CRITICAL PATH METHOD (CPM)

CPM is used for optimising resource allocation and minimising overall cost for a given project. CPM uses two times and cost estimates for each activity i.e., one for normal situation other for the crash situation. CPM assume each activity is precise and known. It assumed to be known the relation between resources employed and time needed.

FOLLOWING ARE THE STEPS INVOLVED IN CPM TECHNIQUE

1. CPM provides systematical division of various activities of the project.
2. It helps in determining the time for each activity and event.
3. It helps to calculate earliest start and finish time and latest start and finish time.
4. It determines the float for each activity on the basis of difference between the earliest time and latest time.
5. It determines the total duration of the project.
6. Optimise the cost by shifting resources.
7. It updates the network resources.

CPM calculation

- Path
  - A connected sequence of activities leading from the starting event to the ending event
- Critical Path
  - The longest path (time); determines the project duration
- Critical Activities
  - All of the activities that make up the critical path

Forward Pass

- Earliest Start Time (ES)
  - earliest time an activity can start
  - ES = maximum EF of immediate predecessors
- Earliest finish time (EF)
  - earliest time an activity can finish
  - earliest start time plus activity time

EF= ES + *t*

Backward Pass

- Latest Start Time (LS)

  Latest time an activity can start without delaying critical path time

  LS= LF - *t*

- Latest finish time (LF)

  latest time an activity can be completed without delaying critical path time

  LS = minimum LS of immediate predecessors

CPM analysis

- Draw the CPM network
- Analyze the paths through the network
- Determine the float for each activity
  - Compute the activity's float

    $$float = LS - ES = LF - EF$$

  - Float is the maximum amount of time that this activity can be delay in its completion before it becomes a critical activity, i.e., delays completion of the project
- Find the critical path is that the sequence of activities and events where there is no "slack" i.e.. Zero slack
  - Longest path through a network
- Find the project duration is minimum project completion time

Project management generally consists of three phases.

Planning:

Planning involves setting the objectives of the project. Identifying various activities to be performed and determining the requirement of resources such as men, materials, machines, etc.

The cost and time for all the activities are estimated, and a network diagram is developed showing sequential interrelationships (predecessor and successor) between various activities during the planning stage.

Scheduling:

Based on the time estimates, the start and finish times for each activity are worked out by applying forward and backward pass techniques, critical path is identified, along with the slack and float for the non-critical paths.

Controlling:

Controlling refers to analyzing and evaluating the actual progress against the plan. Reallocation of resources, crashing and review of projects with periodical reports are carried out.

COMPONENTS of PERT/CPM NETWORK

PERT / CPM networks contain two major components

i. Activities, and

ii. Events

Activity: *An activity represents an action and consumption of resources (time, money,* energy) required to complete a portion of a project. Activity is represented by an arrow, (Figure 8.1).

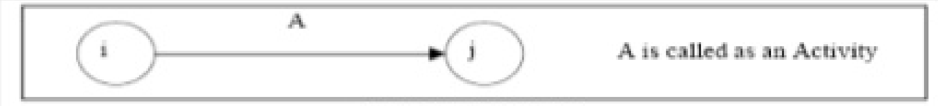


Figure 8.1: An Activity

Event: *An event (or node) will always occur at the beginning and end of an activity. The* event has no resources and is represented by a circle. The ith event and jth event are the tail event and head event respectively, (Figure 8.2).

Figure 8.2: An Event

Merge and Burst Events

One or more activities can start and end simultaneously at an event (Figure 8.3 a, b).

Figure 8.3

Preceding and Succeeding Activities

Activities performed before given events are known as *preceding activities* (Figure 8.4), and activities performed after a given event are known as *succeeding activities*.

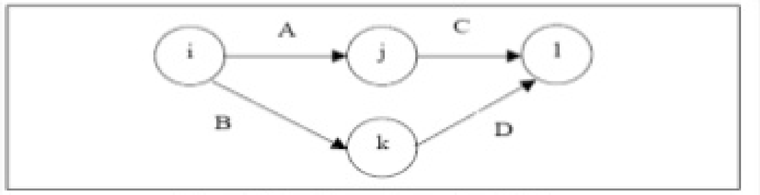


Figure 8.4: Preceding and Succeeding Activities

Activities A and B precede activities C and D respectively.

Dummy Activity

An imaginary activity which does not consume any resource and time is called a ***dummy activity. Dummy activities are simply used to represent a connection between events in*** order to maintain a logic in the network. It is represented by a dotted line in a network, see Figure 8.5.

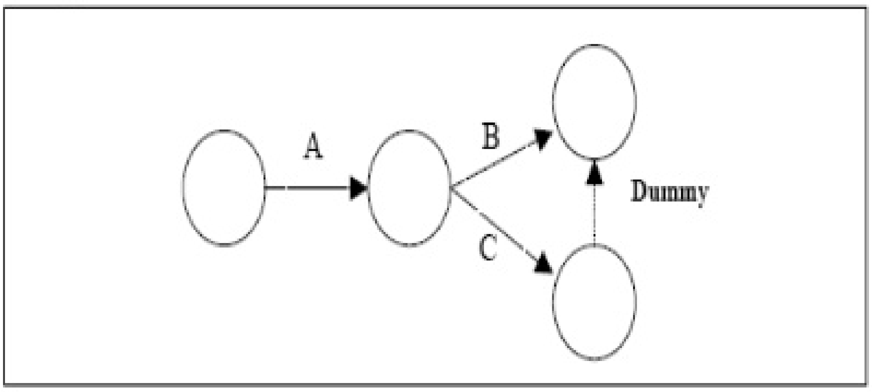


Figure 8.5: Dummy Activity

ERRORS TO BE AVOIDED IN CONSTRUCTING A NETWORK

a. Two activities starting from a tail event must not have a same end event. To ensure this, it is absolutely necessary to introduce a dummy activity, as shown in Figure 8.6.

