PROJECT EVALUATION REVIEW TECHNIQUE, (PERT)

In the critical path method, the time estimates are assumed to be known with certainty. In certain projects like research and development, new product introductions, it is difficult to estimate the time of various activities.

Hence PERT is used in such projects with a probabilistic method using three time estimates for an activity, rather than a single estimate, as shown in Figure 8.22.

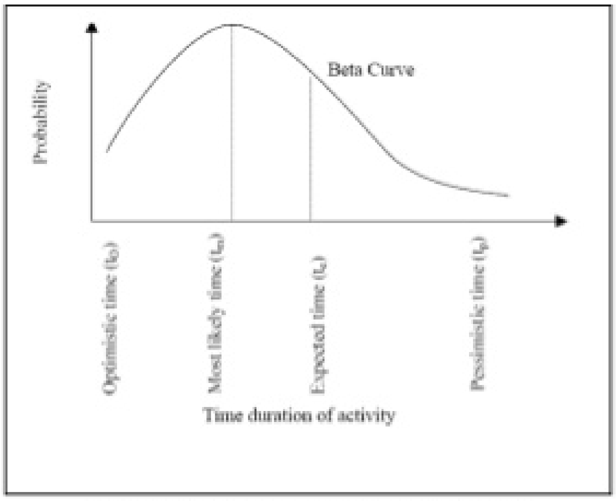


Figure 8.22: PERT Using Probabilistic Method with 3 Time Estimates

Optimistic time to:

It is the shortest time taken to complete the activity. It means that if everything goes well then there is more chance of completing the activity within this time.

Most likely time tm:

It is the normal time taken to complete an activity, if the activity were frequently repeated under the same conditions.

Pessimistic time tp:

It is the longest time that an activity would take to complete. It is the worst time estimate that an activity would take if unexpected problems are faced.

Taking all these time estimates into consideration, the expected time of an activity is arrived at.

The average or mean (ta) value of the activity duration is given by,

$$T_a = \frac{t_0 + 4t_m + t_p}{6} \qquad \text{.......................(5)}$$

The variance of the activity time is calculated using the formula,

$$\sigma_i^2 = \left(\frac{t_p - t_0}{6}\right)^2$$

Probability for Project Duration

The probability of completing the project within the scheduled time (Ts) or contracted time may be obtained by using the standard normal deviate where Te is the expected time of project completion.

$$Z_0 = \frac{T_s - T_e}{\sqrt{\Sigma\sigma^2} \text{ in critical path}}$$

Probability of completing the project within the scheduled time is,

$$P\,(T \leq T_s) = P\,(Z \leq Z_0) \text{ (from normal tables)} \quad \text{....}$$

PERT

- PERT is based on the assumption that an activity's duration follows a probability distribution instead of being a single value
- Three time estimates are required to compute the parameters of an activity's duration distribution:
  - pessimistic time (t_p) - the time the activity would take if things did not go well
  - most likely time (t_m) - the consensus best estimate of the activity's duration
  - optimistic time (t_o) - the time the activity would take if things did go well

Mean (expected time): $t_e = \frac{t_p + 4\,t_m + t_o}{6}$

Variance: $V_t = \sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$

PERT analysis

- Draw the network.
- Analyze the paths through the network and find the critical path.
- The length of the critical path is the mean of the project duration probability distribution which is assumed to be normal
- The standard deviation of the project duration probability distribution is computed by adding the variances of the critical activities (all of the activities that make up the critical path) and taking the square root of that sum
- Probability computations can now be made using the normal distribution table.

