32.16 Crankshaft

A crankshaft (*i.e.* a shaft with a crank) is used to convert reciprocating motion of the piston into rotatory motion or vice versa. The crankshaft consists of the shaft parts which revolve in the main bearings, the crankpins to which the big ends of the connecting rod are connected, the crank arms or webs (also called cheeks) which connect the crankpins and the shaft parts. The crankshaft, depending upon the position of crank, may be divided into the following two types :

1. Side crankshaft or overhung crankshaft, as shown in Fig. 32.15 (*a*), and
2. Centre crankshaft, as shown in Fig. 32. 15 (*b*).

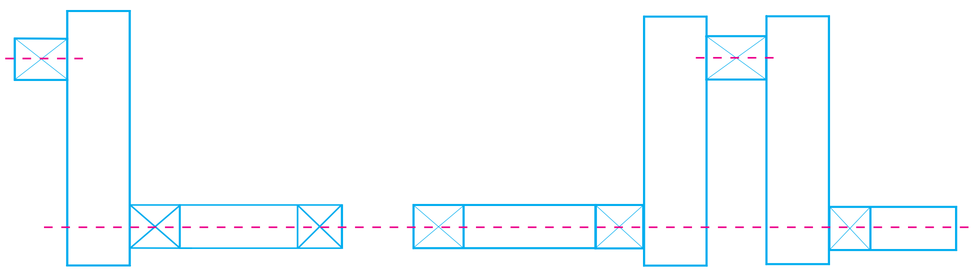


Fig. 32.15. Types of crankshafts.

The crankshaft, depending upon the number of cranks in the shaft, may also be classfied as single throw or multi-throw crankshafts. A crankhaft with only one side crank or centre crank is called a ***single throw crankshaft*** whereas the crankshaft with two side cranks, one on each end or with two or more centre cranks is known as ***multi-throw crankshaft.***

The side crankshafts are used for medium and large size horizontal engines.

32.17 Material and manufacture of Crankshafts

The crankshafts are subjected to shock and fatigue loads. Thus material of the crankshaft should be tough and fatigue resistant. The crankshafts are generally made of carbon steel, special steel or special cast iron.

In industrial engines, the crankshafts are commonly made from carbon steel such as 40 C 8, 55 C 8 and 60 C 4. In transport engines, manganese steel such as 20 Mn 2, 27 Mn 2 and 37 Mn 2 are generally used for the making of crankshaft. In aero engines, nickel chromium steel such as 35 Ni 1 Cr 60 and 40 Ni 2 Cr 1 Mo 28 are extensively used for the crankshaft.

The crankshafts are made by drop forging or casting process but the former method is more common. The surface of the crankpin is hardened by case carburizing, nitriding or induction hardening.

32.18 Bearing Pressures and Stresses in Crankshaft

The bearing pressures are very important in the design of crankshafts. The *maximum permissible bearing pressure depends upon the maximum gas pressure, journal velocity, amount and method of lubrication and change of direction of bearing pressure.

The following two types of stresses are induced in the crankshaft.

1. Bending stress ; and 2. Shear stress due to torsional moment on the shaft.

* The values of maximum permissible bearing pressures for different types of engines are given in Chapter 26, Table 26.3.

Most crankshaft failures are caused by a progressive fracture due to repeated bending or reversed torsional stresses. Thus the crankshaft is under fatigue loading and, therefore, its design should be based upon endurance limit. Since the failure of a crankshaft is likely to cause a serious engine destruction and neither all the forces nor all the stresses acting on the crankshaft can be determined accurately, therefore a high factor of safety from 3 to 4, based on the endurance limit, is used.

The following table shows the allowable bending and shear stresses for some commonly used materials for crankshafts :

Table 32.2. Allowable bending and shear stresses.

Material	*Endurance limit in MPa*		*Allowable stress in MPa*	
	Bending	*Shear*	*Bending*	*Shear*
Chrome nickel	525	290	130 to 175	72.5 to 97
Carbon steel and cast steel	225	124	56 to 75	31 to 42
Alloy cast iron	140	140	35 to 47	35 to 47

32.19 Design Procedure for Crankshaft

The following procedure may be adopted for designing a crankshaft.

1. First of all, find the magnitude of the various loads on the crankshaft.

2. Determine the distances between the supports and their position with respect to the loads.

3. For the sake of simplicity and also for safety, the shaft is considered to be supported at the centres of the bearings and all the forces and reactions to be acting at these points. The distances between the supports depend on the length of the bearings, which in turn depend on the diameter of the shaft because of the allowable bearing pressures.

4. The thickness of the cheeks or webs is assumed to be from 0.4 d_s to 0.6 d_s, where d_s is the diameter of the shaft. It may also be taken as 0.22D to 0.32 D, where D is the bore of cylinder in mm.

5. Now calculate the distances between the supports.

6. Assuming the allowable bending and shear stresses, determine the main dimensions of the crankshaft.

Notes: 1. The crankshaft must be designed or checked for at least two crank positions. Firstly, when the crankshaft is subjected to maximum bending moment and secondly when the crankshaft is subjected to maximum twisting moment or torque.

2. The additional moment due to weight of flywheel, belt tension and other forces must be considered.

3. It is assumed that the effect of bending moment does not exceed two bearings between which a force is considered.

32.20 Design of Centre Crankshaft

We shall design the centre crankshaft by considering the two crank possitions, *i.e.* when the crank is at dead centre (or when the crankshaft is subjected to maximum bending moment) and when the crank is at angle at which the twisting moment is maximum. These two cases are discussed in detail as below :

1. *When the crank is at dead centre*. At this position of the crank, the maximum gas pressure on the piston will transmit maximum force on the crankpin in the plane of the crank causing only bending of the shaft. The crankpin as well as ends of the crankshaft will be only subjected to bending moment. Thus, when the crank is at the dead centre, the bending moment on the shaft is maximum and the twisting moment is zero.

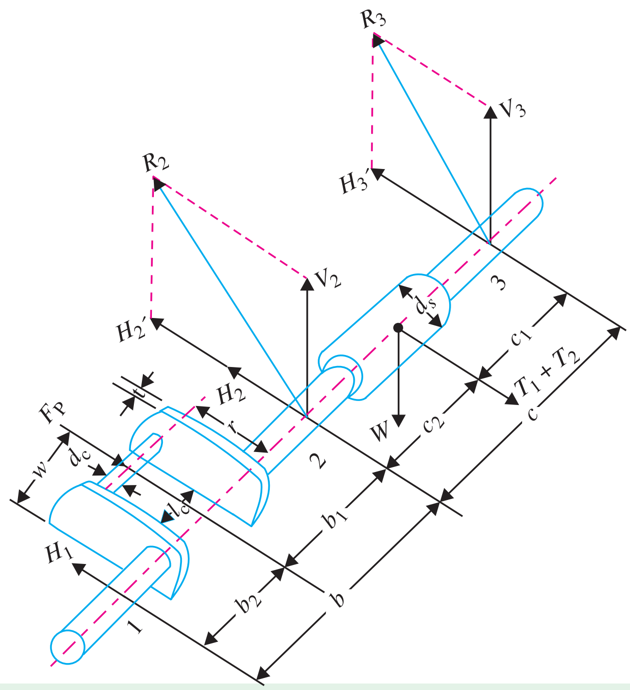


Fig. 32.16. Centre crankshaft at dead centre.

Consider a single throw three bearing crankshaft as shown in Fig. 32.16.

Let D = Piston diameter or cylinder bore in mm,

p = Maximum intensity of pressure on the piston in N/mm^2,

W = Weight of the flywheel acting downwards in N, and

*$T_1 + T_2$ = Resultant belt tension or pull acting horizontally in N.

