed to the final delivery pressure (p3) and supplied to the air receiver tank.





Advantages:

1. **Saving in work input**: The air is cooled in an intercooler before entering the high pressure cylinder. Hence less power is required to drive a multistage compressor as compared to a single stage compressor for delivering same quantity of air at the same delivery pressure.

2. **Better balancing**: When the air is sucked in one cylinder, there is compression in the other cylinder. This provides more uniform torque. Hence size of the flywheel is reduced.

3. **No leakage and better lubrication**: The pressure and temperature ranges are kept within desirable limits. This results in a) Minimum air leakage through the piston of the cylinder and b) effective lubrication due to lower temperature.

4. **More volumetric efficiency**: For small pressure range, effect of expansion of the remnant air (high pressure air in the clearance space) is less. Thus by increasing number of stages, volumetric efficiency is improved.

5. **High delivery pressure**: The delivery pressure of air is high with reasonable volumetric efficiency.

6. **Simple construction of LP cylinder**: The maximum pressure in the low pressure cylinder is less. Hence, low pressure cylinder can be made lighter in construction.

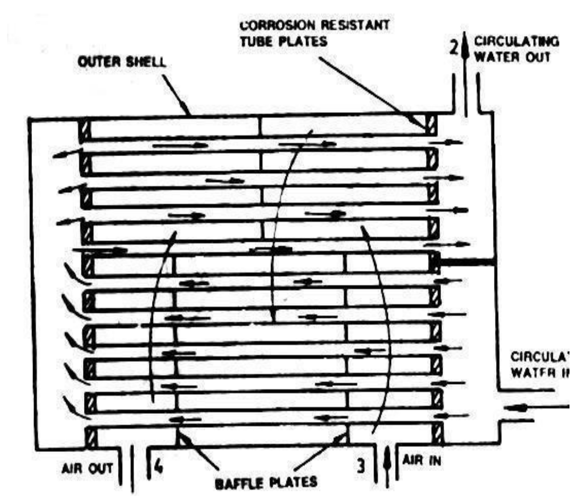
7. **Cheaper materials**: Lower operating temperature permits the use of cheaper materials for construction.

Disadvantages:

1. More than one cylinder is required.
2 An intercooler is required. This increases initial cost. Also space required is more.
3. Continuous flow of cooling water is required.
4. Complicated in construction.

4.14 Intercoolers:

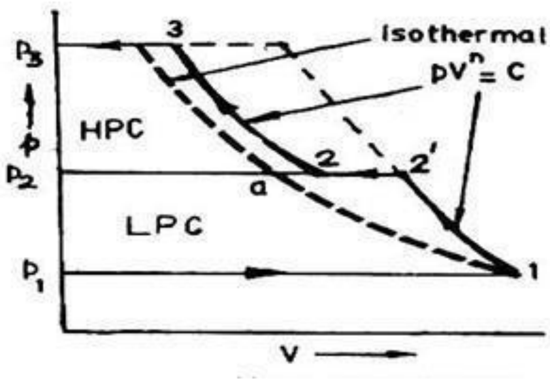
An intercooler is a simple heat exchanger. It exchanges the heat of compressed air from the LP compressor to the circulating water before the air enters the HP compressor. It consists of a number of special metal tubes connected to corrosion resistant plates at both ends. The entire nest of tubes is covered by an outer shell



Working: Cold water enters the bottom of the intercooler through water inlet (1) and flows into the bottom tubes. Then they pass through the top tubes and leaves through the water outlet (2) at the top. Air from LP compressor enters through the air inlet (3) of the intercooler and passes over the tubes. While passing over the tubes, the air is cooled (by the cold water circulated through the tubes). This cold air leaves the intercooler through the air outlet (4). Baffle plates are provided in the intercooler to change the direction of air. This provides a better heat transfer from air to the circulating water.

4.15 Work input required in multistage compressor:

The following assumptions are made for calculating the work input in multistage compression.



1. Pressure during suction and delivery remains constant in each stage.

2. Intercooling takes place at constant pressure in each stage.
3. The compression process is same for each stage.
4. The mass of air handled by LP cylinder and HP cylinder is same.
5. There is no clearance volume in each cylinder.

6 There is no pressure drop between the two stages, i.e., exhaust pressure of one stage is Work required to drive the multi-stage compressor can be calculated from the area of the p - V diagram .

Let, p1,V1 and T1 be the condition of air entering the LP cylinder. P2, V2 and T2 be the condition of air entering the HP cylinder. p3 be the final delivery pressure of air.

Then,

Total work input = Work input for LP compressor + Work input for HP compressor.

