Wire frame modelling

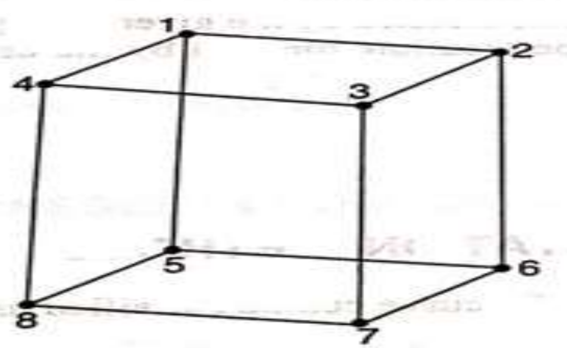
1. A wire frame model consists of points and curves only.
2. A wire frame model consists of 2 tables the vertex table and edge table.
3. A wire frame model does not face information.
4. ***Example*** : Representing a cube defined by 8 vertices and 12 edges

Vertex

Vertex No.	*X*	*Y*	*Z*
1	1	1	1
2	1	−1	1
3	−1	−1	1
4	−1	1	1
5	1	1	−1
6	1	−1	−1
7	−1	−1	−1
8	−1	−1	−1

Edge

Edge No.	*Start Vertex*	*End Vertex*
1	1	2
2	2	3
3	3	4
4	4	1
5	5	6
6	6	7
7	7	8
8	8	5
9	1	5
10	2	6
11	3	7
12	4	8



A cube.

Advantages & Dis-advantages of wire frame modelling

Curves plays important role in generating a wire frame modelling

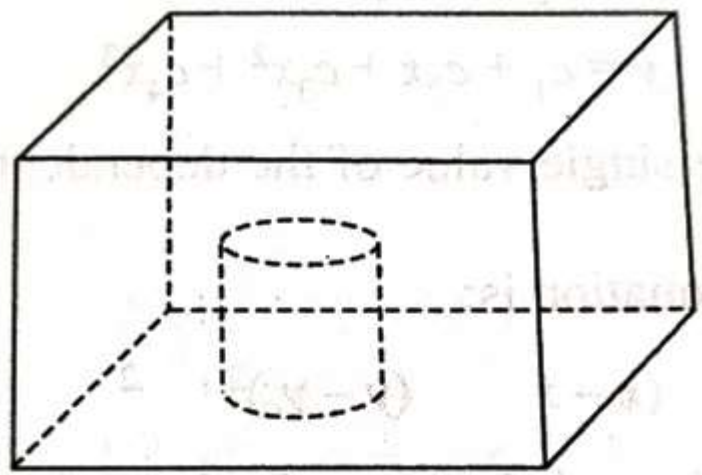
Advantages :

1. Ease of creation.
2. Low level of hardware and software requirements.
3. Data storage requirement is low.

Dis-advantages:

1. It can very confusing to visualise.

example : A blind hole in a box may be look like a solid cylinder



A wireframe model of a solid object with a blind hole.

Classification of wire frame entities

Curves are classified as

1. **Analytical curves:**

- This types of curve can be represented by a **simple mathematical equation** such as circle or an line
- It has a fixed form **& cannot be modified to achieve a shape that violates the mathematical equations**
- The analytical curves are :

1. line
2. arc
3. circle
4. ellipse
5. parabola
6. Hyperbola

2.Synthetic curves:

- An interpolated curve is drawn by **interpolating the given data points** and has a fixed form, dictated by the given data points.
- These curves **have some limited flexibility in shape creation**, dictated by the given data points. The synthetic curves are:

1. Hermite cubic spine or parametric cubic curve or cubic spline.
2. Bezier.
3. B- spline.

Curves representation methods

The mathematical representation of a curves can be classified as:

1. **Non- parametric**
- Explicit
- Implicit
2. **Parametric**

Non- Parametric Representation:

a) The **explicit non- parametric equation** is given by :

$$Y = C_1+C_2X+C_3X^2+C_4X^3$$

- **I**n this equation , there is a unique single value of the dependent variable for each value of the independent variable.

b) The implicit non-parametric equation is:

$$(X-X_C)^2 + (Y-Y_C)^2 = r^2$$

- In this equation , no difference is made between the dependent & the independent variable.