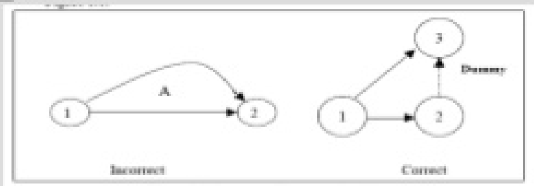


Figure 8.6: Correct and Incorrect Activities

b. Looping error should not be formed in a network, as it represents performance of activities repeatedly in a cyclic manner, as shown below in Figure 8.7.

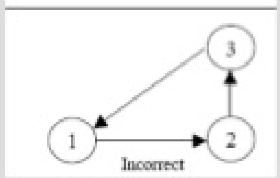


Figure 8.7: Looping Error

c. In a network, there should be only one start event and one ending event as shown below, in Figure 8.8.

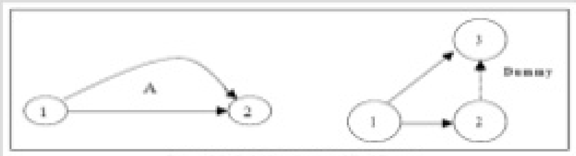


Figure 8.8: Only One Start and End Event

d. The direction of arrows should flow from left to right avoiding mixing of direction as shown in Figure 8.9.

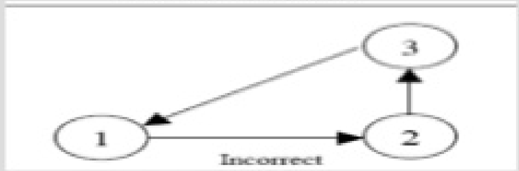


Figure 8.9: Wrong Direction of Arrows

RULES IN CONSTRUCTING A NETWORK

1. **No single activity can be represented more than once in a network. The length of an arrow has no significance.**

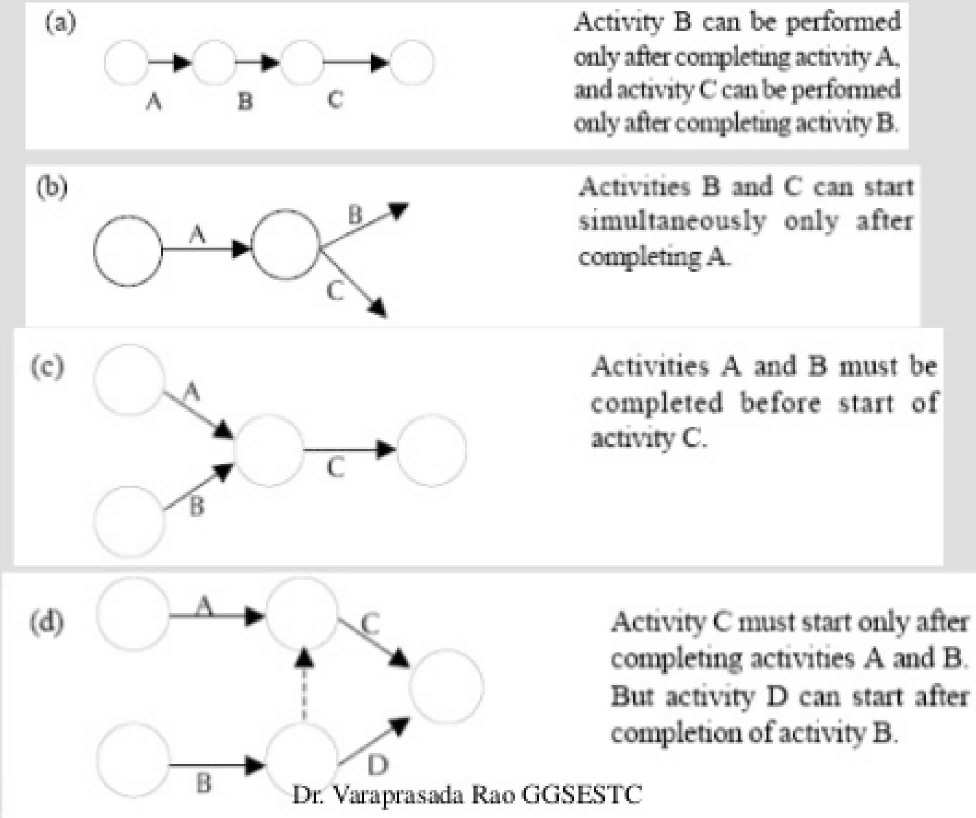
2. The event numbered 1 is the start event and an event with highest number is the end event. Before an activity can be undertaken, all activities preceding it must be completed. That is, the activities must follow a logical sequence (or – interrelationship) between activities.

3. In assigning numbers to events, there should not be any duplication of event numbers in a network.

4. Dummy activities must be used only if it is necessary to reduce the complexity of a network.

5. A network should have only one start event and one end event.

Some conventions of network diagram are shown in Figure 8.10 (a), (b), (c), (d) below:



PROCEDURE FOR NUMBERING THE EVENTS USING FULKERSON'S RULE

Step1: Number the start or initial event as 1.

