Boundary representation (B-Rep)

- Solid model is defined by their enclosing surfaces or boundaries. This technique consists of the geometric information about the faces, edges and vertices of an object with the topological data on how these are connected.
- Why B-Rep includes such topological information?
  - A solid is represented as a closed space in 3D space (surface connect without gaps)
  - The boundary of a solid separates points inside from points outside solid.

- B- Rep model
  – Technique guarantees that surfaces definitively divide model space into solid and even after model modification commands.
  – B-Rep graph store face, edge and vertices as nodes, with pointers, or branches between the nodes to indicate connectivity.

Boundary representation- validity

- System must validate topology of created solid.
- B-Rep has to fulfill certain conditions to disallow self-intersecting and open objects
- This condition include
  - Each edge should adjoin exactly two faces and have a vertex at each end.
  - Vertices are geometrically described by point coordinates
  - At least three edges must meet at each vertex.
  - Faces are described by surface equations
  - The set of faces forms a complete skin of the solid with no missing parts.
  - Each face is bordered by an ordered set of edges forming a
    closed loop.
  - Faces must only intersect at common edges or vertices.
  - The boundaries of faces do not intersect themselves

<u>B-reps (Boundary Representations)</u>

Object is defined in terms of its surface boundaries, vertices, edges and faces. Curved surfaces are always approximated with polygons – piecewise linear/planar.

Very commonly used, in practice.

Use planar, polygonal boundaries, but may also use convex polygons or triangles.

Polyhedron is a solid that is bounded by a set of polygons whose edges are each a member of an even number of polygons. Additional constraints will be discussed later on.

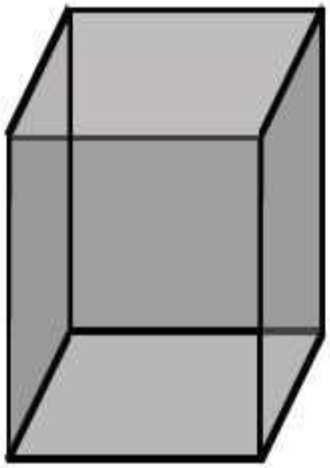
A simple polyhedron can always be deformed into a sphere. Polyhedron has no holes (not a torus).

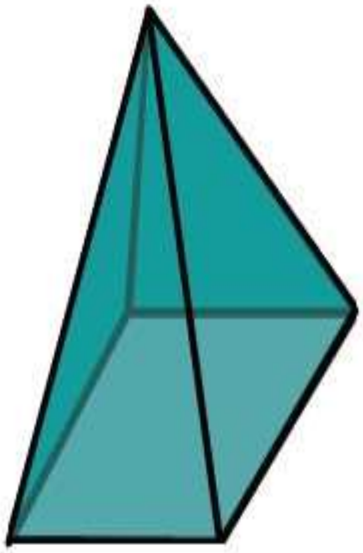
Euler's formula:

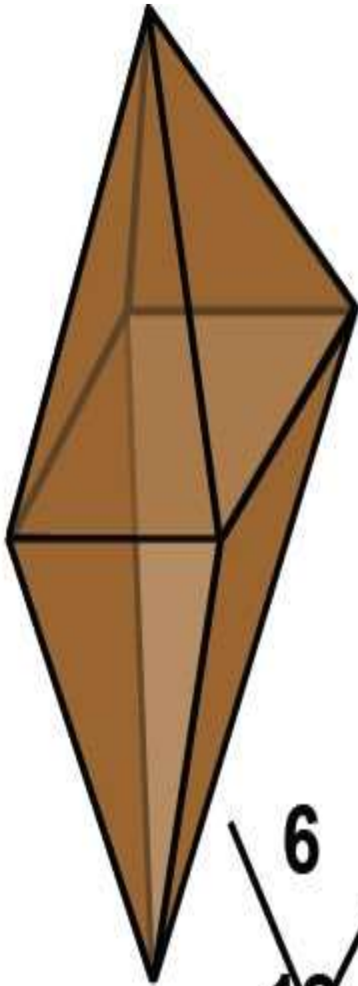
Let V be the number of vertices, E the number of edges, and F the number of faces of a simple polyhedron. Then

$$V - E + F = 2$$

Verify Euler's formula with these examples:







V =	8	5	~~6~~
E =	12	8	~~12~~
F =	6	5	~~9~~

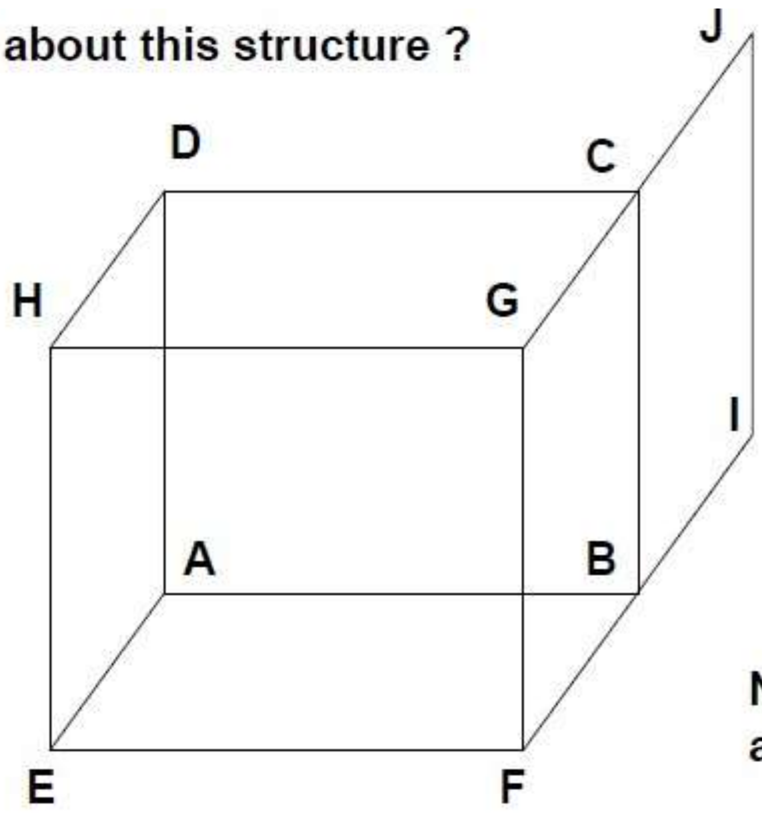
Also applicable for curved edges and non-planar faces

6

Actual: 12

8

What about this structure ?



V = 10;

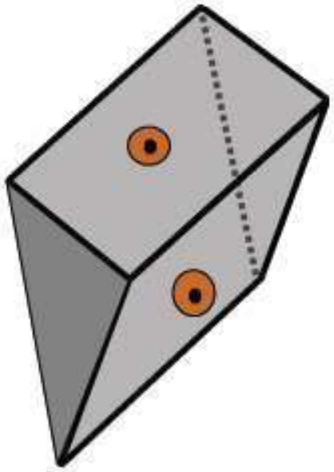
E = 15;

F = 7.

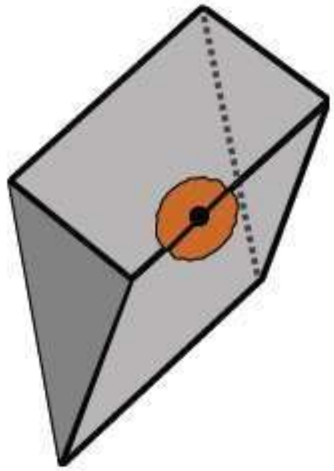
Formula still holds good,
but this is not a bound volume.