Limitations of non-parametric representation are:

1. Explicit non- parametric representation is based on one – to- mapping.
2. This cannot be used for representation of closed curves such as circle or multi- valued curves such as parabola.
3. If the gradient or slope of the curve at a point is vertical , its value is infinity, which cannot be incorporated in the computer programming.

Representation of curves

Types of Curve Equations

- Explicit (non-parametric) $Y = f(X), Z = g(X)$
-
- Implicit (non-parametric) $f(X,Y,Z) = 0$
- Parametric

$X = X(t), Y = Y(t), Z = Z(t)$

Analytic Curves vs. Synthetic Curves

- Analytic Curves are points, lines, arcs and circles, fillets and chamfers, and conics (ellipses, parabolas, and hyperbolas)

- Synthetic curves include various types of splines (cubic spline, B-spline, Beta-spline) and Bezier curves.

Parametric representation of Analytic curves

1. Line

A line is defining by connecting two points P_1 & P_2 . A parameter u is defined such that it has the values 0 & 1 at P_1 & P_2 respectivel

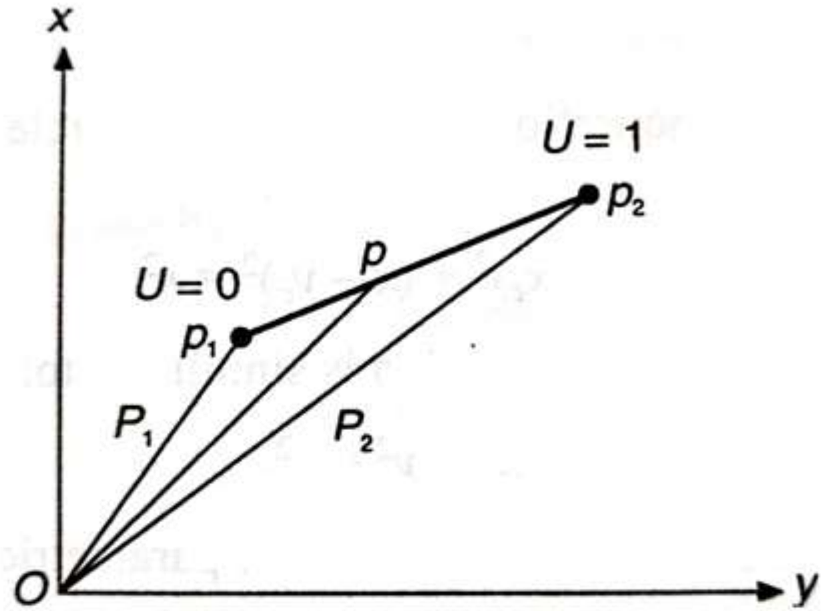
the equation of line is given by:

$$P = P_1 + u (P_1 - P_2) \quad 0 \le u \le 1$$

The length of the line is given by :

L

the unit vector is given by : n = P_2-P_1/2



Line connecting two points.

The equation of the line is given by:

$$P = P_1 + u\,(P_2 - P_1) \qquad 0 \le u \le 1$$

The length of the line is given by:

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The unit vector is given by:

$$n = \frac{P_2 - P_1}{L}$$

In matrix form,

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} x_1 \\ y_1 \\ z_1 \end{Bmatrix} + u \left[\begin{Bmatrix} x_2 \\ y_2 \\ z_2 \end{Bmatrix} - \begin{Bmatrix} x_1 \\ y_1 \\ z_2 \end{Bmatrix} \right]$$