Step2: From event 1, strike off all outgoing activities. This would have made one or more events as initial events (event which do not have incoming activities). Number that event as 2.

Step3: Repeat step 2 for event 2, event 3 and till the end event. The end event must have the highest number

Example 1:

Draw a network for a house construction project. The sequence of activities with their predecessors are given in Table 8.1, below.

CRITICAL PATH ANALYSIS

The critical path for any network is the longest path through the entire network.

Since all activities must be completed to complete the entire project, the length of the critical path is also the shortest time allowable for completion of the project.

Thus if the project is to be completed in that shortest time, all activities on the critical path must be started as soon as possible.

These activities are called critical activities.

If the project has to be completed ahead of the schedule, then the time required for at least one of the critical activity must be reduced.

Further, any delay in completing the critical activities will increase the project duration.

The activity, which does not lie on the critical path, is called non-critical activity.

These non-critical activities may have some slack time.

The slack is the amount of time by which the start of an activity may be delayed without affecting the overall completion time of the project.

But a critical activity has no slack.

To reduce the overall project time, it would require more resources (at extra cost) to reduce the time taken by the critical activities to complete.

Scheduling of Activities: Earliest Time (TE) and Latest Time(TL)

Before the critical path in a network is determined, it is necessary to find the earliest and latest time of each event to know the earliest expected time (TE) at which the activities originating from the event can be started and to know the latest allowable time (TL) at which activities terminating at the event can be completed.

Forward Pass Computations (to calculate Earliest, Time TE)

Step 1: Begin from the start event and move towards the end event.

Step 2: Put TE = 0 for the start event.

Step 3: *Go to the next event (i.e node 2) if there is an incoming activity for event 2,* add calculate TE of previous event (i.e event 1) and activity time.

Note: If there are more than one incoming activities, calculate TE for all incoming activities and take the maximum value. This value is the TE for event 2.

Step 4: Repeat the same procedure from step 3 till the end event.

Backward Pass Computations (to calculate Latest Time TL)

Procedure :

Step 1: **Begin from end event and move towards the start event. Assume that the direction of arrows is reversed.**

Step 2: **Latest Time TL for the last event is the earliest time. TE of the last event.**

Step 3: **Go to the next event, if there is an incoming activity, subtract the value of TL of previous event from the activity duration time. The arrived value is TL for that event. If there are more than one incoming activities, take the minimum TE value.**

Step 4: **Repeat the same procedure from step 2 till the start event.**

DETERMINATION OF FLOAT AND SLACK TIMES

As discussed earlier, the non – critical activities have some slack or float. The *float of an* activity is the amount of time available by which it is possible to delay its completion time without extending the overall project completion time.

tij = duration of activity
TE = earliest expected time
TL = latest allowable time
ES_{ij} = earliest start time of the activity
EF_{ij} = earliest finish time of the activity
LS_{ij} = latest start time of the activity
LF_{ij} = latest finish time of the activity

Total Float TFij: The total float of an activity is the difference between the latest start time and the earliest start time of that activity.

$$TFij = LS_{ij} - ES_{ij} \text{ ....(1)}$$

or

$$TF_{ij} = (TL - TE) - t_{ij} \text{ ....(ii)}$$

Free Float FFij: ***The time by which the completion of an activity can be delayed from*** **its earliest finish time without affecting the earliest start time of the succeeding activity is called free float.**

$FF_{ij} = (Ej – Ei) – t_{ij}$ …………………(3)

FF_{ij} = Total float – Head event slack

Independent Float IFij: **The amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activities, assuming that the preceding activity has finished at its latest finish time.**

IF ij = (Ej – Li) – tij …………………(4)

IFij = Free float – Tail event slack

Where tail event slack = Li – Ei

The negative value of independent float is considered to be zero.

Critical Path:

After determining the earliest and the latest scheduled times for various activities, the minimum time required to complete the project is calculated. In a network, among various paths, the longest path which determines the total time duration of the project is called the critical path. The following conditions must be satisfied in locating the critical path of a network.

An activity is said to be critical only if both the conditions are satisfied.

1. TL – TE = 0
2. TLj – tij – TEj = 0

Example :

A project schedule has the following characteristics as shown in Table

Table 8.5: Project Schedule

Activity	Name	Time	Activity	Name	Time (days)
1-2	A	4	5-6	G	4
1-3	B	1	5-7	H	8
2-4	C	1	6-8	I	1
3-4	D	1	7-8	J	2
3-5	E	6	8-10	K	5
4-9	F	5	9-10	L	7

i. Construct PERT network.

ii. Compute TE and TL for each activity.

iii. Find the critical path.

