In order to have the exact velocity ratio of 4.5, we shall take

$$T_P = 18 \text{ and } T_G = 81 \textbf{ Ans.}$$

∴ Pitch circle diameter of the pinion,

$$D_P = m \times T_P = 10 \times 18 = 180 \text{ mm } \textbf{Ans.}$$

and pitch circle diameter of the gear,

$$D_G = m \times T_G = 10 \times 81 = 810 \text{ mm } \textbf{Ans.}$$

28.24 Spur Gear Construction

The gear construction may have different designs depending upon the size and its application. When the dedendum circle diameter is slightly greater than the shaft diameter, then the pinion teeth are cut integral with the shaft as shown in Fig. 28.13 (*a*). If the pitch circle diameter of the pinion is less than or equal to 14.75 m + 60 mm (where m is the module in mm), then the pinion is made solid with uniform thickness equal to the face width, as shown in Fig. 28.13 (*b*). Small gears upto 250 mm pitch circle diameter are built with a web, which joins the hub and the rim. The web thickness is generally equal to half the circular pitch or it may be taken as 1.6 m to 1.9 m, where m is the module. The web may be made solid as shown in Fig. 28.13 (*c*) or may have recesses in order to reduce its weight.

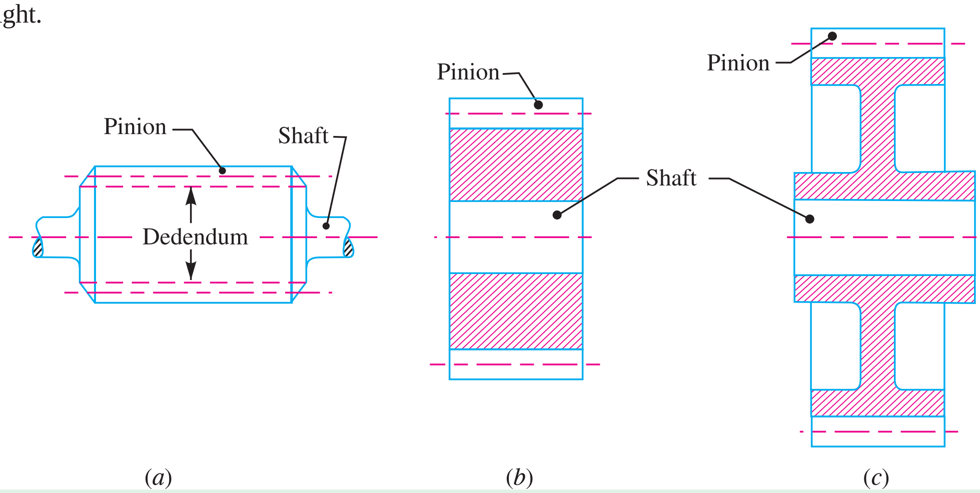


Fig. 28.13. Construction of spur gears.

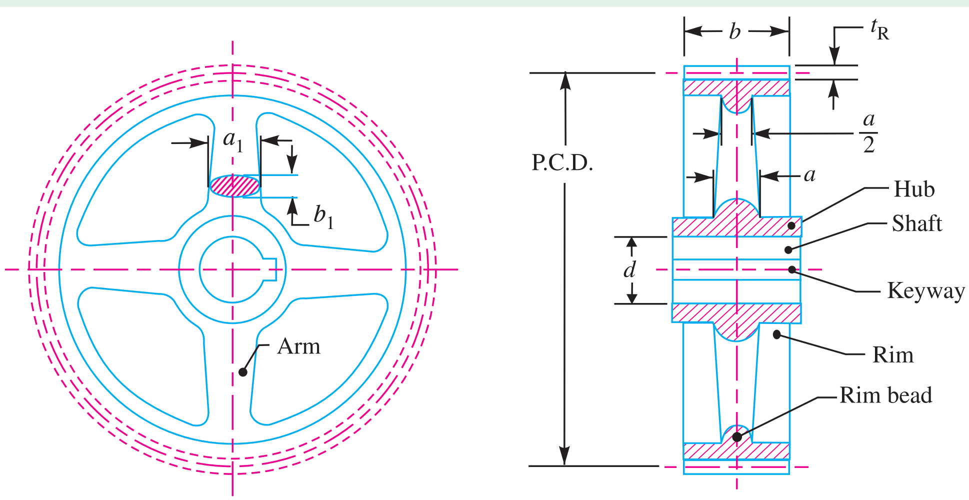


Fig. 28.14. Gear with arms.