2.Circle

- A circle with centre **(X_C, Y_C)** & radius r has an equation as follows:

$$(X-X_C)^2 + (Y-Y_C)^2 = r^2$$

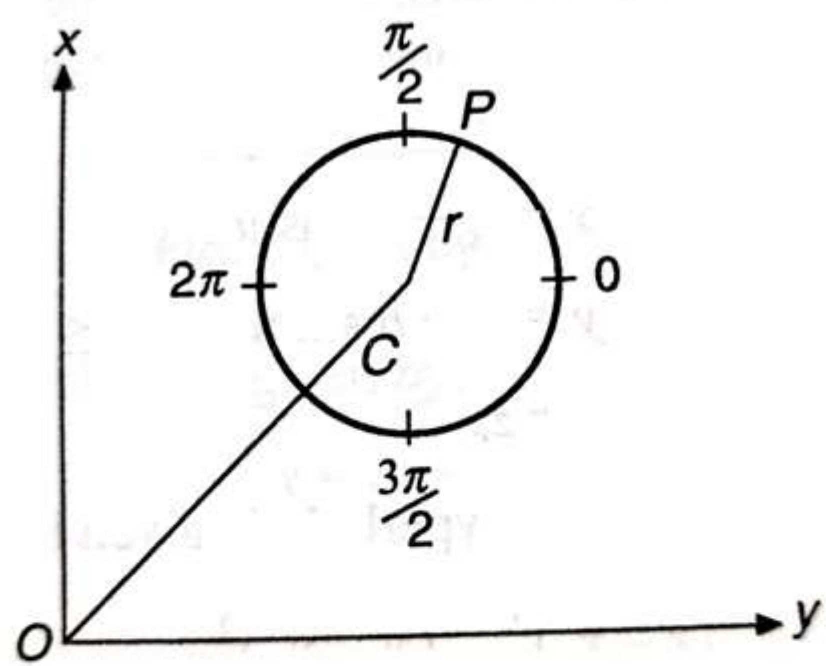
- If the centre is the origin , the above equation is simplified to :

$$X^2 + Y^2 = r^2$$

- From the above equation referred to as the non- parametric (implicit) form of the circle.

Parametric form of circle is:

$$X = x_c + r \cos u$$
$$Y = y_c + r \sin u \qquad 0 \le u \le 1$$
$$Z = Z_c$$



Circle defined by centre and radius.

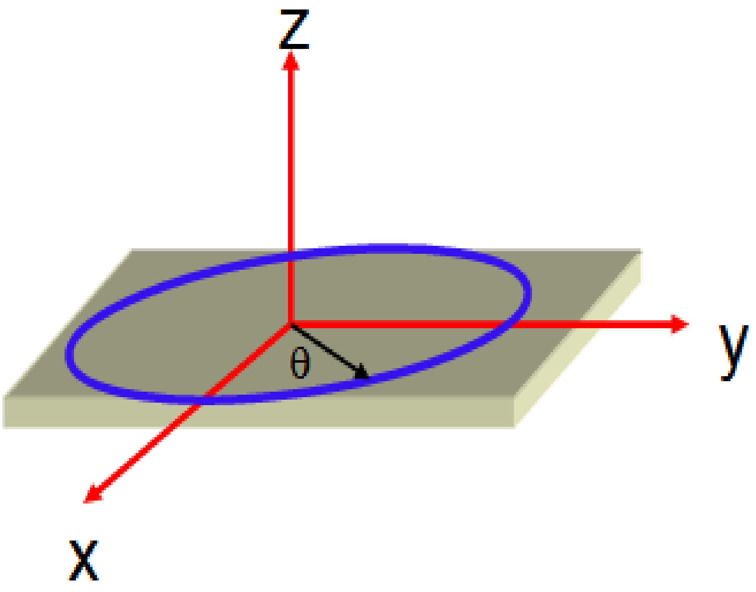
Example: A Circle of radius R

- Implicit:

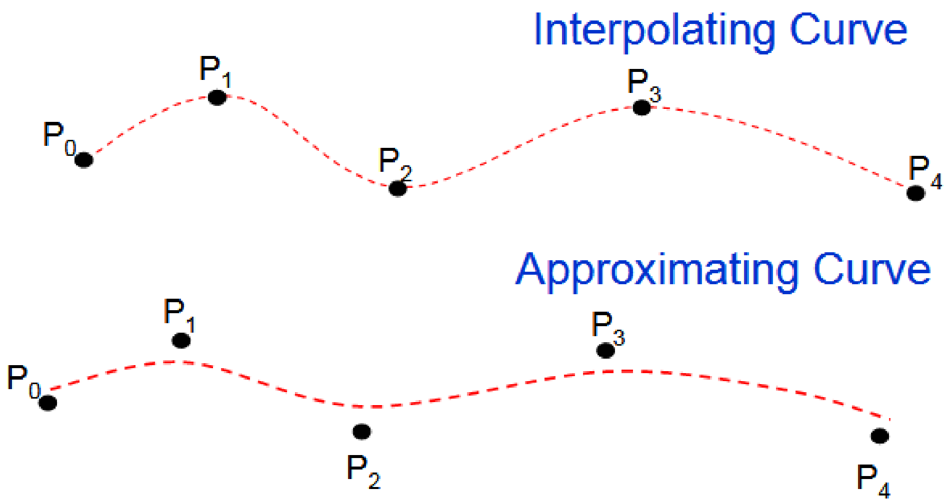
  $$x^2 + y^2 + z^2 - R^2 = 0 \quad \& \quad z = 0$$

- Parametric:

  $$x(\theta) = R\cos(\theta)$$
  $$y(\theta) = R\sin(\theta)$$
  $$z(\theta) = 0$$

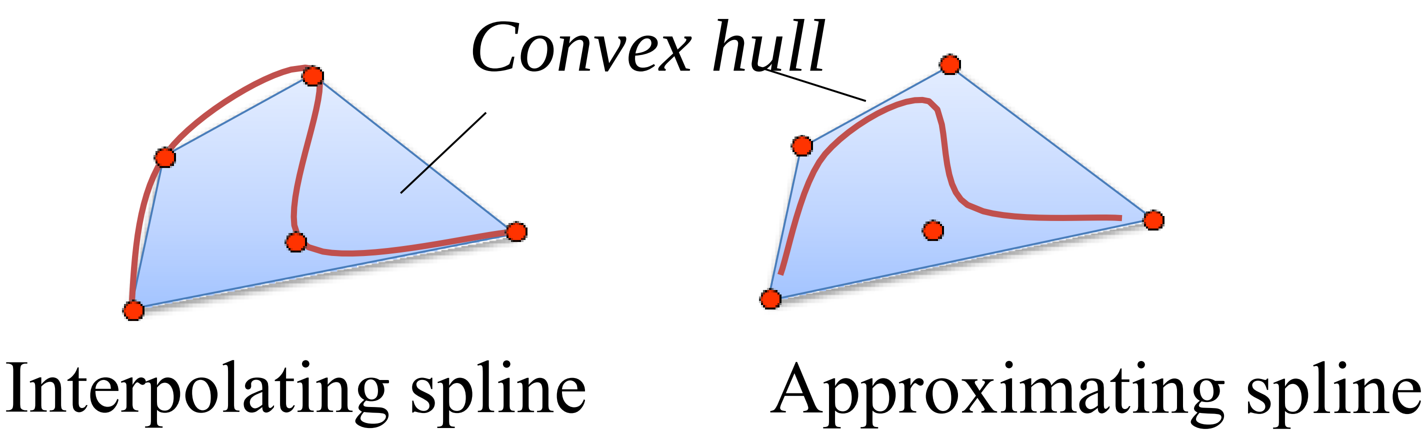


Interpolating and approximating curve:



Convex hull

The convex hull property ensures that a parametric curve will never pass outside of the convex hull formed by the four control vertices.