Table 8.5: Project Schedule

Activity	Name	Time	Activity	Name	Time (days)
1-2	A	4	5-6	G	4
1-3	B	1	5-7	H	8
2-4	C	1	6-8	I	1
3-4	D	1	7-8	J	2
3-5	E	6	8-10	K	5
4-9	F	5	9-10	L	7

(i) From the data given in the problem, the activity network is constructed as shown in Figure given below

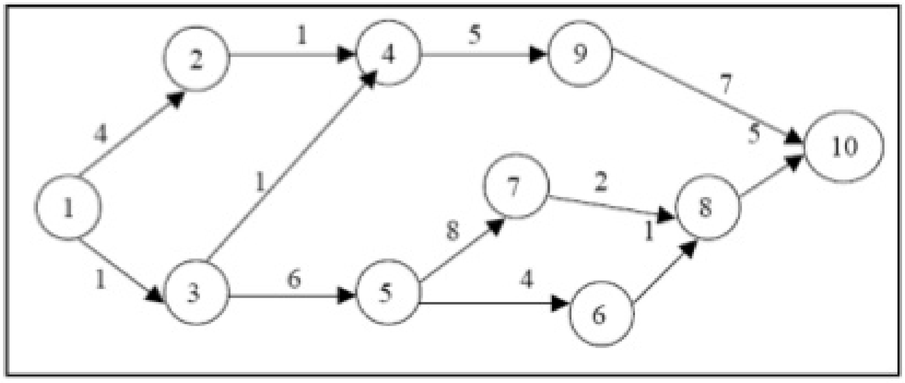


Figure 8.16: Activity Network Diagram

(ii) **To determine the critical path, compute the earliest time TE and latest time TL for each of the activity of the project. The calculations of TE and TL are as follows:,**

To calculate TE for all activities

$TE_1 = 0$

$TE_2 = TE_1 + t_{1,2} = 0 + 4 = 4$

$TE_3 = TE_1 + t_{1,3} = 0 + 1 = 1$

$TE_4 = \max(TE_2 + t_{2,4} \text{ and } TE_3 + t_{3,4})$

$= \max(4 + 1 \text{ and } 1 + 1) = \max(5, 2)$

$= 5$ days

$TE_5 = TE_3 + t_{3,6} = 1 + 6 = 7$

$TE_6 = TE_5 + t_{5,6} = 7 + 4 = 11$

$TE_7 = TE_5 + t_{5,7} = 7 + 8 = 15$

$TE_8 = \max(TE_6 + t_{6,8} \text{ and } TE_7 + t_{7,8})$

$= \max(11 + 1 \text{ and } 15 + 2) = \max(12, 17)$

$= 17$ days

$TE_9 = TE_4 + t_{4,9} = 5 + 5 = 10$

$TE_{10} = \max(TE_9 + t_{9,10} \text{ and } TE_8 + t_{8,10})$

$= \max(10 + 7 \text{ and } 17 + 5) = \max(17, 22)$

$= 22$ days

To calculate TL for all activities

$TL_{10} = TE_{10} = 22$

$TL_9 = TE_{10} - t_{9,10} = 22 - 7 = 15$

$TL_8 = TE_{10} - t_{8,10} = 22 - 5 = 17$

$TL_7 = TE_8 - t_{7,8} = 17 - 2 = 15$

$TL_6 = TE_8 - t_{6,8} = 17 - 1 = 16$

$TL_5 = \min(TE_6 - t_{5,6} \text{ and } TE_7 - t_{5,7})$

$= \min(16 - 4 \text{ and } 15 - 8) = \min(12, 7)$

$= 7$ days

$TL_4 = TL_9 - t_{4,9} = 15 - 5 = 10$

$TL_3 = \min(TL_4 - t_{3,4} \text{ and } TL_5 - t_{3,5})$

$= \min(10 - 1 \text{ and } 7 - 6) = \min(9, 1)$

$= 1$ day

$TL_2 = TL_4 - t_{2,4} = 10 - 1 = 9$

$TL_1 = \text{Min}(TL_2 - t_{1,2} \text{ and } TL_3 - t_{1,3})$

$= \text{Min}(9 - 4 \text{ and } 1 - 1) = 0$

Table 8.6: Various Activities and their Floats

Activity	Activity Name	Normal Time (tij)	Earliest Time (TE)		Latest Time (TL)		Total Float
			Start (ES)	Finish (EF)	Start (LS)	Finish (LF)	
1-2	A	4	0	4	5	9	5
1-3	B	1	0	1	0	1	0
2-4	C	1	4	5	9	10	5
3-4	D	1	1	2	9	10	8
3-5	E	6	1	7	1	7	0
4-9	F	5	5	10	10	15	5
5-6	G	4	7	11	12	16	5
5-7	H	8	7	15	7	15	0
6-8	I	1	11	12	16	17	5
7-8	J	2	15	17	15	17	0
8-10	K	5	17	22	17	22	0
9-10	L	7	10	17	15	22	5

(iii) From the Table 8.6, we observe that the activities 1 – 3, 3 – 5, 5 – 7,7 – 8 and 8 – 10 are critical activities as their floats are zero.