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] + \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right] \text{ kJ/cycle}$$

$$W = \frac{n}{n-1} m R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] + \frac{n}{n-1} m R T_2 \left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right] \text{ kJ/cycle}$$

If intercooling is perfect, $T_2 = T_1$, therefore,

$$W = \frac{n}{n-1} m R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] + \frac{n}{n-1} m R T_1 \left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right] \text{ kJ/cycle}$$

$$W = \frac{n}{n-1} m R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right] \text{ kJ/cycle}$$

4.16 Condition for maximum efficiency (or)
Condition for minimum work input (or)
To prove that for minimum work input the intermediate pressure of a two-stage compressor with perfect intercooling is the geometric mean of the intake pressure and delivery pressure (or)
To prove $p_2 = \sqrt{p_1 \, p_3}$

Work input for a two-stage air compressor with perfect intercooling is given by,

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2 \right] \text{ kJ/cycle}$$

If the initial pressure (**p_1**) and final pressure (**p_3**) are fixed, the value of intermediate pressure (**p_2**) can be determined by differentiating the above equation of work input in terms of **p_2** and equating it to zero.

Let, $\frac{n}{n-1} p_1 V_1 = k\ (constant)$ and $\frac{n-1}{n} = a$

then,

$$W = k \left[\left(\frac{p_2}{p_1}\right)^{a} + \left(\frac{p_3}{p_2}\right)^{a} - 2 \right]$$

or

$$W = k(p_2^a \, p_1^{-a} + p_3^a p_2^{-a} - 2) \quad \text{-------------------- (1)}$$

Differentiating the above equation (1) with respect to p_2 and equating it to zero,

$$\frac{dW}{dp_2} = k \, a \, p_2^{a-1} \, p_1^{-a} + k\,(-a) p_3^a \, p_2^{-a-1} = 0$$

$$k \, a \, \frac{p_2^a}{p_2 p_1^a} - k \, a \, p_3^a \, \frac{1}{p_2^a \, p_2} = 0$$

or

$$\frac{k \, a \, p_2^a}{p_2 \, p_1^a} = \frac{k \, a \, p_3^a}{p_2 \, p_2^a}$$

$$\left(\frac{p_2}{p_1}\right)^{a} = \left(\frac{p_3}{p_2}\right)^{a}$$

=> $p_2^2 = p_1 p_3$

or

intermediate pressure, $\mathbf{p_2} = \sqrt{\mathbf{p_1 p_3}}$

Thus for maximum efficiency the intermediate pressure is the geometric mean of the initial and final pressures.

4.17 Minimum work input for multistage compression with perfect intercooling:

Work input for a two-stage compressor with perfect intercooling is given by

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right] \text{ ----------------------- } (1)$$

Work input will be minimum if $\frac{p_2}{p_1} = \frac{p_3}{p_2}$ ----------------------------- (2)

$$p_2^2 = p_1\, p_3$$

Dividing both sides by p_1^2 ,

$$\left(\frac{p_2}{p_1}\right)^2 = \frac{p_3}{p_1} \quad \Rightarrow \frac{p_2}{p_1} = \left(\frac{p_3}{p_1}\right)^{1/2} \text{ ------------------------------- } (3)$$

From (2), $\frac{p_3}{p_2} = \frac{p_2}{p_1} = \left(\frac{p_3}{p_1}\right)^{1/2}$ -------------------------------------- (4)

Substituting the equation (4) in equation (1), work input for a two stage compressor,

$$W_{min} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_3}{p_1}\right)^{½\left[\frac{n-1}{n}\right]} + \left(\frac{p_3}{p_1}\right)^{½\left[\frac{n-1}{n}\right]} - 2\right]$$

$$= \frac{n}{n-1} p_1 V_1 \left[2\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 2\right]$$

$$W_{min} = \frac{2n}{n-1} p_1 V_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1\right]$$

or

$$W_{min} = \frac{2n}{n-1} mRT_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1\right]$$

For a three stage compressor,

$$W_{min} = \frac{3n}{n-1} p_1 V_1 \left[\left(\frac{p_4}{p_1}\right)^{\frac{n-1}{3n}} - 1\right]$$

or

$$W_{min} = \frac{3n}{n-1} mRT_1 \left[\left(\frac{p_4}{p_1}\right)^{\frac{n-1}{3n}} - 1\right]$$

Generally, the minimum work input for a multistage reciprocating air compressor with x number of stages is given by,

$$W_{min} = \frac{xn}{n-1} p_1 V_1 \left[\left(\frac{p_{x+1}}{p_1}\right)^{\frac{n-1}{xn}} - 1\right]$$

Minimum work input required for a two stage reciprocating air compressor with perfect intercooling is given by,

$$W_{min} = \frac{2n}{n-1} p_1 V_1 \left[\left(\frac{p_3}{p_1} \right)^{\frac{n-1}{2n}} - 1 \right] kJ$$

But,from equation (4), $\left(\frac{p_3}{p_1}\right)^{1/2} = \frac{p_2}{p_1}$

Therefore,

$$\boldsymbol{W_{min} = \frac{2n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] kJ}$$

So, for maximum efficiency ie., for minimum work input, the work required for each stage is same.