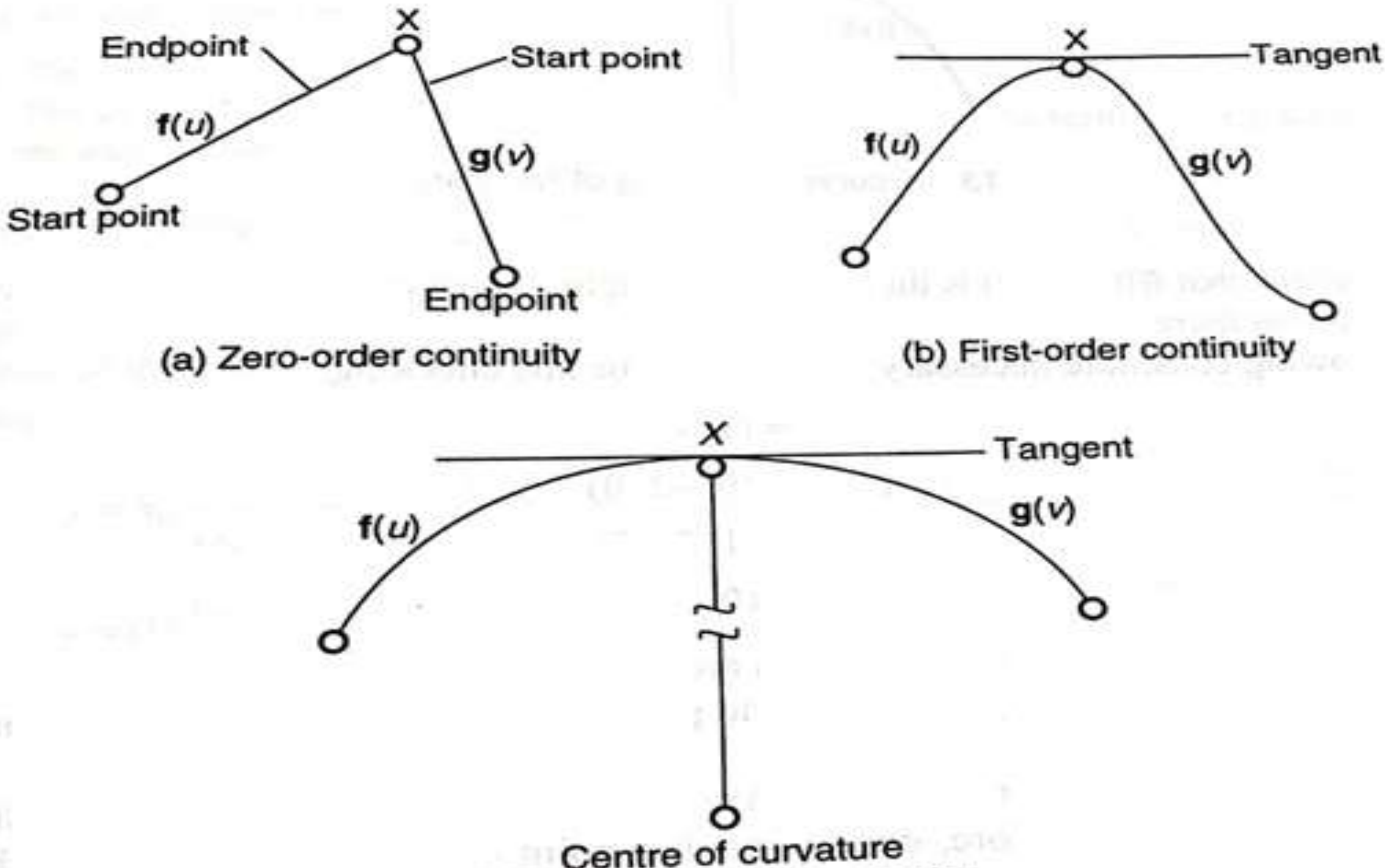
Basic Concepts :

The order of continuity is a term usually used to measure the degree of continuous derivatives (C^0 , C^1 , C^2).

c ^{0} **-** Zero-order parametric continuity - the two curves sections must have the same coordinate position at the boundary point.

c ^{1} **-** First-order parametric continuity - tangent lines of the coordinate functions for two successive curve sections are equal at their joining point.

c ^{2} **-** second-order parametric continuity **-** both the first and second parametric derivativesof the two curve sections are the same at the intersection,



(a) Zero-order continuity

(b) First-order continuity

(c) Second-order continuity

Continuity of curves.