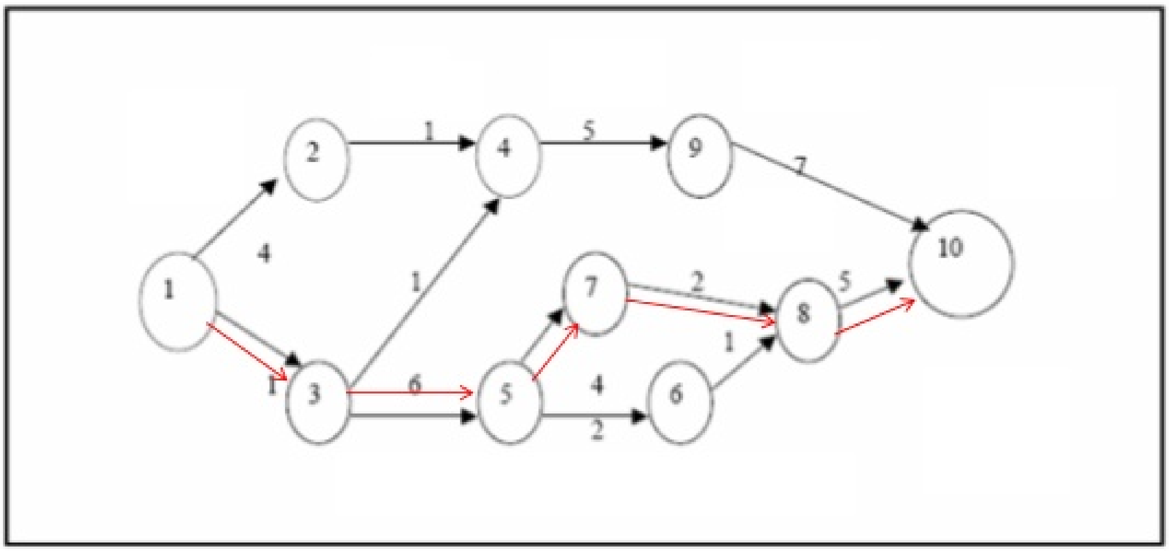
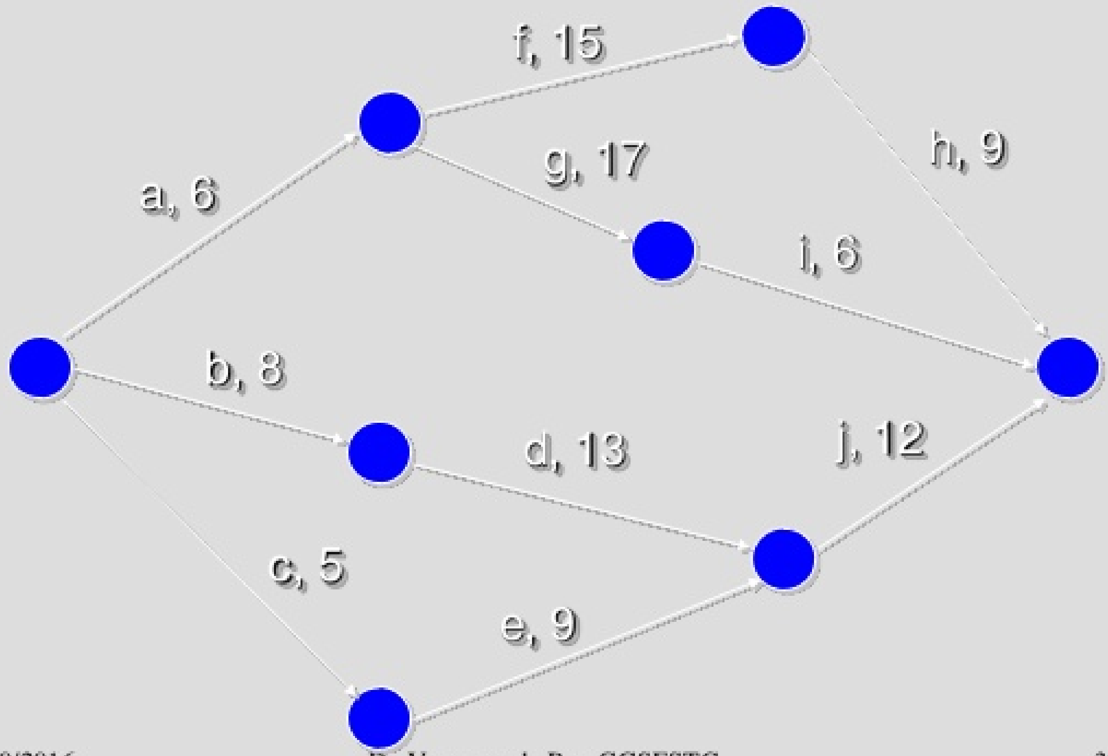


Figure 8.17: Critical Path of the Project

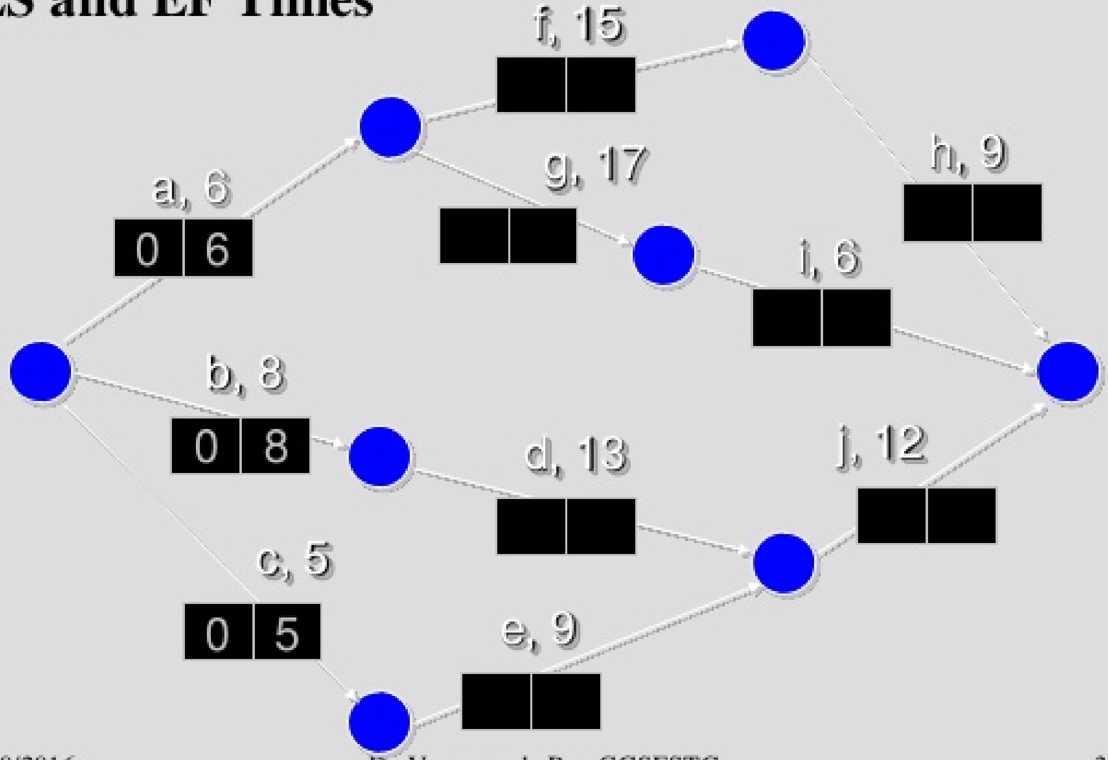
CPM Example:

- **CPM Network**



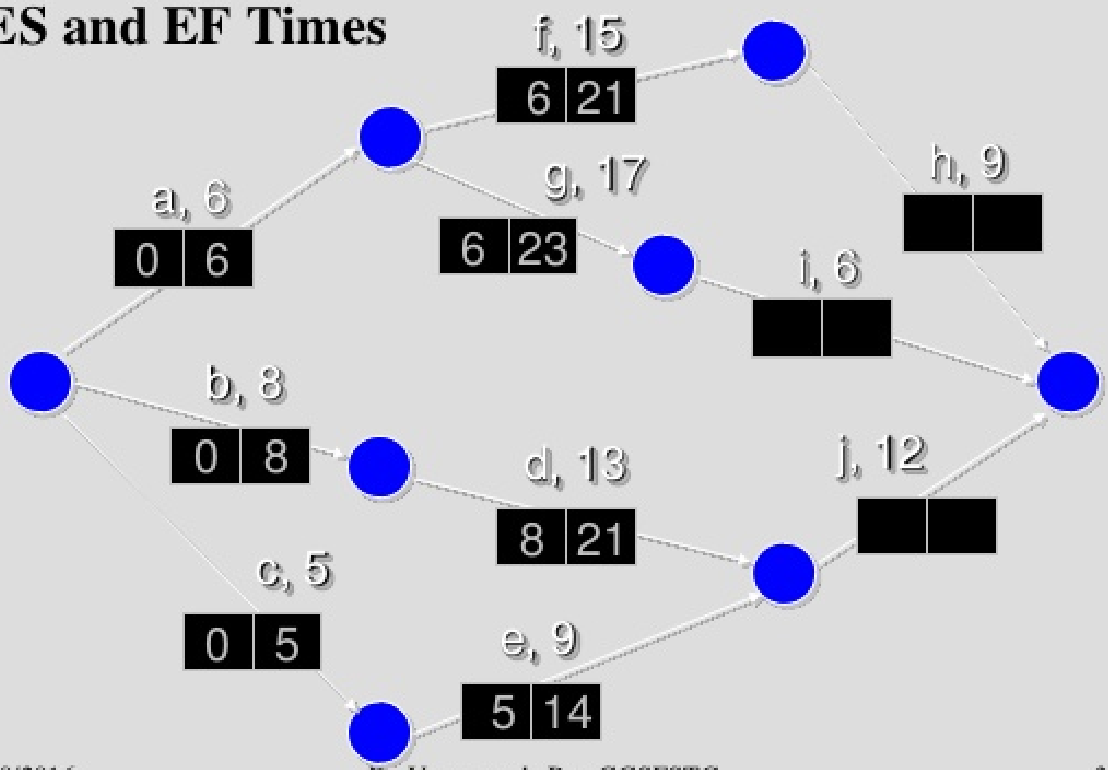
CPM Example

- **ES and EF Times**



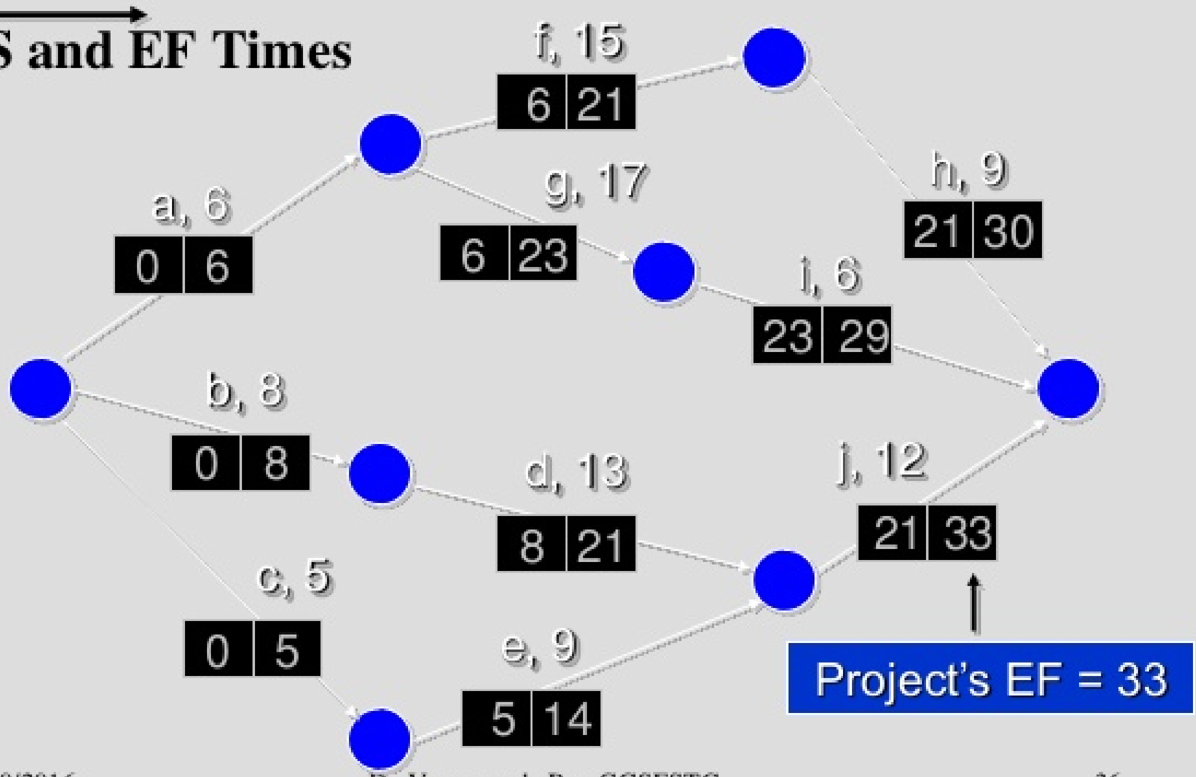
CPM Example