For maximum efficiency, the following conditions must be satisfied:

1. The air is cooled to the initial temperature between the stages (Perfect cooling between stages).
2. In each stage, the pressure ratio is same. $\left(\frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \cdots\right)$
3. The work input for each stage is same.

4.18 Rotary compressors:

Rotary compressors have a rotor to develop pressure. They are classified as
(1) Positive displacement compressors and (2) Non positive displacement (Dynamic) compressors

In positive displacement compressors, the air is trapped in between two sets of engaging surfaces. The pressure rise is obtained by the back flow of air (as in the case of Roots blower) or both by squeezing action and back flow of air (as in the case of vane blower). Example: (1) Roots blower, (2) Vane blower, (3) Screw compressor.
In dynamic compressors, there is a continuous steady flow of air. The air is not positively contained within certain boundaries. Energy is transferred from the rotor of the compressor to the air. The pressure rise is primarily due to dynamic effects. Example: (1) Centrifugal compressor, (2) Axial flow compressor.

4.18.1 Roots blower: The Roots blower is a development of the gear pump.

Construction: It consists of two lobed rotors placed in separate parallel axis of a casing as shown in fig:4.11. The two rotors are driven by a pair of gears (which are driven by the prime mover) and they revolve in opposite directions. The lobes of the rotor are of cycloid shape to ensure correct mating. A small clearance of 0.1 mm to 0.2 mm is provided between the lobe and casing. This reduces the wear of moving parts.
Working: When the rotor is driven by the gear, air is trapped between the lobes and the casing. the trapped air moves along the casing and discharged into the

receiver. There is no increase in pressure since the flow area from entry to exit remains constant. But, when the outlet is opened, there is a back flow of high pressure air in the receiver. This creates the rise in pressure of the air delivered. These types of blowers are used in automobiles for supercharging.

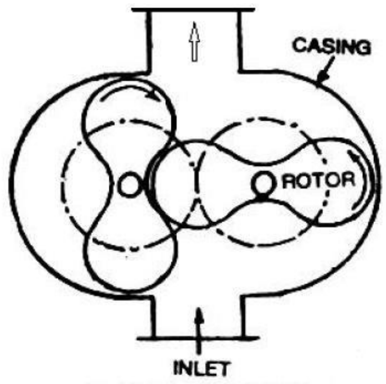


Fig:4.11 Roots blower

Construction: A vane blower consists of (1) a rotor, (2) vanes mounted on the rotor, (3) inlet and outlet ports and (4) casing. The rotor is placed eccentrically in the outer casing. Concentric vanes (usually 6 to 8 nos.) are mounted on the rotor. The vanes are made of fiber or carbon. Inlet suction area is greater than outlet delivery area.

Vane blower

Working: When the rotor is rotated by the prime mover, air is entrapped between two consecutive vanes. This air is gradually compressed due to decreasing volume between the rotor and the outer casing. This air is delivered to the receiver. This partly compressed air is further increased in pressure due to the back flow of high pressure air from the receiver.

Advantages: 1. Very simple and compact, 2. High efficiency 3. Higher speeds are possible