The thrust in the connecting rod will be equal to the gas load on the piston (F_P). We know that gas load on the piston,

$$F_P = \frac{\pi}{4} \times D^2 \times p$$

Due to this piston gas load (F_P) acting horizontally, there will be two horizontal reactions H_1 and H_2 at bearings 1 and 2 respectively, such that

$$H_1 = \frac{F_P \times b_1}{b}; \quad \text{and} \quad H_2 = \frac{F_P \times b_2}{b}$$

Due to the weight of the flywheel (W) acting downwards, there will be two vertical reactions V_2 and V_3 at bearings 2 and 3 respectively, such that

$$V_2 = \frac{W \times c_1}{c}; \quad \text{and} \quad V_3 = \frac{W \times c_2}{c}$$

Now due to the resultant belt tension ($T_1 + T_2$), acting horizontally, there will be two horizontal reactions H_2' and H_3' at bearings 2 and 3 respectively, such that

$$H_2' = \frac{(T_1 + T_2)\, c_1}{c}; \quad \text{and} \quad H_3' = \frac{(T_1 + T_2)\, c_2}{c}$$

The resultant force at bearing 2 is given by

$$R_2 = \sqrt{(H_2 + H_2')^2 + (V_2)^2}$$

* T_1 is the belt tension in the tight side and T_2 is the belt tension in the slack side.

and the resultant force at bearing 3 is given by

$$R_3 = \sqrt{(H_3)^2 + (V_3)^2}$$

Now the various parts of the centre crankshaft are designed for bending only, as discussed below:

(a) Design of crankpin

Let d_c = Diameter of the crankpin in mm,

l_c = Length of the crankpin in mm,

σ_b = Allowable bending stress for the crankpin in N/mm^2.

We know that bending moment at the centre of the crankpin,

$$M_C = H_1 \cdot b_2 \quad \text{...}(i)$$

We also know that

$$M_C = \frac{\pi}{32}(d_c)^3 \sigma_b \quad \text{...}(ii)$$

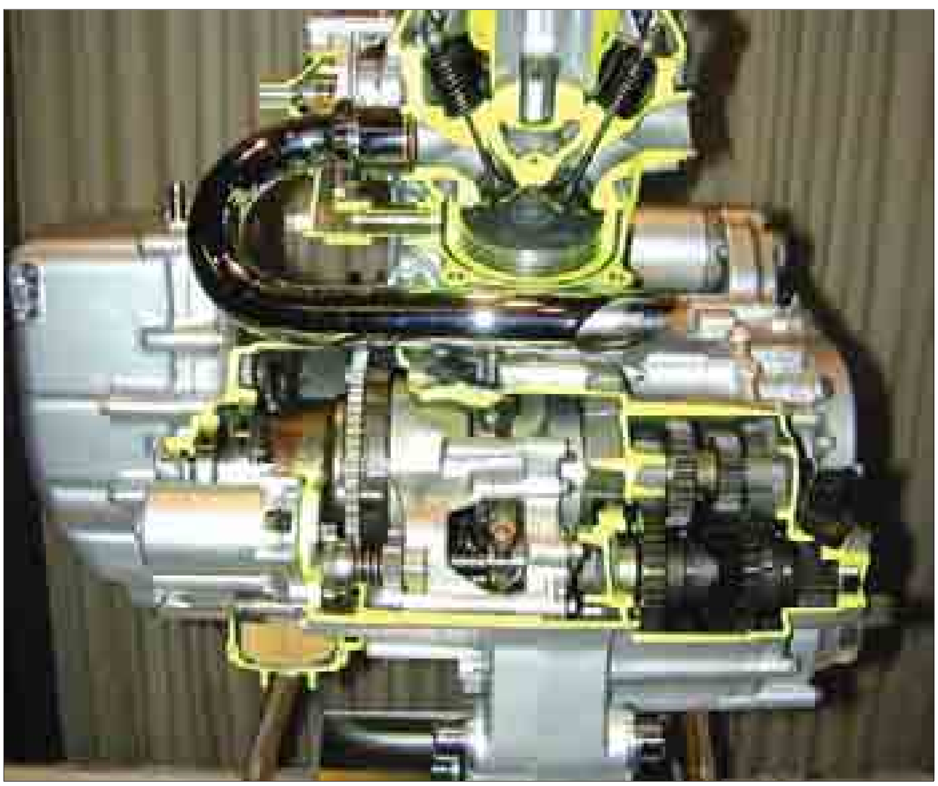
From equations (*i*) and (*ii*), diameter of the crankpin is determined. The length of the crankpin is given by

$$l_c = \frac{F_P}{d_c . p_b}$$

where p_b = Permissible bearing pressure in N/mm^2.

(b) Design of left hand crank web

The crank web is designed for eccentric loading. There will be two stresses acting on the crank web, one is direct compressive stress and the other is bending stress due to piston gas load (F_P).



Water cooled 4-cycle diesel engine

The thickness (t) of the crank web is given empirically as

$$t = 0.4\, d_s \text{ to } 0.6\, d_s$$
$$= 0.22D \text{ to } 0.32D$$
$$= 0.65\, d_c + 6.35 \text{ mm}$$

where d_s = Shaft diameter in mm,

D = Bore diameter in mm, and

d_c = Crankpin diameter in mm,

The width of crank web (w) is taken as

$$w = 1.125\, d_c + 12.7 \text{ mm}$$

We know that maximum bending moment on the crank web,

$$M = H_1\left(b_2 - \frac{l_c}{2} - \frac{t}{2}\right)$$

and section modulus, $Z = \frac{1}{6} \times w.\, t^2$

$\therefore$ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{6H_1\left(b_2 - \frac{l_c}{2} - \frac{t}{2}\right)}{w\,.\,t^2}$

and direct compressive stress on the crank web,

$$\sigma_c = \frac{H_1}{w\,.\,t}$$

$\therefore$ Total stress on the crank web

$$= \text{Bending stress} + \text{Direct stress} = \sigma_b + \sigma_c$$
$$= \frac{6H_1\left(b_2 - \frac{l_c}{2} - \frac{t}{2}\right)}{w\,.\,t^2} + \frac{H_1}{w\,.\,t}$$

This total stress should be less than the permissible bending stress.

(c) Design of right hand crank web

The dimensions of the right hand crank web (*i.e.* thickness and width) are made equal to left hand crank web from the balancing point of view.

(d) Design of shaft under the flywheel

Let d_s = Diameter of shaft in mm.

We know that bending moment due to the weight of flywheel,

$$M_W = V_3\,.\,c_1$$

and bending moment due to belt tension,

$$M_T = H_3'\,.\,c_1$$

These two bending moments act at right angles to each other. Therefore, the resultant bending moment at the flywheel location,

$$M_S = \sqrt{(M_W)^2 + (M_T)^2} = \sqrt{(V_3 \cdot c_1)^2 + (H_3 \cdot c_1)^2} \quad \text{... }(i)$$

We also know that the bending moment at the shaft,

$$M_S = \frac{\pi}{32}(d_s)^3\, \sigma_b \quad \text{... }(ii)$$

where σ_b = Allowable bending stress in N/mm^2.

From equations ***(i)*** and ***(ii)***, we may determine the shaft diameter (d_s).

2. *When the crank is at an angle of maximum twisting moment*

The twisting moment on the crankshaft will be maximum when the tangential force on the crank (F_T) is maximum. The maximum value of tangential force lies when the crank is at angle of 25° to 30° from the dead centre for a constant volume combustion engines (*i.e.*, petrol engines) and 30° to 40° for constant pressure combustion engines (*i.e.*, diesel engines).

Consider a position of the crank at an angle of maximum twisting moment as shown in Fig. 32.17 (*a*). If p' is the intensity of pressure on the piston at this instant, then the piston gas load at this position of crank,

$$F_P = \frac{\pi}{4} \times D^2 \times p'$$

and thrust on the connecting rod,

$$F_Q = \frac{F_P}{\cos\phi}$$

where ϕ = Angle of inclination of the connecting rod with the line of stroke *PO*.