Large gears are provided with arms to join the hub and the rim, as shown in Fig. 28.14. The number of arms depends upon the pitch circle diameter of the gear. The number of arms may be selected from the following table.

Table 28.11. Number of arms for the gears.

S. No.	*Pitch circle diameter*	*Number of arms*
1.	Up to 0.5 m	4 or 5
2.	0.5 – 1.5 m	6
3.	1.5 – 2.0 m	8
4.	Above 2.0 m	10

The cross-section of the arms is most often elliptical, but other sections as shown in Fig. 28.15 may also be used.

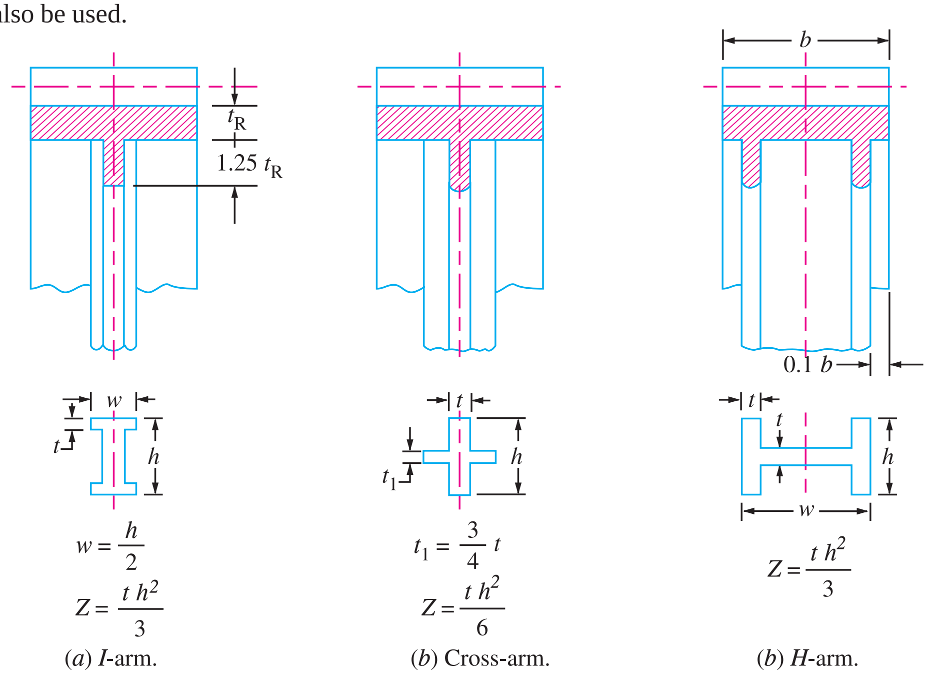


Fig. 28.15. Cross-section of the arms.

The hub diameter is kept as 1.8 times the shaft diameter for steel gears, twice the shaft diameter for cast iron gears and 1.65 times the shaft diameter for forged steel gears used for light service. The length of the hub is kept as 1.25 times the shaft diameter for light service and should not be less than the face width of the gear.

The thickness of the gear rim should be as small as possible, but to facilitate casting and to avoid sharp changes of section, the minimum thickness of the rim is generally kept as half of the circular pitch (or it may be taken as 1.6 *m* to 1.9 *m*, where *m* is the module). The thickness of rim (t_R) may also be calculated by using the following relation, *i.e.*

$$t_R = m\sqrt{\frac{T}{n}}$$

where T = Number of teeth, and

n = Number of arms.

The rim should be provided with a circumferential rib of thickness equal to the rim thickness.

28.25 Design of Shaft for Spur Gears

In order to find the diameter of shaft for spur gears, the following procedure may be followed.