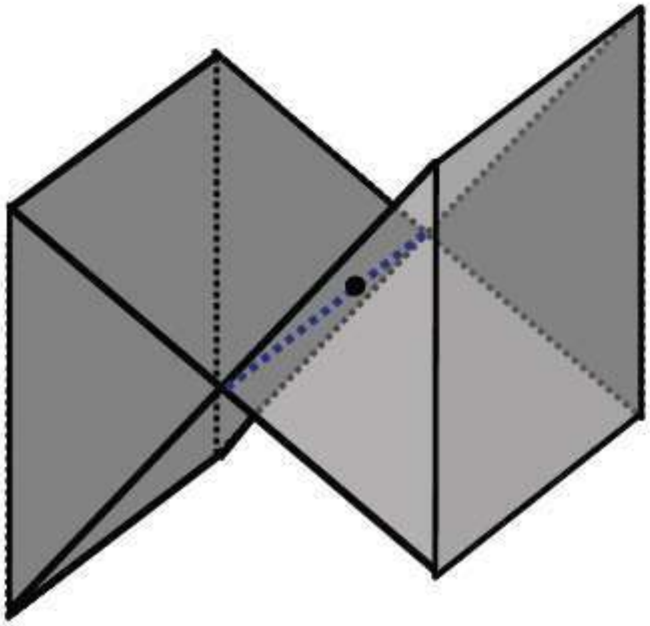
Need additional constraints to guarantee a solid object:

- Each edge must contain two vertices
- Must be shared exactly by two faces
- At least three edges must meet to form a vertex
- Faces must not interpenetrate



Solids with boundaries with 2-manifolds





Solids with boundaries but not 2-manifolds

Generalized Euler's formula for objects with 2-manifolds, and faces which may have holes:

$$V - E + F - H = 2(C - G)$$

where, H - No. of <u>HOLES</u> in the face;

G - No. of holes that pass through the object;

C - No. of separate <u>COMPONENTS</u> (or parts) of the object.

If C = 1, G is called as *<u>GENUS.</u>*   If C > 1, G is the sum of the <u>GENERA</u> of its components.