The *thrust in the connecting rod (F_Q) may be divided into two components, one perpendicular to the crank and the other along the crank. The component of F_Q perpendicular to the crank is the tangential force (F_T) and the component of F_Q along the crank is the radial force (F_R) which produces thrust on the crankshaft bearings. From Fig. 32.17 (*b*), we find that

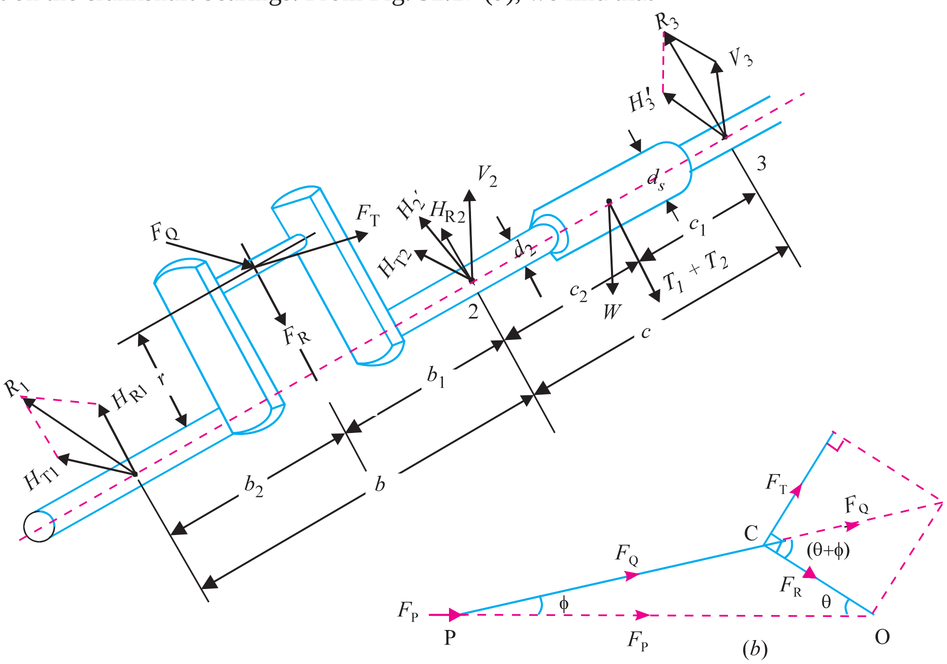


Fig. 32.17. (*a*) Crank at an angle of maximum twisting moment. (*b*) Forces acting on the crank.

$$F_T = F_Q \sin(\theta + \phi)$$

and

$$F_R = F_Q \cos(\theta + \phi)$$

It may be noted that the tangential force will cause twisting of the crankpin and shaft while the radial force will cause bending of the shaft.

* For further details, see Author's popular book on **'Theory of Machines'.**

Due to the tangential force (F_T), there will be two reactions at bearings 1 and 2, such that

$$H_{T1} = \frac{F_T \times b_1}{b}; \quad \text{and} \quad H_{T2} = \frac{F_T \times b_2}{b}$$

Due to the radial force (F_R), there will be two reactions at the bearings 1 and 2, such that

$$H_{R1} = \frac{F_R \times b_1}{b}; \quad \text{and} \quad H_{R2} = \frac{F_R \times b_2}{b}$$

Pull-start motor in an automobile

The reactions at the bearings 2 and 3, due to the flywheel weight (W) and resultant belt pull ($T_1 + T_2$) will be same as discussed earlier.

Now the various parts of the crankshaft are designed as discussed below :

(*a*) *Design of crankpin*

Let d_c = Diameter of the crankpin in mm.

We know that bending moment at the centre of the crankpin,

$$M_C = H_{R1} \times b_2$$

and twisting moment on the crankpin,

$$T_C = H_{T1} \times r$$

∴ Equivalent twisting moment on the crankpin,

$$T_e = \sqrt{(M_C)^2 + (T_C)^2} = \sqrt{(H_{R1} \times b_2)^2 + (H_{T1} \times r)^2} \quad ...(i)$$

We also know that twisting moment on the crankpin,

$$T_e = \frac{\pi}{16}(d_c)^3 \tau \quad ...(ii)$$

where τ = Allowable shear stress in the crankpin.

From equations (*i*) and (*ii*), the diameter of the crankpin is determined.

(b) Design of shaft under the flywheel

Let d_s = Diameter of the shaft in mm.

We know that bending moment on the shaft,

$$M_S = R_3 \times c_1$$

and twisting moment on the shaft,

$$T_S = F_T \times r$$

∴ Equivalent twisting moment on the shaft,

$$T_e = \sqrt{(M_S)^2 + (T_S)^2} = \sqrt{(R_3 \times c_1)^2 + (F_T \times r)^2} \quad \text{... (i)}$$

We also know that equivalent twisting moment on the shaft,

$$T_e = \frac{\pi}{16}(d_s)^3 \tau \quad \text{... (ii)}$$

where τ = Allowable shear stress in the shaft.

From equations **(*i*)** and **(*ii*)**, the diameter of the shaft is determined.

(c) Design of shaft at the juncture of right hand crank arm

Let d_{s1} = Diameter of the shaft at the juncture of right hand crank arm.

We know that bending moment at the juncture of the right hand crank arm,

$$M_{S1} = R_1\left(b_2 + \frac{l_c}{2} + \frac{t}{2}\right) - F_Q\left(\frac{l_c}{2} + \frac{t}{2}\right)$$

and the twisting moment at the juncture of the right hand crank arm,

$$T_{S1} = F_T \times r$$

∴ Equivalent twisting moment at the juncture of the right hand crank arm,

$$T_e = \sqrt{(M_{S1})^2 + (T_{S1})^2} \quad \text{... (i)}$$

We also know that equivalent twisting moment,

$$T_e = \frac{\pi}{16}(d_{s1})^3 \tau \quad \text{... (ii)}$$

where τ = Allowable shear stress in the shaft.

From equations **(*i*)** and **(*ii*)**, the diameter of the shaft at the juncture of the right hand crank arm is determined.

(d) Design of right hand crank web

The right hand crank web is subjected to the following stresses:

(*i*) Bending stresses in two planes normal to each other, due to the radial and tangential components of F_Q,

(*ii*) Direct compressive stress due to F_R, and

(*iii*) Torsional stress.

The bending moment due to the radial component of F_Q is given by,

$$M_R = H_{R2}\left(b_1 - \frac{l_c}{2} - \frac{t}{2}\right) \quad \text{... (i)}$$

We also know that $M_R = \sigma_{bR} \times Z = \sigma_{bR} \times \frac{1}{6} \times w \cdot t^2$ ... **(*ii*)** where σ_{bR} = Bending stress in the radial direction, and

$$Z = \text{Section modulus} = \frac{1}{6} \times w \cdot t^2$$

From equations *(i)* and *(ii)*, the value of bending stress σ_{bR} is determined.

The bending moment due to the tangential component of F_Q is maximum at the juncture of crank and shaft. It is given by

$$M_T = F_T \left[r - \frac{d_{s1}}{2} \right] \quad \text{... (iii)}$$

where d_{s1} = Shaft diameter at juncture of right hand crank arm, *i.e.* at bearing 2.

We also know that

$$M_T = \sigma_{bT} \times Z = \sigma_{bT} \times \frac{1}{6} \times t.w^2 \quad \text{... (iv)}$$

where σ_{bT} = Bending stress in tangential direction.

From equations *(iii)* and *(iv)*, the value of bending stress σ_{bT} is determined.