1. First of all, find the normal load (W_N), acting between the tooth surfaces. It is given by

$$W_N = W_T / \cos \phi$$

where W_T = Tangential load, and

ϕ = Pressure angle.

A thrust parallel and equal to W_N will act at the gear centre as shown in Fig. 28.16.

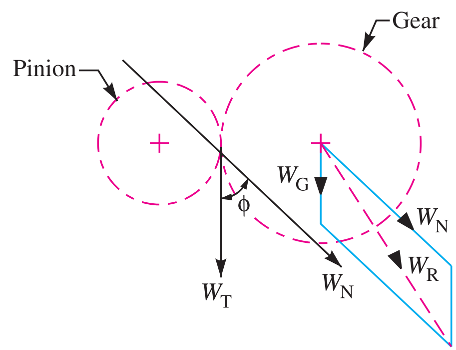


Fig. 28.16. Load acting on the gear.

2. The weight of the gear is given by

$$W_G = 0.001\,18\ T_G.b.m^2 \text{ (in N)}$$

where T_G = No. of teeth on the gear,

b = Face width in mm, and

m = Module in mm.

3. Now the resultant load acting on the gear,

$$W_R = \sqrt{(W_N)^2 + (W_G)^2 + 2\, W_N \times W_G \cos \phi}$$

4. If the gear is overhung on the shaft, then bending moment on the shaft due to the resultant load,

$$M = W_R \times x$$

where x = Overhang *i.e.* the distance between the centre of gear and the centre of bearing.

5. Since the shaft is under the combined effect of torsion and bending, therefore we shall determine the equivalent torque. We know that equivalent torque,

$$T_e = \sqrt{M^2 + T^2}$$

where T = Twisting moment = $W_T \times D_G / 2$

6. Now the diameter of the gear shaft (d) is determined by using the following relation, *i.e.*

$$T_e = \frac{\pi}{16} \times \tau \times d^3$$

where τ = Shear stress for the material of the gear shaft.

Note : Proceeding in the similar way as discussed above, we may calculate the diameter of the pinion shaft.

28.26 Design of Arms for Spur Gears

The cross-section of the arms is calculated by assuming them as a cantilever beam fixed at the hub and loaded at the pitch circle. It is also assumed that the load is equally distributed to all the arms. It may be noted that the arms are designed for the stalling load. The ***stalling load*** is a load that will develop the maximum stress in the arms and in the teeth. This happens at zero velocity, when the drive just starts operating.

The stalling load may be taken as the design tangential load divided by the velocity factor.

Let W_S = Stalling load = $\dfrac{\text{Design tangential load}}{\text{Velocity factor}} = \dfrac{W_T}{C_v}$,

D_G = Pitch circle diameter of the gear,

n = Number of arms, and

σ_b = Allowable bending stress for the material of the arms.

Now, maximum bending moment on each arm,

$$M = \frac{W_S \times D_G / 2}{n} = \frac{W_S \times D_G}{2n}$$

and the section modulus of arms for elliptical cross-section,

$$Z = \frac{\pi (a_1)^2 b_1}{32}$$

where a_1 = Major axis, and b_1 = Minor axis.

The major axis is usually taken as twice the minor axis. Now, using the relation, $\sigma_b = M / Z$, we can calculate the dimensions a_1 and b_1 for the gear arm at the hub end.

Note : The arms are usually tapered towards the rim about 1/16 per unit length of the arm (or radius of the gear).

∴ Major axis of the section at the rim end

$$= a_1 - \text{Taper} = a_1 - \frac{1}{16} \times \text{Length of the arm} = a_1 - \frac{1}{16} \times \frac{D_G}{2} = a_1 - \frac{D_G}{32}$$

Example 28.7. *A motor shaft rotating at 1500 r.p.m. has to transmit 15 kW to a low speed shaft with a speed reduction of 3:1. The teeth are $14\frac{1}{2}°$ involute with 25 teeth on the pinion. Both the pinion and gear are made of steel with a maximum safe stress of 200 MPa. A safe stress of 40 MPa may be taken for the shaft on which the gear is mounted and for the key.*

Design a spur gear drive to suit the above conditions. Also sketch the spur gear drive. Assume starting torque to be 25% higher than the running torque.