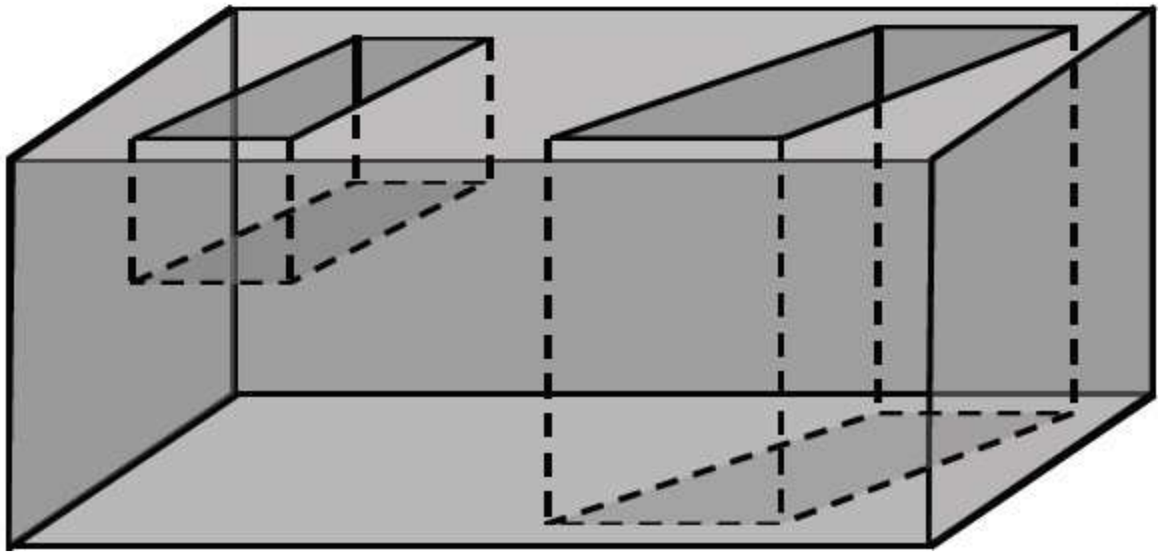
V = 24

E = 36

F = 15

H = 3

C = G = 1

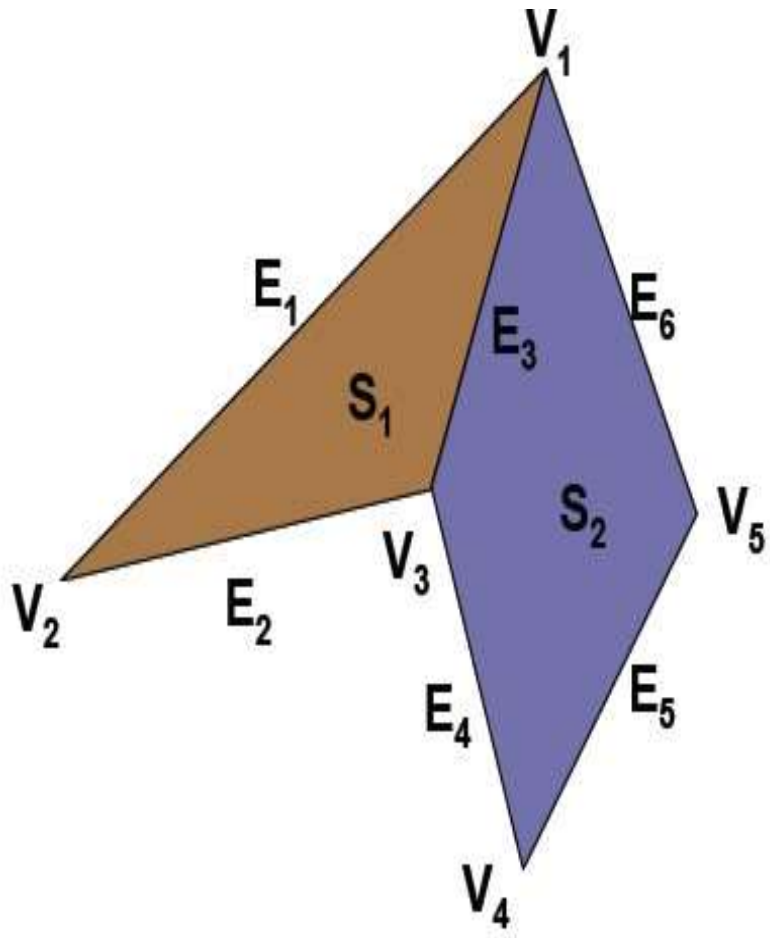


Vertex Table

V_1: X_1, Y_1, Z_1
V_2: X_2, Y_2, Z_2
V_3: X_3, Y_3, Z_3
V_4: X_4, Y_4, Z_4
V_5: X_5, Y_5, Z_5
V_6: X_6, Y_6, Z_6

Edge Table

E_1: V_1, V_2
E_2: V_2, V_3
E_3: V_3, V_1
E_4: V_3, V_4
E_5: V_4, V_5
E_6: V_5, V_1



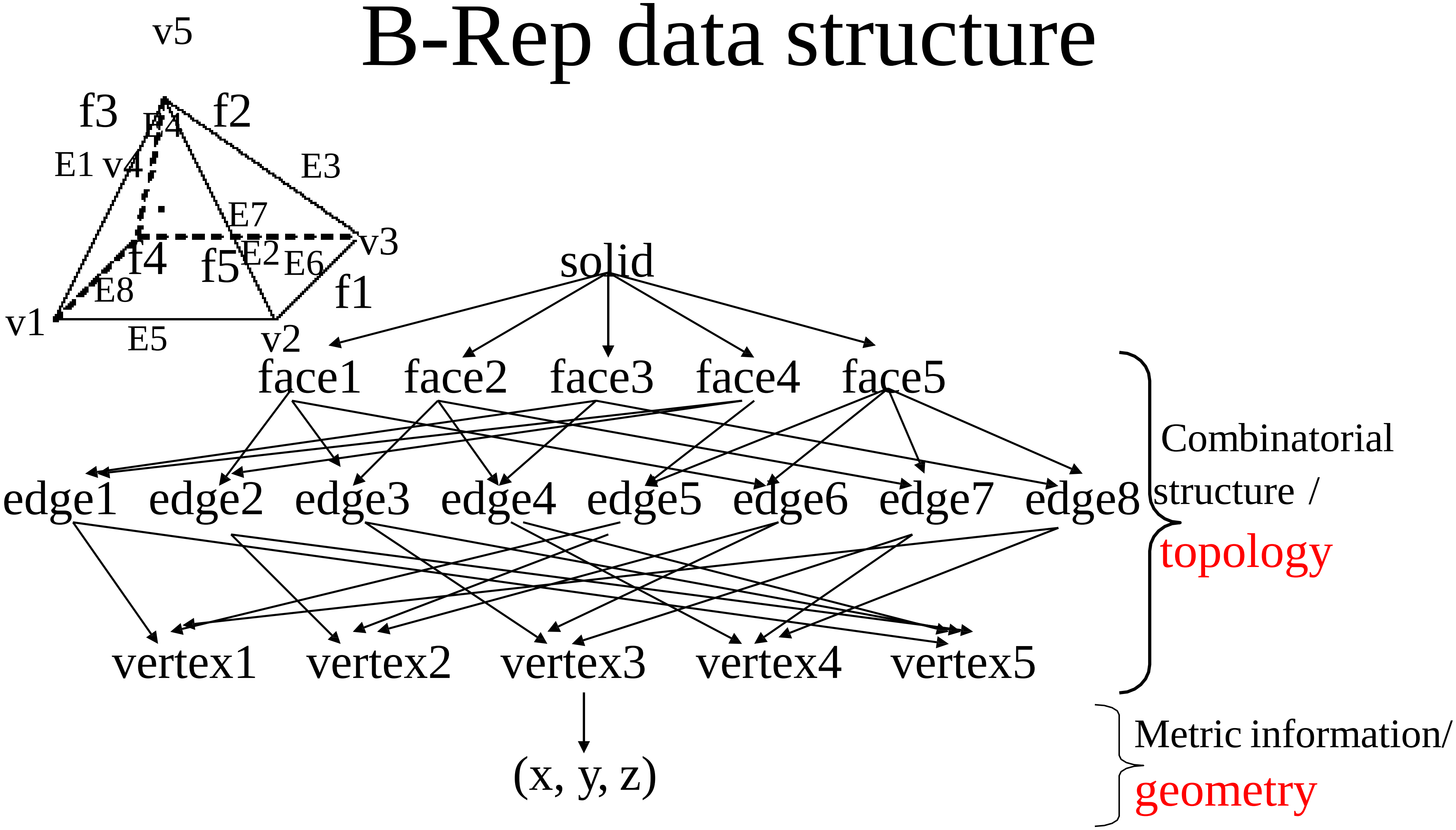
Polygon Surface Table

S_1: E_1, E_2, E_3
S_2: E_3, E_4, E_5, E_6

Complete (expanded) Edge Table

E_1: V_1, V_2, S_1
E_2: V_2, V_3, S_1
E_3: V_3, V_1, S_1, S_2
E_4: V_3, V_4, S_2
E_5: V_4, V_5, S_2
E_6: V_5, V_1, S_2

B-Rep data structure



Boundary representation- ambiguity and uniqueness

- Valid B-Reps are unambiguous
- Not fully unique, but much more so than CSG
- Potential difference exists in division of
  - Surfaces into faces.
  - Curves into edges
- Capability to construct unusual shapes that would not be possible with
  the available CSG aircraft fuselages, swing shapes
- Less computational time to reconstruct the image
- Requires more storage
- More prone to validity failure than CSG
- Model display limited to planar faces and linear edges
  - complex curve and surfaces only approximated