The direct compressive stress is given by,

$$\sigma_d = \frac{F_R}{2w.t}$$

The maximum compressive stress (σ_c) will occur at the upper left corner of the cross-section of the crank.

$$\therefore \quad \sigma_c = \sigma_{bR} + \sigma_{bT} + \sigma_d$$

Now, the twisting moment on the arm,

$$T = H_{T1}\left(b_2 + \frac{l_c}{2}\right) - F_T \times \frac{l_c}{2} = H_{T2}\left(b_1 - \frac{l_c}{2}\right)$$

We know that shear stress on the arm,

$$\tau = \frac{T}{Z_P} = \frac{4.5\,T}{w.t^2}$$

where Z_P = Polar section modulus = $\dfrac{w.t^2}{4.5}$

∴ Maximum or total combined stress,

$$(\sigma_c)_{max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2}$$

Snow blower on a railway track

The value of $(\sigma_c)_{max}$ should be within safe limits. If it exceeds the safe value, then the dimension *w* may be increased because it does not affect other dimensions.

(e) Design of left hand crank web

Since the left hand crank web is not stressed to the extent as the right hand crank web, therefore, the dimensions for the left hand crank web may be made same as for right hand crank web.

(f) Design of crankshaft bearings

The bearing 2 is the most heavily loaded and should be checked for the safe bearing pressure.

We know that the total reaction at the bearing 2,

$$R_2 = \frac{F_P}{2} + \frac{W}{2} + \frac{T_1 + T_2}{2}$$

$$\therefore \text{ Total bearing pressure } = \frac{R_2}{l_2 \cdot d_{s1}}$$

where l_2 = Length of bearing 2.

32.21 Side or Overhung Crankshaft

The side or overhung crankshafts are used for medium size and large horizontal engines. Its main advantage is that it requires only two bearings in either the single or two crank construction. The design procedure for the side or overhung crankshaft is same as that for centre crankshaft. Let us now design the side crankshaft by considering the two crank positions, *i.e.* when the crank is at dead centre (or when the crankshaft is subjected to maximum bending moment) and when the crank is at an angle at which the twisting moment is maximum. These two cases are discussed in detail as below:

1. *When the crank is at dead centre.* Consider a side crankshaft at dead centre with its loads and distances of their application, as shown in Fig. 32.18.

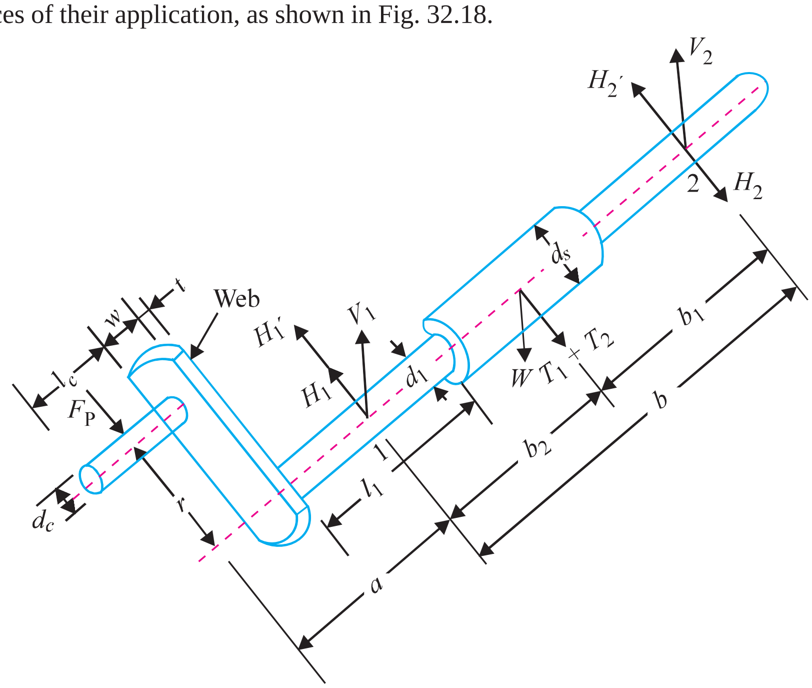


Fig. 32.18. Side crankshaft at dead centre.

Let D = Piston diameter or cylinder bore in mm,

p = Maximum intensity of pressure on the piston in N/mm^2,

W = Weight of the flywheel acting downwards in N, and

$T_1 + T_2$ = Resultant belt tension or pull acting horizontally in N.

We know that gas load on the piston,

$$F_P = \frac{\pi}{4} \times D^2 \times p$$

Due to this piston gas load (F_P) acting horizontally, there will be two horizontal reactions H_1 and H_2 at bearings 1 and 2 respectively, such that

$$H_1 = \frac{F_P\,(a + b)}{b}\,; \quad \text{and} \quad H_2 = \frac{F_P \times a}{b}$$

Due to the weight of the flywheel (W) acting downwards, there will be two vertical reactions V_1 and V_2 at bearings l and 2 respectively, such that

$$V_1 = \frac{W\,.\,b_1}{b}\,; \quad \text{and} \quad V_2 = \frac{W\,.\,b_2}{b}$$

Now due to the resultant belt tension ($T_1 + T_2$) acting horizontally, there will be two horizontal reactions H_1' and H_2' at bearings 1 and 2 respectively, such that

$$H_1' = \frac{(T_1 + T_2)\,b_1}{b}\,; \quad \text{and} \quad H_2' = \frac{(T_1 + T_2)\,b_2}{b}$$

The various parts of the side crankshaft, when the crank is at dead centre, are now designed as discussed below:

(a) Design of crankpin. The dimensions of the crankpin are obtained by considering the crankpin in bearing and then checked for bending stress.

Let d_c = Diameter of the crankpin in mm,

l_c = Length of the crankpin in mm, and

p_b = Safe bearing pressure on the pin in N/mm^2. It may be between 9.8 to 12.6 N/mm^2.

We know that $F_P = d_c\,.\,l_c\,.\,p_b$

From this expression, the values of d_c and l_c may be obtained. The length of crankpin is usually from 0.6 to 1.5 times the diameter of pin.

The crankpin is now checked for bending stress. If it is assumed that the crankpin acts as a cantilever and the load on the crankpin is uniformly distributed, then maximum bending moment will be $\frac{F_P \times l_c}{2}$. But in actual practice, the bearing pressure on the crankpin is not uniformly distributed and may, therefore, give a greater value of bending moment ranging between $\frac{F_P \times l_c}{2}$ and $F_P \times l_c$. So, a mean value of bending moment, *i.e.* $\frac{3}{4} F_P \times l_c$ may be assumed.

Close-up view of an automobile piston

∴ Maximum bending moment at the crankpin,

$$M = \frac{3}{4} F_P \times l_c \qquad \text{... (Neglecting pin collar thickness)}$$

Section modulus for the crankpin,

$$Z = \frac{\pi}{32}(d_c)^3$$

∴ Bending stress induced,

$$\sigma_b = M / Z$$

This induced bending stress should be within the permissible limits.

(b) Design of bearings. The bending moment at the centre of the bearing 1 is given by

$$M = F_P (0.75\, l_c + t + 0.5\, l_1) \qquad \text{...}(i)$$

where l_c = Length of the crankpin,

t = Thickness of the crank web = 0.45 d_c to 0.75 d_c, and

l_1 = Length of the bearing = 1.5 d_c to 2 d_c.

We also know that

$$M = \frac{\pi}{32}(d_1)^3 \sigma_b \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*), the diameter of the bearing 1 may be determined.

Note : The bearing 2 is also made of the same diameter. The length of the bearings are found on the basis of allowable bearing pressures and the maximum reactions at the bearings.

(c) Design of crank web. When the crank is at dead centre, the crank web is subjected to a bending moment and to a direct compressive stress.