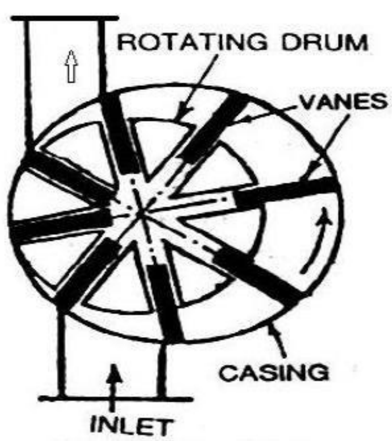


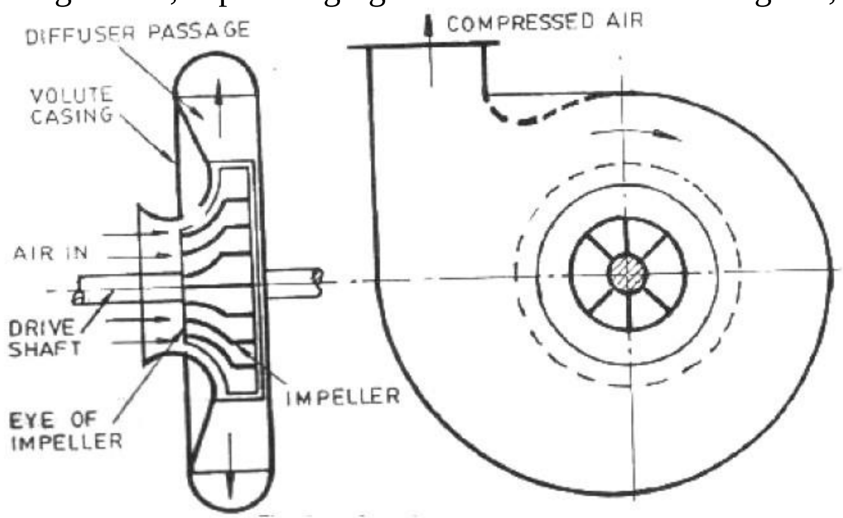
Fig: 4.12 Vane blower

4.18.3 Centrifugal compressor

Construction: It consists of an impeller, a casing and a diffuser. The impeller consists of a number of blades or vanes, is mounted on the compressor shaft inside the casing. The impeller is surrounded by the casing.

Working: In this compressor air enters axially and leaves radially. When the impeller rotates, air enters axially through the eye of the impeller with a low velocity. This air moves over the impeller vanes. Then, it flows radially outwards from the impeller. The velocity and pressure increases in the impeller. The air then enters the diverging passage known as diffuser. In the diffuser, kinetic energy is converted into pressure energy and the pressure of the air further increases. It is shown in fig:4.14. Finally, high pressure air is delivered to the receiver. Generally half of the total pressure rise takes place in the impeller and the other half in the diffuser.

Applications: Centrifugal compressors are used for low pressure units such as for refrigeration, supercharging of internal combustion engines, etc.



4.18.4 Axial flow compressor

In this air compressor, air enters and leaves axially.

Construction: It consists of two sets of blades: Rotor blades and stator blades. The blades are so arranged that the unit consists of adjacent rows of rotor blades and stator blades as shown in fig:4.15. The stator blades are fixed to the casing. The rotor blades are fixed on the rotating drum. The drum is rotated by a prime mover through a driving shaft. Single stage compressor consists of a row of rotor blades followed by a row of stator blades. Compression of air takes place in each pair of blades (one rotor blade and one stator blade). Hence there are many stages of compression in this type of compressor.

Working: When the switch is switched on, the prime mover rotates the drum. Air enters through the compressor inlet and passes through the rotor and stator blades. While passing

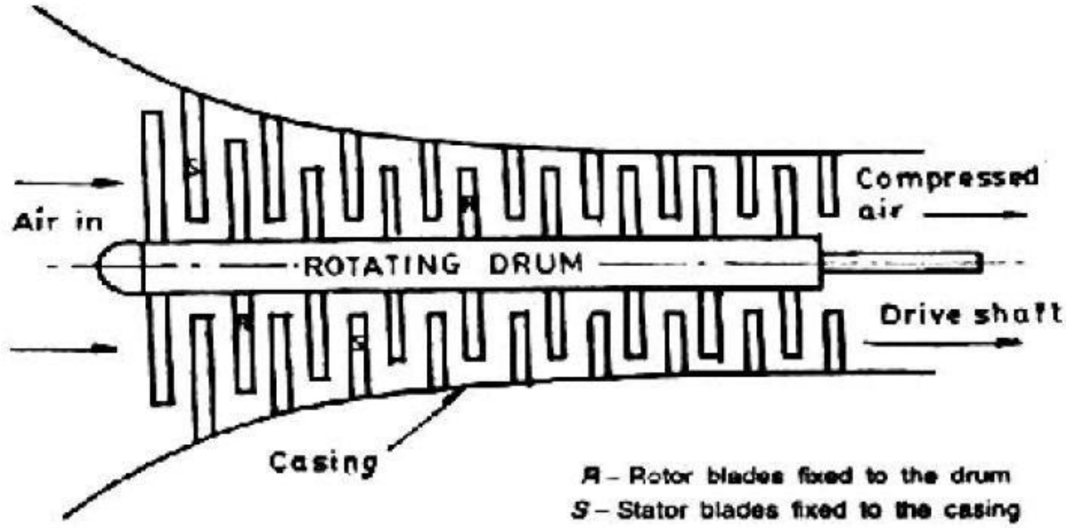


Fig:4.15 Axial flow compressor

Applications:

1. They are widely used in high pressure units such as industrial and marine gas turbine plants,
2. They are most suitable for aircraft work (Jet propulsion) since they require less frontal area.