Solution : Given : N_P = 1500 r.p.m. ; P = 15 kW = 15 × 10^3 W ; *V.R.* = T_G/T_P = 3 ; $\phi = 14\frac{1}{2}°$; T_P = 25 ; $\sigma_{OP} = \sigma_{OG}$ = 200 MPa = 200 N/mm^2 ; τ = 40 MPa = 40 N/mm^2

Design for spur gears

Since the starting torque is 25% higher than the running torque, therefore the spur gears should be designed for power,

$$P_1 = 1.25\, P = 1.25 \times 15 \times 10^3 = 18\,750 \text{ W}$$

We know that the gear reduction ratio (T_G / T_P) is 3. Therefore the number of teeth on the gear,

$$T_G = 3\, T_P = 3 \times 25 = 75$$

Let us assume that the module (m) for the pinion and gear is 6 mm.

∴ Pitch circle diameter of the pinion,

$$D_P = m.T_P = 6 \times 25 = 150 \text{ mm} = 0.15 \text{ m}$$

and pitch circle diameter of the gear,

$$D_G = m.\, T_G = 6 \times 75 = 450 \text{ mm}$$

We know that pitch line velocity,

$$v = \frac{\pi D_P . N_P}{60} = \frac{\pi \times 0.15 \times 1500}{60} = 11.8 \text{ m/s}$$

Assuming steady load conditions and 8–10 hours of service per day, the service factor (C_S) from Table 28.10 is given by

$$C_S = 1$$

∴ Design tangential tooth load,

$$W_T = \frac{P_1}{v} \times C_S = \frac{18\,750}{11.8} \times 1 = 1590 \text{ N}$$

We know that for ordinary cut gears and operating at velocities upto 12.5 m/s, the velocity factor,

$$C_v = \frac{3}{3 + v} = \frac{3}{3 + 11.8} = 0.203$$

Since both the pinion and the gear are made of the same material, therefore the pinion is the weaker.

We know that for $14\frac{1}{2}°$ involute teeth, tooth form factor for the pinion,

$$y_P = 0.124 - \frac{0.684}{T_P} = 0.124 - \frac{0.684}{25} = 0.0966$$

Let b = Face width for both the pinion and gear.

We know that the design tangential tooth load (W_T),

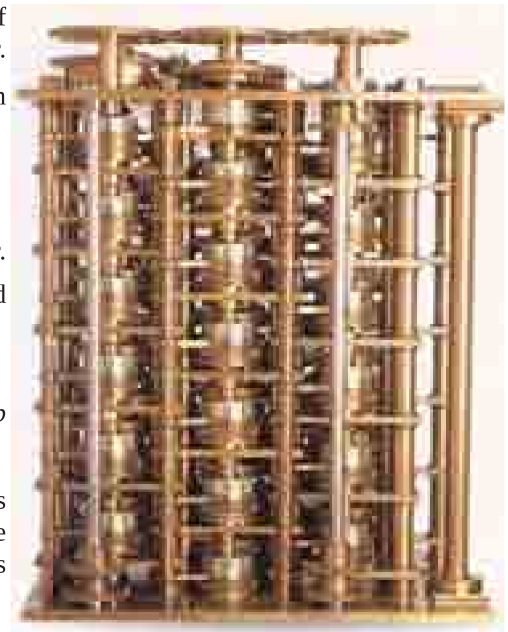
$$1590 = \sigma_{wP}.b.\pi\, m.y_P = (\sigma_{OP}.C_v)\, b.\pi\, m.y_P$$

$$= (200 \times 0.203)\, b \times \pi \times 6 \times 0.0966 = 74\, b$$

$\therefore$ $b = 1590 / 74 = 21.5$ mm

In actual practice, the face width (b) is taken as 9.5 m to 12.5 m, but in certain cases, due to space limitations, it may be taken as 6 m. Therefore let us take the face width,

$$b = 6\, m = 6 \times 6 = 36 \text{ mm}$$ **Ans.**



This mathematical machine called difference engine, assembled in 1832, used 2,000 levers, cams and gears.