We know that bending moment on the crank web,

$$M = F_P (0.75\, l_c + 0.5\, t)$$

and section modulus, $Z = \frac{1}{6} \times w \,.\, t^2$

∴ Bending stress, $\sigma_b = \frac{M}{Z}$

We also know that direct compressive stress,

$$\sigma_d = \frac{F_P}{w \,.\, t}$$

∴ Total stress on the crank web,

$$\sigma_T = \sigma_b + \sigma_d$$

This total stress should be less than the permissible limits.

(d) Design of shaft under the flywheel. The total bending moment at the flywheel location will be the resultant of horizontal bending moment due to the gas load and belt pull and the vertical bending moment due to the flywheel weight.

Let d_s = Diameter of shaft under the flywheel.

We know that horizontal bending moment at the flywheel location due to piston gas load,

$$M_1 = F_P (a + b_2) - H_1 \,.b_2 = H_2 \,.b_1$$

and horizontal bending moment at the flywheel location due to belt pull,

$$M_2 = H_1'.b_2 = H_2'.b_1 = \frac{(T_1 + T_2)\ b_1.b_2}{b}$$

∴ Total horizontal bending moment,

$$M_H = M_1 + M_2$$

We know that vertical bending moment due to flywheel weight,

$$M_V = V_1.b_2 = V_2.b_1 = \frac{W b_1 b_2}{b}$$

∴ Resultant bending moment,

$$M_R = \sqrt{(M_H)^2 + (M_V)^2} \quad \text{...}(i)$$

We also know that

$$M_R = \frac{\pi}{32}(d_s)^3 \sigma_b \quad \text{...}(ii)$$

From equations (*i*) and (*ii*), the diameter of shaft (d_s) may determined.

2. *When the crank is at an angle of maximum twisting moment.* Consider a position of the crank at an angle of maximum twisting moment as shown in Fig. 32.19. We have already discussed in the design of a centre crankshaft that the thrust in the connecting rod (F_Q) gives rise to the tangential force (F_T) and the radial force (F_R).

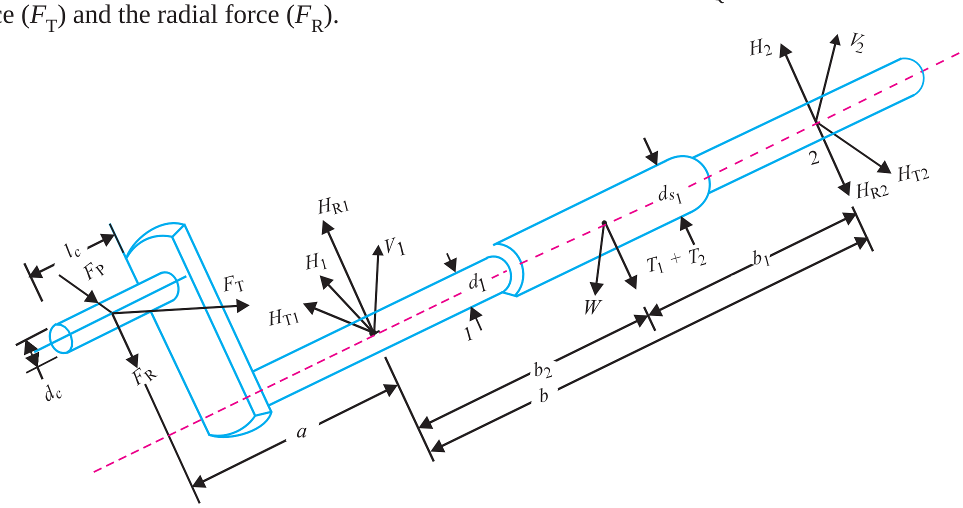


Fig. 32.19. Crank at an angle of maximum twisting moment.

Due to the tangential force (F_T), there will be two reactions at the bearings 1 and 2, such that

$$H_{T1} = \frac{F_T(a+b)}{b}; \quad \text{and} \quad H_{T2} = \frac{F_T \times a}{b}$$

Due to the radial force (F_R), there will be two reactions at the bearings 1 and 2, such that

$$H_{R1} = \frac{F_R(a+b)}{b}; \quad \text{and} \quad H_{R2} = \frac{F_R \times a}{b}$$

The reactions at the bearings 1 and 2 due to the flywheel weight (W) and resultant belt pull ($T_1 + T_2$) will be same as discussed earlier.

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crank web. The most critical section is where the web joins the shaft. This section is subjected to the following stresses :

(*i*) Bending stress due to the tangential force F_T ;

(*ii*) Bending stress due to the radial force F_R ;

(*iii*) Direct compressive stress due to the radial force F_R ; and

(*iv*) Shear stress due to the twisting moment of F_T.

We know that bending moment due to the tangential force,

$$M_{bT} = F_T\left(r - \frac{d_1}{2}\right)$$

where d_1 = Diameter of the bearing 1.

Diesel, petrol and steam engines have crank shaft

∴ Bending stress due to the tangential force,

$$\sigma_{bT} = \frac{M_{bT}}{Z} = \frac{6M_{bT}}{t.w^2} \qquad \ldots\left(\because Z = \frac{1}{6} \times t \cdot w^2\right) \ldots(i)$$

We know that bending moment due to the radial force,

$$M_{bR} = F_R\,(0.75\,l_c + 0.5\,t)$$

∴ Bending stress due to the radial force,

$$\sigma_{bR} = \frac{M_{bR}}{Z} = \frac{6\,M_{bR}}{w.t^2} \qquad \ldots\left(\text{Here } Z = \frac{1}{6} \times w \cdot t^2\right) \ldots(ii)$$

We know that direct compressive stress,

$$\sigma_d = \frac{F_R}{w.t} \qquad \ldots(iii)$$

∴ Total compressive stress,

$$\sigma_c = \sigma_{bT} + \sigma_{bR} + \sigma_d \qquad \ldots(iv)$$

We know that twisting moment due to the tangential force,

$$T = F_T\,(0.75\,l_c + 0.5\,t)$$

∴ Shear stress, $$\tau = \frac{T}{Z_P} = \frac{4.5T}{w.t^2}$$

where Z_P = Polar section modulus = $\dfrac{w.t^2}{4.5}$

Now the total or maximum stress is given by

$$\sigma_{max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2} \quad ...(v)$$

This total maximum stress should be less than the maximum allowable stress.

(b) Design of shaft at the junction of crank

Let d_{s1} = Diameter of the shaft at the junction of the crank.

We know that bending moment at the junction of the crank,

$$M = F_Q (0.75 l_c + t)$$

and twisting moment on the shaft

$$T = F_T \times r$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} \quad ...(i)$$

We also know that equivalent twisting moment,

$$T_e = \frac{\pi}{16}(d_{s1})^3 \tau \quad ...(ii)$$

From equations ***(i)*** and ***(ii)***, the diameter of the shaft at the junction of the crank (d_{s1}) may be determined.

(c) Design of shaft under the flywheel

Let d_s = Diameter of shaft under the flywheel.

The resultant bending moment (M_R) acting on the shaft is obtained in the similar way as discussed for dead centre position.

We know that horizontal bending moment acting on the shaft due to piston gas load,

$$M_1 = F_P (a + b_2) - \left[\sqrt{(H_{R1})^2 + (H_{T1})^2}\right] b_2$$

and horizontal bending moment at the flywheel location due to belt pull,

$$M_2 = H_1'.b_2 = H_2'.b_1 = \frac{(T_1 + T_2)\, b_1.b_2}{b}$$

∴ Total horizontal bending moment,

$$M_H = M_1 + M_2$$

Vertical bending moment due to the flywheel weight,

$$M_V = V_1 \cdot b_2 = V_2 \cdot b_1 = \frac{W b_1 b_2}{b}$$

∴ Resultant bending moment,

$$M_R = \sqrt{(M_H)^2 + (M_V)^2}$$

We know that twisting moment on the shaft,

$$T = F_T \times r$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{(M_R)^2 + T^2} \quad ...(i)$$

We also know that equivalent twisting moment,

$$T_e = \frac{\pi}{16}(d_s)^3 \tau \quad ...(ii)$$

From equations ***(i)*** and ***(ii)***, the diameter of shaft under the flywheel (d_s) may be determined.