Probability computation

Determine probability that project is completed within specified time

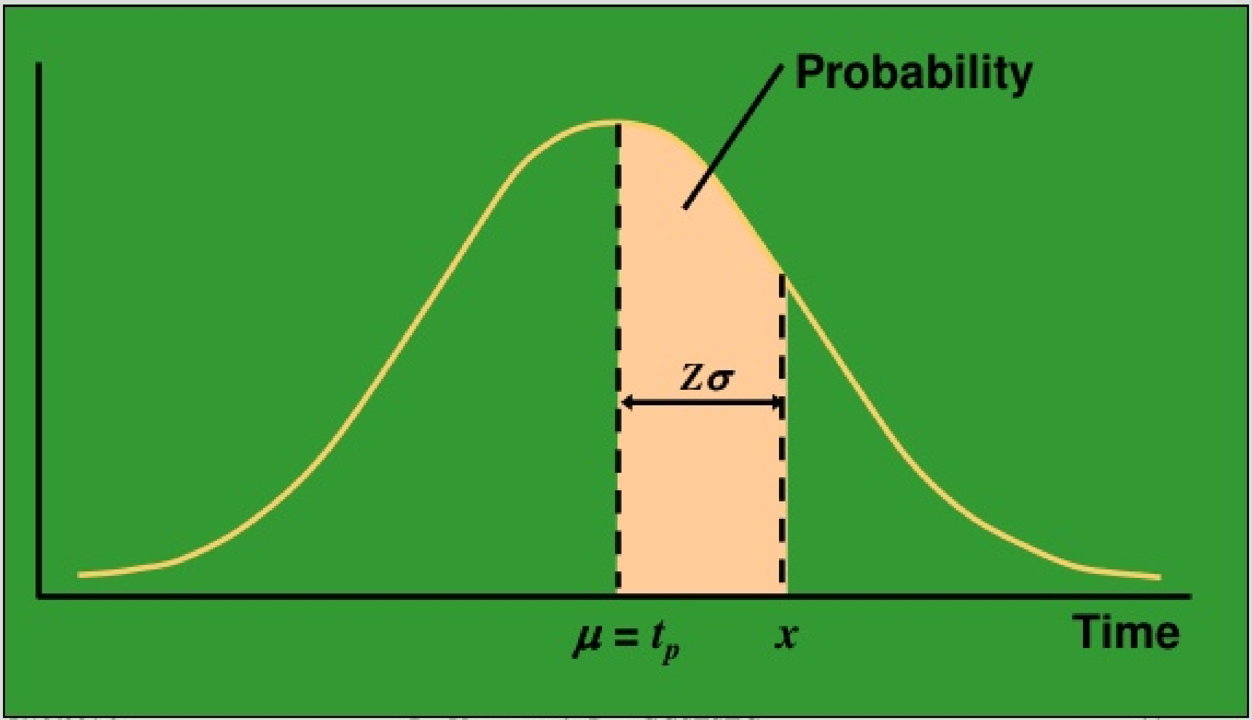
$$Z = \frac{x - \mu}{\sigma}$$

where $\mu = t_p$ = project mean time

σ = project standard mean time

x = (proposed) specified time

Normal Distribution of Project Time



Comparing CPM and PERT

Both CPM and PERT (Program Evaluation and Review Technique) provide the user with project management tools to plan, monitor, and update their project as it progresses. There are many similarities and differences between the two, however.

Similarities between PERT and CPM

- Both follow the same steps and use network diagrams
- Both are used to plan the scheduling of individual activities that make up a project
- They can be used to determine the earliest/latest start and finish times for each activity

Differences between PERT and CPM

- PERT is probabilistic whereas CPM is deterministic
- In CPM, estimates of activity duration are based on historical data
- In PERT, estimates are uncertain and we talk of ranges of duration and the probability that an activity duration will fall into that range
- CPM concentrates on Time/Cost trade off.

Benefits of CPM/PERT

- Useful at many stages of project management
- Mathematically simple
- Give critical path and slack time
- Provide project documentation
- Useful in monitoring costs

CPM/PERT can answer the following important questions:

•How long will the entire project take to be completed? What are the risks involved?
•Which are the critical activities or tasks in the project which could delay the entire project if they were not completed on time?
•Is the project on schedule, behind schedule or ahead of schedule?
•If the project has to be finished earlier than planned, what is the best way to do this at the least cost?