From Table 28.1, the other proportions, for the pinion and the gear having $14\frac{1}{2}°$ involute teeth, are as follows :

Addendum	=	$1\, m = 6$ mm **Ans.**
Dedendum	=	$1.25\, m = 1.25 \times 6 = 7.5$ mm **Ans.**
Working depth	=	$2\, m = 2 \times 6 = 12$ mm **Ans.**
Minimum total depth	=	$2.25\, m = 2.25 \times 6 = 13.5$ mm **Ans.**
Tooth thickness	=	$1.5708\, m = 1.5708 \times 6 = 9.4248$ mm **Ans.**
Minimum clearance	=	$0.25\, m = 0.25 \times 6 = 1.5$ mm **Ans.**

Design for the pinion shaft

We know that the normal load acting between the tooth surfaces,

$$W_N = \frac{W_T}{\cos\phi} = \frac{1590}{\cos 14^1/_2°} = \frac{1590}{0.9681} = 1643 \text{ N}$$

and weight of the pinion,

$$W_P = 0.00118\, T_P.b.m^2 = 0.001\,18 \times 25 \times 36 \times 6^2 = 38 \text{ N}$$

$\therefore$ Resultant load acting on the pinion,

$$^*W_R = \sqrt{(W_N)^2 + (W_P)^2 + 2W_N.W_P.\cos\phi}$$

$$= \sqrt{(1643)^2 + (38)^2 + 2 \times 1643 \times 38 \times \cos 14^1/_2°} = 1680 \text{ N}$$

Assuming that the pinion is overhung on the shaft and taking overhang as 100 mm, therefore

Bending moment on the shaft due to the resultant load,

$$M = W_R \times 100 = 1680 \times 100 = 168\,000 \text{ N-mm}$$

* Since the weight of the pinion (W_P) is very small as compared to the normal load (W_N), therefore it may be neglected. Thus the resultant load acting on the pinion (W_R) may be taken equal to W_N.

and twisting moment on the shaft,

$$T = W_T \times \frac{D_P}{2} = 1590 \times \frac{150}{2} = 119\ 250 \text{ N-mm}$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(168\ 000)^2 + (119\ 250)^2} = 206 \times 10^3 \text{ N-mm}$$

Let d_P = Diameter of the pinion shaft.

We know that equivalent twisting moment (T_e),

$$206 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_P)^2 = \frac{\pi}{16} \times 40\,(d_P)^3 = 7.855\,(d_P)^3$$

∴ $(d_P)^3 = 206 \times 10^3 / 7.855 = 26.2 \times 10^3$ or $d_P = 29.7$ say 30 mm **Ans.**

We know that the diameter of the pinion hub

$= 1.8\, d_P = 1.8 \times 30 = 54$ mm **Ans.**

and length of the hub $= 1.25\, d_P = 1.25 \times 30 = 37.5$ mm

Since the length of the hub should not be less than that of the face width *i.e.* 36 mm, therefore let us take length of the hub as 36 mm. **Ans.**

Note : Since the pitch circle diameter of the pinion is 150 mm, therefore the pinion should be provided with a web and not arms. Let us take thickness of the web as 1.8 *m*, where *m* is the module.

∴ Thickness of the web $= 1.8\, m = 1.8 \times 6 = 10.8$ mm **Ans.**

Design for the gear shaft

We have calculated above that the normal load acting between the tooth surfaces,

$$W_N = 1643 \text{ N}$$

We know that weight of the gear,

$$W_G = 0.001\ 18\ T_G.b.m^2 = 0.001\ 18 \times 75 \times 36 \times 6^2 = 115 \text{ N}$$

∴ Resulting load acting on the gear,

$$W_R = \sqrt{(W_N)^2 + (W_G)^2 + 2W_N \times W_G \cos\phi}$$

$$= \sqrt{(1643)^2 + (115)^2 + 2 \times 1643 \times 115 \cos 14{}^1\!/_2{}^\circ} = 1755 \text{ N}$$

Assuming that the gear is overhung on the shaft and taking the overhang as 100 mm, therefore bending moment on the shaft due to the resultant load,

$$M = W_R \times 100 = 1755 \times 100 = 175\ 500 \text{ N-mm}$$

and twisting moment on the shaft,

$$T = W_T \times \frac{D_G}{2} = 1590 \times \frac{450}{2} = 357\ 750 \text{ N-mm}$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(175\ 500)^2 + (357\ 750)^2} = 398 \times 10^3 \text{ N-mm}$$

Let d_G = Diameter of the gear shaft.