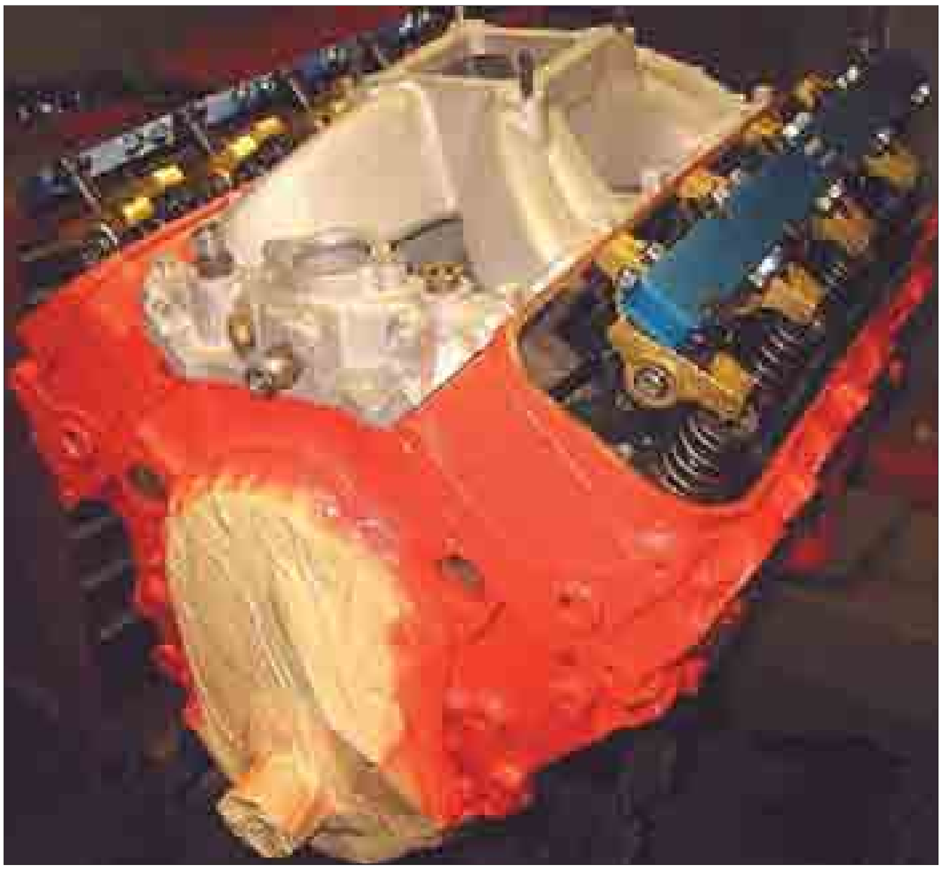
Example 32.4. *Design a plain carbon steel centre crankshaft for a single acting four stroke single cylinder engine for the following data:*

Bore = 400 mm ; Stroke = 600 mm ; Engine speed = 200 r.p.m. ; Mean effective pressure = 0.5 N/mm²; Maximum combustion pressure = 2.5 N/mm²; Weight of flywheel used as a pulley = 50 kN; Total belt pull = 6.5 kN.

When the crank has turned through 35° from the top dead centre, the pressure on the piston is 1N/mm² and the torque on the crank is maximum. The ratio of the connecting rod length to the crank radius is 5. Assume any other data required for the design.

Solution. Given : $D = 400$ mm ; $L = 600$ mm or $r = 300$ mm ; $p_m = 0.5$ N/mm² ; $p = 2.5$ N/mm² ; $W = 50$ kN ; $T_1 + T_2 = 6.5$ kN ; $\theta = 35°$; $p' = 1$N/mm² ; $l / r = 5$

We shall design the crankshaft for the two positions of the crank, *i.e.* firstly when the crank is at the dead centre ; and secondly when the crank is at an angle of maximum twisting moment.



Part of a car engine

1. ***Design of the crankshaft when the crank is at the dead centre*** (See Fig. 32.18)

We know that the piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4}(400)^2 2.5 = 314\,200 \text{ N} = 314.2 \text{ kN}$$

Assume that the distance (b) between the bearings 1 and 2 is equal to twice the piston diameter (D).

$\therefore \qquad b = 2D = 2 \times 400 = 800$ mm and $$b_1 = b_2 = \frac{b}{2} = \frac{800}{2} = 400 \text{ mm}$$

We know that due to the piston gas load, there will be two horizontal reactions H_1 and H_2 at bearings 1 and 2 respectively, such that

$$H_1 = \frac{F_P \times b_1}{b} = \frac{314.2 \times 400}{800} = 157.1 \text{ kN}$$

and $$H_2 = \frac{F_P \times b_2}{b} = \frac{314.2 \times 400}{800} = 157.1 \text{ kN}$$

Assume that the length of the main bearings to be equal, *i.e.*, $c_1 = c_2 = c / 2$. We know that due to the weight of the flywheel acting downwards, there will be two vertical reactions V_2 and V_3 at bearings 2 and 3 respectively, such that

$$V_2 = \frac{W \times c_1}{c} = \frac{W \times c/2}{c} = \frac{W}{2} = \frac{50}{2} = 25 \text{ kN}$$

and $$V_3 = \frac{W \times c_2}{c} = \frac{W \times c/2}{c} = \frac{W}{2} = \frac{50}{2} = 25 \text{ kN}$$

Due to the resultant belt tension $(T_1 + T_2)$ acting horizontally, there will be two horizontal reactions H_2' and H_3' respectively, such that

$$H_2' = \frac{(T_1 + T_2)\, c_1}{c} = \frac{(T_1 + T_2)\, c/2}{c} = \frac{T_1 + T_2}{2} = \frac{6.5}{2} = 3.25 \text{ kN}$$

and $$H_3' = \frac{(T_1 + T_2)\, c_2}{c} = \frac{(T_1 + T_2)\, c/2}{c} = \frac{T_1 + T_2}{2} = \frac{6.5}{2} = 3.25 \text{ kN}$$

Now the various parts of the crankshaft are designed as discussed below:

(*a*) *Design of crankpin*

Let d_c = Diameter of the crankpin in mm ;

l_c = Length of the crankpin in mm ; and

σ_b = Allowable bending stress for the crankpin. It may be assumed as 75 MPa or N/mm^2.