- **ES and EF Times**



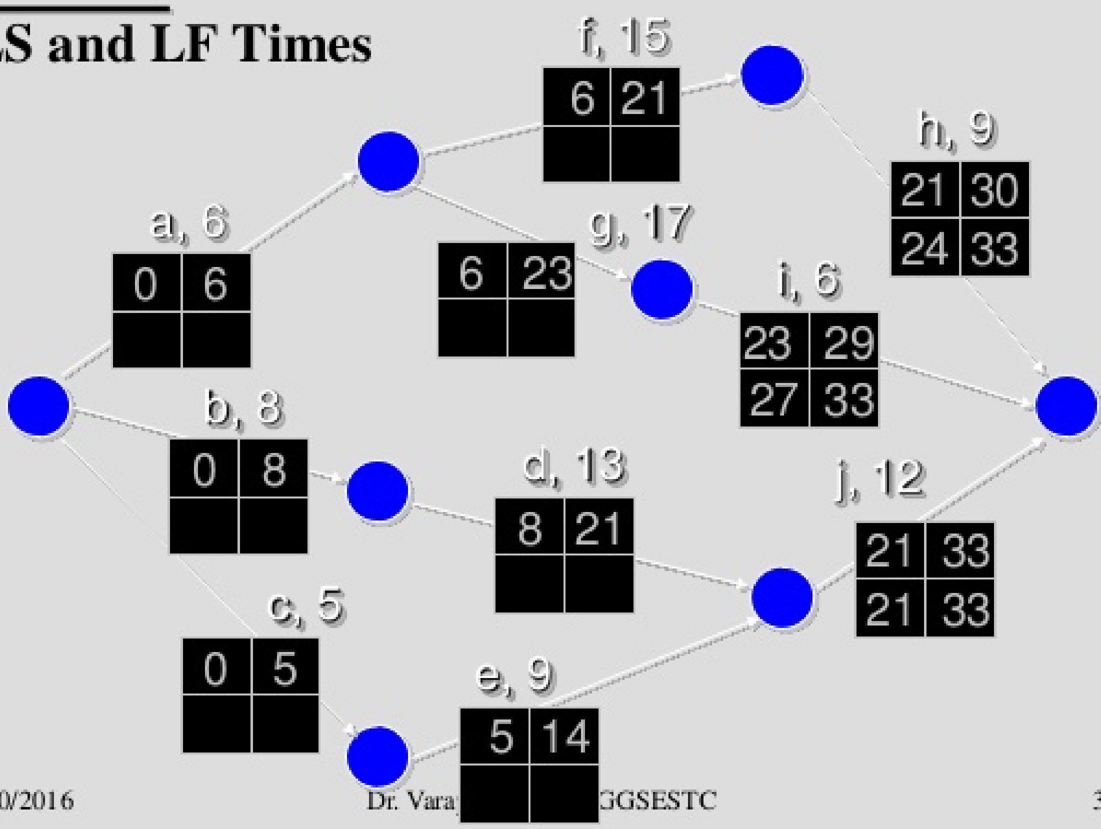
CPM Example

- **ES and EF Times**



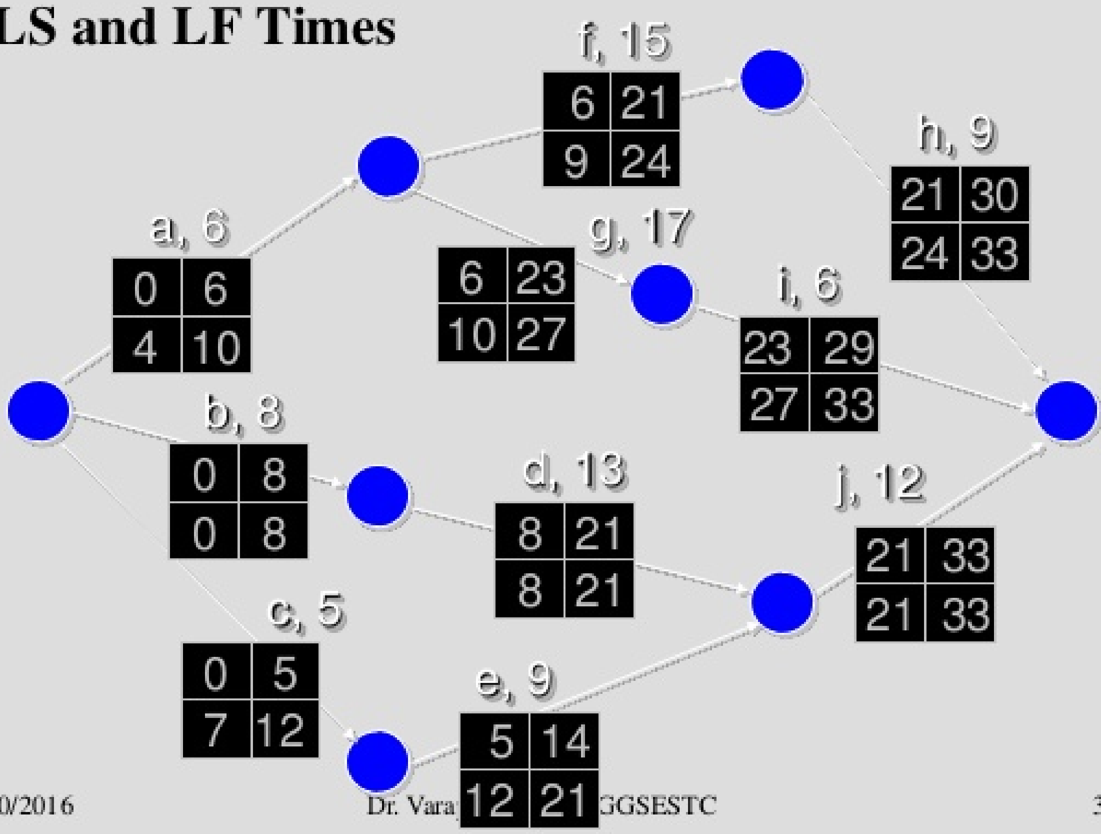
CPM Example