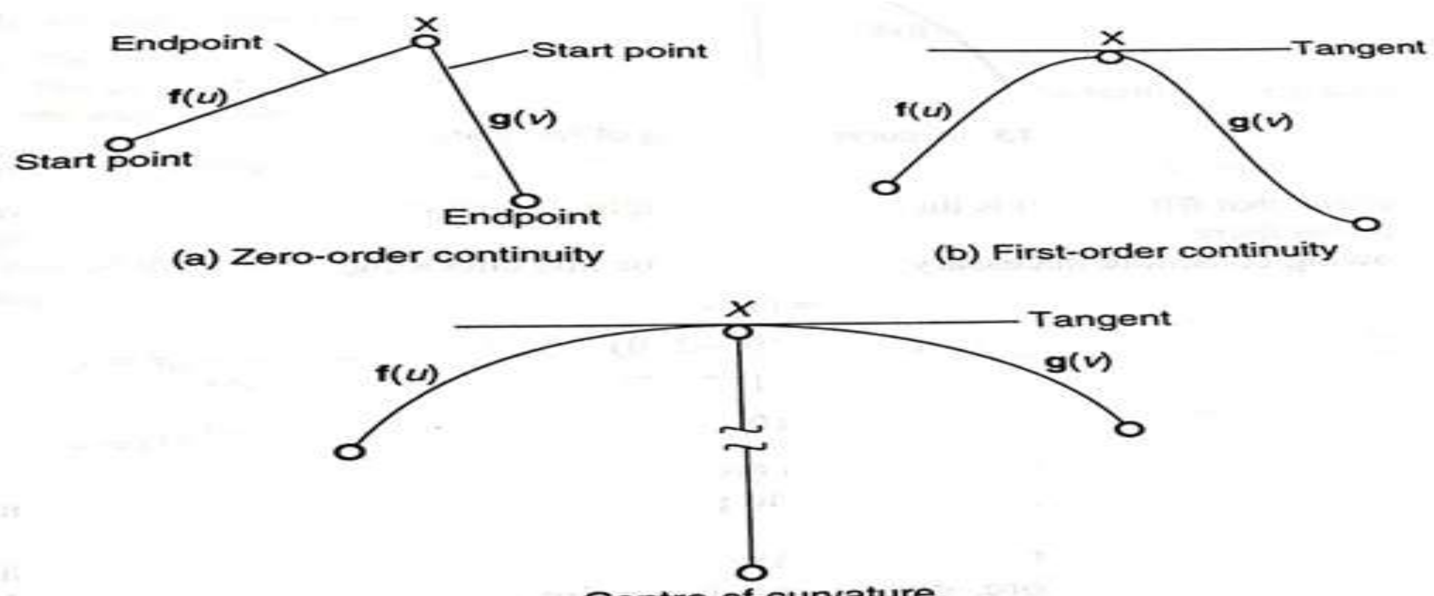
Curvature continuity

C^0 Continuity :

- Consider The end Point of curve f(b) & the start point of the curve g(m) .
- If f(b) & g(m) are equal as shown , we shall say curves are C^0 continuous at f(b) = g (m).

C^1 Continuity : Two curves are C^1 Continuous at the joining point if the first derivative does not change when crossing one to the other.

C^2 Continuity : Two curves are C^2 Continuous at the joining point if in addition to the first derivative , the second derivative is also same when one curve is crossed to the other.



(a) Zero-order continuity

(b) First-order continuity

(c) Second-order continuity

Continuity of curves.