We know that the bending moment at the centre of the crankpin,

$$M_C = H_1 \cdot b_2 = 157.1 \times 400 = 62\,840 \text{ kN-mm} \quad ...(i)$$

We also know that

$$M_C = \frac{\pi}{32}(d_c)^3 \sigma_b = \frac{\pi}{32}(d_c)^3 75 = 7.364 (d_c)^3 \text{ N-mm}$$

$$= 7.364 \times 10^{-3}\, (d_c)^3 \text{ kN-mm} \quad ...(ii)$$

Equating equations (*i*) and (*ii*), we have

$$(d_c)^3 = 62\,840 / 7.364 \times 10^{-3} = 8.53 \times 10^6$$

or d_c = 204.35 say 205 mm **Ans.**

We know that length of the crankpin,

$$l_c = \frac{F_P}{d_c \cdot p_b} = \frac{314.2 \times 10^3}{205 \times 10} = 153.3 \text{ say } 155 \text{ mm} \quad \textbf{Ans.}$$

...(Taking p_b = 10 N/mm^2)

(*b*) *Design of left hand crank web*

We know that thickness of the crank web,

$$t = 0.65\, d_c + 6.35 \text{ mm}$$

$$= 0.65 \times 205 + 6.35 = 139.6 \text{ say } 140 \text{ mm} \quad \textbf{Ans.}$$

and width of the crank web, $w = 1.125\ d_c + 12.7$ mm

$$= 1.125 \times 205 + 12.7 = 243.3 \text{ say } 245 \text{ mm } \textbf{Ans.}$$

We know that maximum bending moment on the crank web,

$$M = H_1\left(b_2 - \frac{l_c}{2} - \frac{t}{2}\right)$$

$$= 157.1\left(400 - \frac{155}{2} - \frac{140}{2}\right) = 39\ 668 \text{ kN-mm}$$

Section modulus, $Z = \frac{1}{6} \times w.t^2 = \frac{1}{6} \times 245\ (140)^2 = 800 \times 10^3 \text{ mm}^3$

$\therefore$ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{39\ 668}{800 \times 10^3} = 49.6 \times 10^{-3} \text{ kN/mm}^2 = 49.6 \text{ N/mm}^2$

We know that direct compressive stress on the crank web,

$$\sigma_c = \frac{H_1}{w.t} = \frac{157.1}{245 \times 140} = 4.58 \times 10^{-3} \text{ kN/mm}^2 = 4.58 \text{ N/mm}^2$$

$\therefore$ Total stress on the crank web

$$= \sigma_b + \sigma_c = 49.6 + 4.58 = 54.18 \text{ N/mm}^2 \text{ or MPa}$$

Since the total stress on the crank web is less than the allowable bending stress of 75 MPa, therefore, the design of the left hand crank web is safe.

(c) Design of right hand crank web

From the balancing point of view, the dimensions of the right hand crank web (*i.e.* thickness and width) are made equal to the dimensions of the left hand crank web.

(d) Design of shaft under the flywheel

Let d_s = Diameter of the shaft in mm.

Since the lengths of the main bearings are equal, therefore

$$l_1 = l_2 = l_3 = 2\left(\frac{b}{2} - \frac{l_c}{2} - t\right) = 2\left(400 - \frac{155}{2} - 140\right) = 365 \text{ mm}$$

Assuming width of the flywheel as 300 mm, we have

$$c = 365 + 300 = 665 \text{ mm}$$

Hydrostatic transmission inside a tractor engine

Allowing space for gearing and clearance, let us take c = 800 mm.

$$\therefore \quad c_1 = c_2 = \frac{c}{2} = \frac{800}{2} = 400 \text{ mm}$$

We know that bending moment due to the weight of flywheel,

$$M_W = V_3 \cdot c_1 = 25 \times 400 = 10\ 000 \text{ kN-mm} = 10 \times 10^6 \text{ N-mm}$$

and bending moment due to the belt pull,

$$M_T = H_3' \cdot c_1 = 3.25 \times 400 = 1300 \text{ kN-mm} = 1.3 \times 10^6 \text{ N-mm}$$

∴ Resultant bending moment on the shaft,

$$M_S = \sqrt{(M_W)^2 + (M_T)^2} = \sqrt{(10 \times 10^6)^2 + (1.3 \times 10^6)^2}$$
$$= 10.08 \times 10^6 \text{ N-mm}$$

We also know that bending moment on the shaft (M_S),

$$10.08 \times 10^6 = \frac{\pi}{32}(d_s)^3 \sigma_b = \frac{\pi}{32}(d_s)^3 42 = 4.12\ (d_s)^3$$

$$\therefore \quad (d_s)^3 = 10.08 \times 10^6 / 4.12 = 2.45 \times 10^6 \text{ or } d_s = 134.7 \text{ say } 135 \text{ mm}$$ **Ans.**

2. *Design of the crankshaft when the crank is at an angle of maximum twisting moment*

We know that piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p' = \frac{\pi}{4}(400)^2 1 = 125\ 680 \text{ N} = 125.68 \text{ kN}$$

In order to find the thrust in the connecting rod (F_Q), we should first find out the angle of inclination of the connecting rod with the line of stroke (*i.e.* angle ϕ). We know that

$$\sin \phi = \frac{\sin \theta}{l/r} = \frac{\sin 35°}{5} = 0.1147$$

$$\therefore \quad \phi = \sin^{-1}(0.1147) = 6.58°$$

We know that thrust in the connecting rod,

$$F_Q = \frac{F_P}{\cos \phi} = \frac{125.68}{\cos 6.58°} = \frac{125.68}{0.9934} = 126.5 \text{ kN}$$

Tangential force acting on the crankshaft,

$$F_T = F_Q \sin(\theta + \phi) = 126.5 \sin(35° + 6.58°) = 84 \text{ kN}$$

and radial force,

$$F_R = F_Q \cos(\theta + \phi) = 126.5 \cos(35° + 6.58°) = 94.6 \text{ kN}$$

Due to the tangential force (F_T), there will be two reactions at bearings 1 and 2, such that

$$H_{T1} = \frac{F_T \times b_1}{b} = \frac{84 \times 400}{800} = 42 \text{ kN}$$

and

$$H_{T2} = \frac{F_T \times b_2}{b} = \frac{84 \times 400}{800} = 42 \text{ kN}$$

Due to the radial force (F_R), there will be two reactions at bearings 1 and 2, such that

$$H_{R1} = \frac{F_R \times b_1}{b} = \frac{94.6 \times 400}{800} = 47.3 \text{ kN}$$

$$H_{R2} = \frac{F_R \times b_2}{b} = \frac{94.6 \times 400}{800} = 47.3 \text{ kN}$$

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crankpin

Let d_c = Diameter of crankpin in mm.

We know that the bending moment at the centre of the crankpin,

$$M_C = H_{R1} \times b_2 = 47.3 \times 400 = 18\,920 \text{ kN-mm}$$

and twisting moment on the crankpin,

$$T_C = H_{T1} \times r = 42 \times 300 = 12\,600 \text{ kN-mm}$$

∴ Equivalent twisting moment on the crankpin,

$$T_e = \sqrt{(M_C)^2 + (T_C)^2} = \sqrt{(18\,920)^2 + (12\,600)^2}$$

$$= 22\,740 \text{ kN-mm} = 22.74 \times 10^6 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$22.74 \times 10^6 = \frac{\pi}{16}(d_c)^3 \tau = \frac{\pi}{16}(d_c)^3 35 = 6.873\,(d_c)^3$$

...(Taking τ = 35 MPa or N/mm^2)

∴ $(d_c)^3 = 22.74 \times 10^6 / 6.873 = 3.3 \times 10^6$ or $d_c = 149$ mm

Since this value of crankpin diameter (*i.e.* d_c = 149 mm) is less than the already calculated value of d_c = 205 mm, therefore, we shall take d_c = 205 mm. **Ans.**

(b) Design of shaft under the flywheel

Let d_s = Diameter of the shaft in mm.

The resulting bending moment on the shaft will be same as calculated eariler, *i.e.*

$$M_S = 10.08 \times 10^6 \text{ N-mm}$$

and twisting moment on the shaft,

$$T_S = F_T \times r = 84 \times 300 = 25\,200 \text{ kN-mm} = 25.2 \times 10^6 \text{ N-mm}$$

∴ Equivalent twisting moment on shaft,

$$T_e = \sqrt{(M_S)^2 + (T_S)^2}$$

$$= \sqrt{(10.08 \times 10^6)^2 + (25.2 \times 10^6)^2} = 27.14 \times 10^6 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$27.14 \times 10^6 = \frac{\pi}{16}(d_s)^3 \tau = \frac{\pi}{16}(135)^3 \tau = 483\,156\,\tau$$

∴ $\tau = 27.14 \times 10^6 / 483\,156 = 56.17$ N/mm^2

From above, we see that by taking the already calculated value of *ds* = 135 mm, the induced shear stress is more than the allowable shear stress of 31 to 42 MPa. Hence, the value of d_s is calculated by taking τ = 35 MPa or N/mm^2 in the above equation, *i.e.*

$$27.14 \times 10^6 = \frac{\pi}{16}(d_s)^3 35 = 6.873\,(d_s)^3$$

∴ $(d_s)^3 = 27.14 \times 10^6 / 6.873 = 3.95 \times 10^6$ or d_s = 158 say 160 mm **Ans.**

(c) Design of shaft at the juncture of right hand crank arm

Let d_{s1} = Diameter of the shaft at the juncture of the right hand crank arm.