Limitations to CPM/PERT

- Clearly defined, independent and stable activities
- Specified precedence relationships
- Over emphasis on critical paths
- Deterministic CPM model
- Activity time estimates are subjective and depend on judgment
- PERT assumes a beta distribution for these time estimates, but the actual distribution may be different
- PERT consistently underestimates the expected project completion time due to alternate paths becoming critical

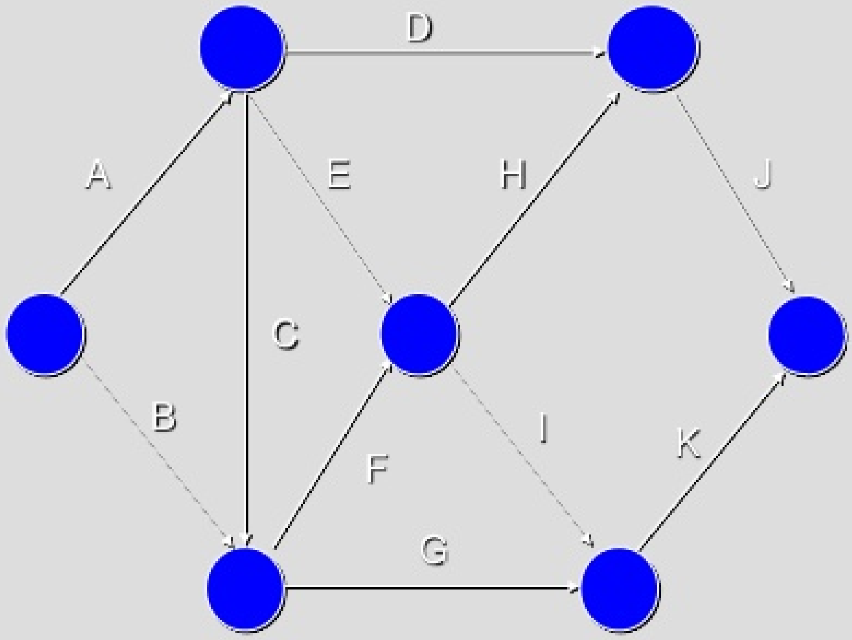
To overcome the limitation, Monte Carlo simulations can be performed on the network to eliminate the optimistic bias

PERT Example

Activity	Immed. Predec.	Optimistic Time (Hr.)	Most Likely Time (Hr.)	Pessimistic Time (Hr.)
A	--	4	6	8
B	--	1	4.5	5
C	A	3	3	3
D	A	4	5	6
E	A	0.5	1	1.5
F	B,C	3	4	5
G	B,C	1	1.5	5
H	E,F	5	6	7
I	E,F	2	5	8
J	D,H	2.5	2.75	4.5
K	G,I	3	5	7

PERT Example

PERT Network



PERT Example

Activity	Expected Time	Variance
A	6	4/9
B	4	4/9
C	3	0
D	5	1/9
E	1	1/36
F	4	1/9
G	2	4/9
H	6	1/9
I	5	1
J	3	1/9
K	5	4/9

PERT Example

Activity	ES	EF	LS	LF	Slack
A	0	6	0	6	0 *critical
B	0	4	5	9	5
C	6	9	6	9	0 *
D	6	11	15	20	9
E	6	7	12	13	6
F	9	13	9	13	0 *
G	9	11	16	18	7
H	13	19	14	20	1
I	13	18	13	18	0 *
J	19	22	20	23	1
K	18	23	18	23	0 *

PERT Example

$$V_{path} = V_A + V_C + V_F + V_I + V_K$$
$$= 4/9 + 0 + 1/9 + 1 + 4/9$$
$$= 2$$
$$\sigma_{path} = 1.414$$
$$z = (24 - 23)/\sigma = (24-23)/1.414 = .71$$

From the Standard Normal Distribution table:

$$P(z \leq .71) = .5 + .2612 = \boxed{.7612}$$

Example

An R & D project has a list of tasks to be performed whose time estimates are given in the Table 8.11, as follows.