- **LS and LF Times**



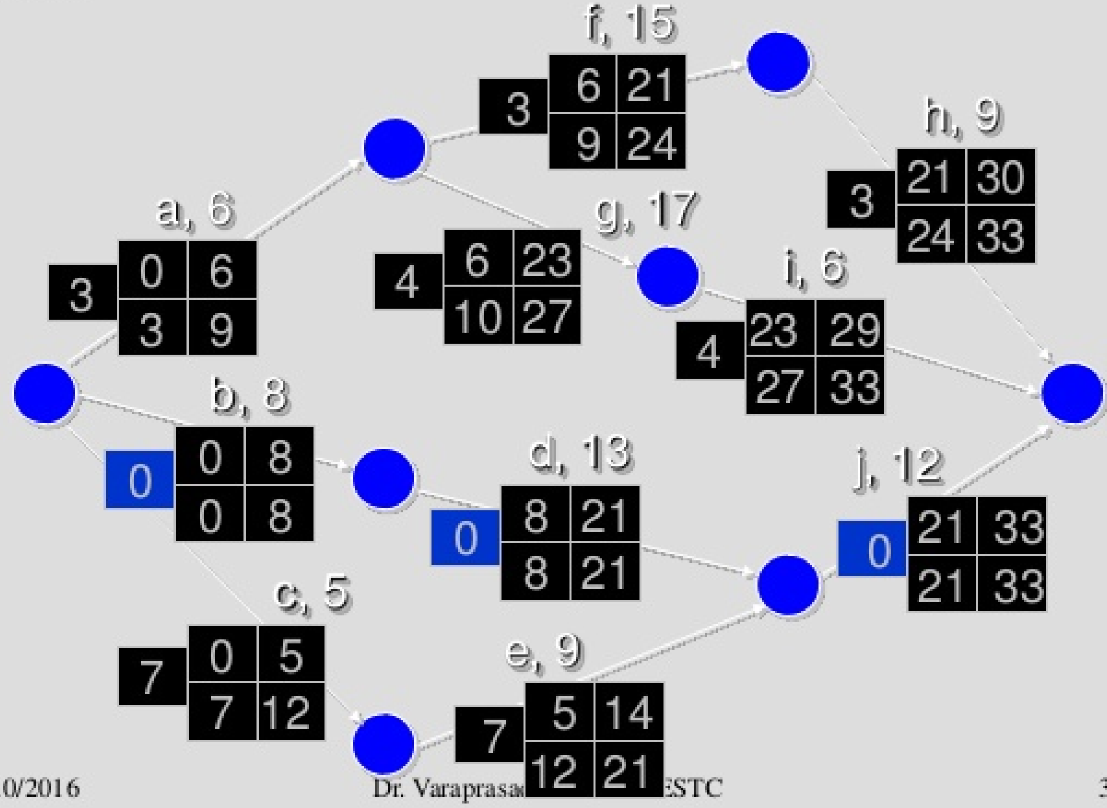
CPM Example

- **LS and LF Times**



CPM Example

- **Float**



CPM Example

- **Critical Path**