We know that equivalent twisting moment (T_e),

$$398 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_G)^3 = \frac{\pi}{16} \times 40\,(d_G)^3 = 7.855\,(d_G)^3$$

∴ $(d_G)^3 = 398 \times 10^3 / 7.855 = 50.7 \times 10^3$ or $d_G = 37$ say 40 mm **Ans.**

We know that diameter of the gear hub

$$= 1.8\, d_G = 1.8 \times 40 = 72 \text{ mm } \textbf{Ans.}$$

and length of the hub $= 1.25\, d_G = 1.25 \times 40 = 50$ mm **Ans.**

Design for the gear arms

Since the pitch circle diameter of the gear is 450 mm, therefore the gear should be provided with four arms. Let us assume the cross-section of the arms as elliptical with major axis (a_1) equal to twice the minor axis (b_1).

∴ Section modulus of arms,

$$Z = \frac{\pi (a_1)^2 b_1}{32} = \frac{\pi (a_1)^2}{32} \times \frac{a_1}{2} = 0.05\,(a_1)^3 \qquad \text{... } (\because b_1 = a_1/2)$$

Since the arms are designed for the stalling load and stalling load is taken as the design tangential load divided by the velocity factor, therefore stalling load,

$$W_S = \frac{W_T}{C_v} = \frac{1590}{0.203} = 7830 \text{ N} \qquad \text{... } (\because C_v = 0.203)$$

∴ Maximum bending moment on each arm,

$$M = \frac{W_S}{n} \times \frac{D_G}{2} = \frac{7830}{4} \times \frac{450}{2} = 440\ 440 \text{ N-mm}$$

We know that bending stress (σ_b),

$$42 = \frac{M}{Z} = \frac{440\ 440}{0.05\,(a_1)^3} = \frac{9 \times 10^6}{(a_1)^3} \qquad \text{... (Taking } \sigma_b = 42 \text{ N/mm}^2\text{)}$$

∴ $(a_1)^3 = 9 \times 10^6 / 42 = 0.214 \times 10^6$ or $a_1 = 60$ mm **Ans.**

and $b_1 = a_1 / 2 = 60 / 2 = 30$ mm **Ans.**

These dimensions refer to the hub end. Since the arms are tapered towards the rim and the taper is 1 / 16 per unit length of the arm (or radius of the gear), therefore

Major axis of the arm at the rim end,

$$a_2 = a_1 - \text{Taper} = a_1 - \frac{1}{16} \times \frac{D_G}{2}$$

$$= 60 - \frac{1}{16} \times \frac{450}{2} = 46 \text{ mm } \textbf{Ans.}$$

and minor axis of the arm at the rim end,

$$b_2 = \frac{\text{Major axis}}{2} = \frac{46}{2} = 23 \text{ mm } \textbf{Ans.}$$

Design for the rim

The thickness of the rim for the pinion (t_{RP}) may be taken as 1.6 *m* to 1.9 *m*, where *m* is the module. Let us take thickness of the rim for the pinion,

$$t_{RP} = 1.6\, m = 1.6 \times 6 = 9.6 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

The thickness of the rim for the gear (t_{RG}) may be obtained by using the relation,

$$t_{RG} = m \sqrt{\frac{T_G}{n}} = 6 \sqrt{\frac{45}{4}} = 20 \text{ mm } \textbf{Ans.}$$

EXERCISES

1. Calculate the power that can be transmitted safely by a pair of spur gears with the data given below. Calculate also the bending stresses induced in the two wheels when the pair transmits this power.

Number of teeth in the pinion = 20

Number of teeth in the gear = 80