Table 8.11: Time Estimates for R & D Project

Activity i j	Activity Name	T_0	t_m (in days)	t_p
1-2	A	4	6	8
1-3	B	2	3	10
1-4	C	6	8	16
2-4	D	1	2	3
3-4	E	6	7	8
3-5	F	6	7	14
4-6	G	3	5	7
4-7	H	4	11	12
5-7	I	2	4	6
6-7	J	2	9	10

a. Draw the project network.
b. Find the critical path.
c. Find the probability that the project is completed in 19 days. If the probability is less than 20%, find the probability of completing it in 24 days.

Time expected for each activity is calculated using the formula (5):
Similarly, the expected time is calculated for all the activities.

$$T_a = \frac{t_0 + 4tm + tp}{6}$$

$$= \frac{4 + 4(6) + 8}{6} = \frac{36}{6} = 6 \text{ days for activity A}$$

The variance of activity time is calculated using the formula (6).
Similarly, variances of all the activities are calculated.

$$\sigma_i^2 = \left(\frac{t_p - t_0}{6}\right)^2$$

$$= \left(\frac{8-4}{6}\right)^2 = 0.444$$

Table 8.12: T_e & s^2 Calculated

Activity	T_o	T_m	T_p	T_a	σ^2
1-2	4	6	8	6	0.444
1-3	2	3	10	4	1.777
1-4	6	8	16	9	2.777
2-4	1	2	3	2	0.111
3-4	6	7	8	7	0.111
3-5	6	7	14	8	1.777
4-6	3	5	7	5	0.444
4-7	4	11	12	10	1.777
5-7	2	4	6	4	0.444
6-7	2	9	10	8	1.777

Construct a network diagram:

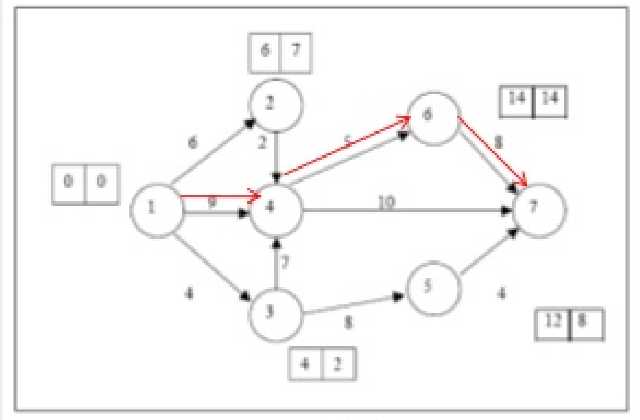


Figure 8.23: Network Diagram

calculate the time earliest (TE) and time Latest (TL) for all the activities.

From the network diagram Figure 8.24, the critical path is identified as
1-4, 4-6, 6-7, with a project duration of 22 days.

The probability of completing the project within 19 days is given by, P (Z< Zo)

To find Zo , $Z_0 = \left(\frac{T_s - T_e}{\sqrt{\Sigma\sigma \text{ in critical path}}}\right)$

$= \left(\frac{19-22}{\sqrt{2.777+0.444+1.777}}\right)$ $= \left(\frac{-3}{\sqrt{5}}\right) = -1.3416$

we know, P (Z <Z Network Model 0) = 0.5 – z (1.3416) (from normal tables, z (1.3416) = 0.4099)
= 0.5 – 0.4099
= 0.0901
= 9.01% Thus, the probability of completing the R & D project in 19 days is 9.01%.

Since the probability of completing the project in 19 days is less than 20% As in question, we find the probability of completing it in 24 days.

$Z_0 = \frac{T_s - T_e}{\sqrt{\Sigma\sigma \text{ in critical path}}}$

$= \left(\frac{24-22}{\sqrt{5}}\right) = \left(\frac{2}{\sqrt{5}}\right) = 0.8944$ days

$P(Z \le Z_0) = 0.5 - \Upsilon\ (0.8944)$ (from normal tables, $\Upsilon\ (0.8944) = 0.3133$)
$= 0.5 + 0.3133$
$= 0.8133$
$= 81.33\%$