We know that the resultant force at the bearing 1,

$$R_1 = \sqrt{(H_{T1})^2 + (H_{R1})^2} = \sqrt{(42)^2 + (47.3)^2} = 63.3 \text{ kN}$$

∴ Bending moment at the juncture of the right hand crank arm,

$$M_{S1} = R_1\left(b_2 + \frac{l_c}{2} + \frac{t}{2}\right) - F_Q\left(\frac{l_c}{2} + \frac{t}{2}\right)$$

$$= 63.3\left(400 + \frac{155}{2} + \frac{140}{2}\right) - 126.5\left(\frac{155}{2} + \frac{140}{2}\right)$$

$$= 34.7 \times 10^3 - 18.7 \times 10^3 = 16 \times 10^3 \text{ kN-mm} = 16 \times 10^6 \text{ N-mm}$$

and twisting moment at the juncture of the right hand crank arm,

$$T_{S1} = F_T \times r = 84 \times 300 = 25\,200 \text{ kN-mm} = 25.2 \times 10^6 \text{ N-mm}$$

∴ Equivalent twisting moment at the juncture of the right hand crank arm,

$$T_e = \sqrt{(M_{S1})^2 + (T_{S1})^2}$$

$$= \sqrt{(16 \times 10^6)^2 + (25.2 \times 10^6)^2} = 29.85 \times 10^6 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$29.85 \times 10^6 = \frac{\pi}{16}(d_{s1})^3 \tau = \frac{\pi}{16}(d_{s1})^3 42 = 8.25\,(d_{s1})^3$$

...(Taking τ = 42 MPa or N/mm^2)

∴ $(d_{s1})^3 = 29.85 \times 10^6 / 8.25 = 3.62 \times 10^6$ or $d_{s1} = 153.5$ say 155 mm **Ans.**

(d) Design of right hand crank web

Let σ_{bR} = Bending stress in the radial direction ; and

σ_{bT} = Bending stress in the tangential direction.

We also know that bending moment due to the radial component of F_Q,

$$M_R = H_{R2}\left(b_1 - \frac{l_c}{2} - \frac{t}{2}\right) = 47.3\left(400 - \frac{155}{2} - \frac{140}{2}\right) \text{kN-mm}$$

$$= 11.94 \times 10^3 \text{ kN-mm} = 11.94 \times 10^6 \text{ N-mm} \quad ...(i)$$

We also know that bending moment,

$$M_R = \sigma_{bR} \times Z = \sigma_{bR} \times \frac{1}{6} \times w.t^2 \quad ...\left(\because Z = \frac{1}{6} \times w.t^2\right)$$

$$11.94 \times 10^6 = \sigma_{bR} \times \frac{1}{6} \times 245\,(140)^2 = 800 \times 10^3 \sigma_{bR}$$

∴ $\sigma_{bR} = 11.94 \times 10^6 / 800 \times 10^3 = 14.9$ N/mm^2 or MPa

We know that bending moment due to the tangential component of F_Q,

$$M_T = F_T\left(r - \frac{d_{s1}}{2}\right) = 84\left(300 - \frac{155}{2}\right) = 18\,690 \text{ kN-mm}$$

$$= 18.69 \times 10^6 \text{ N-mm}$$

We also know that bending moment,

$$M_T = \sigma_{bT} \times Z = \sigma_{bT} \times \frac{1}{6} \times t.w^2 \quad ...\left(\because Z = \frac{1}{6} \times t.w^2\right)$$

$$18.69 \times 10^6 = \sigma_{bT} \times \frac{1}{6} \times 140\,(245)^2 = 1.4 \times 10^6 \sigma_{bT}$$

∴ $\sigma_{bT} = 18.69 \times 10^6 / 1.4 \times 10^6 = 13.35$ N/mm^2 or MPa

Direct compressive stress,

$$\sigma_b = \frac{F_R}{2w \cdot t} = \frac{94.6}{2 \times 245 \times 140} = 1.38 \times 10^{-3} \text{ kN/mm}^2 = 1.38 \text{ N/mm}^2$$

and total compressive stress,

$$\sigma_c = \sigma_{bR} + \sigma_{bT} + \sigma_d$$
$$= 14.9 + 13.35 + 1.38 = 29.63 \text{ N/mm}^2 \text{ or MPa}$$

We know that twisting moment on the arm,

$$T = H_{T2}\left(b_1 - \frac{l_c}{2}\right) = 42\left(400 - \frac{155}{2}\right) = 13\,545 \text{ kN-mm}$$
$$= 13.545 \times 10^6 \text{ N-mm}$$

Piston and piston rod

and shear stress on the arm,

$$\tau = \frac{T}{Z_P} = \frac{4.5T}{w.t^2} = \frac{4.5 \times 13.545 \times 10^6}{245\,(140)^2} = 12.7 \text{ N/mm}^2 \text{ or MPa}$$

We know that total or maximum combined stress,

$$(\sigma_c)_{max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2}$$
$$= \frac{29.63}{2} + \frac{1}{2}\sqrt{(29.63)^2 + 4\,(12.7)^2} = 14.815 + 19.5 = 34.315 \text{ MPa}$$

Since the maximum combined stress is within the safe limits, therefore, the dimension w = 245 mm is accepted.

(e) Design of left hand crank web

The dimensions for the left hand crank web may be made same as for right hand crank web.

(f) Design of crankshaft bearings

Since the bearing 2 is the most heavily loaded, therefore, only this bearing should be checked for bearing pressure.

We know that the total reaction at bearing 2,

$$R_2 = \frac{F_P}{2} + \frac{W}{2} + \frac{T_1 + T_2}{2} = \frac{314.2}{2} + \frac{50}{2} + \frac{6.5}{2} = 185.35 \text{ kN} = 185\,350 \text{ N}$$

∴ Total bearing pressure

$$= \frac{R_2}{l_2 \cdot d_{s1}} = \frac{185\ 350}{365 \times 155} = 3.276 \text{ N/mm}^2$$

Since this bearing pressure is less than the safe limit of 5 to 8 N/mm^2, therefore, the design is safe.

Example 32.5. *Design a side or overhung crankshaft for a 250 mm × 300 mm gas engine. The weight of the flywheel is 30 kN and the explosion pressure is 2.1 N/mm^2. The gas pressure at the maximum torque is 0.9 N/mm^2, when the crank angle is 35° from I. D. C. The connecting rod is 4.5 times the crank radius.*

Solution. Given : D = 250 mm ; L = 300 mm or $r = L / 2 = 300 / 2$ = 150 mm ; W = 30 kN = 30 × 10^3 N ; p = 2.1 N/mm^2, p' = 0.9 N/mm^2 ; $l = 4.5\ r$ or $l / r = 4.5$

We shall design the crankshaft for the two positions of the crank, *i.e.* firstly when the crank is at the dead centre and secondly when the crank is at an angle of maximum twisting moment.

Valve guides of an IC engine

1. ***Design of crankshaft when the crank is at the dead centre*** (See Fig. 32.18)

We know that piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p$$

$$= \frac{\pi}{4}(250)^2 2.1 = 103 \times 10^3 \text{ N}$$

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crankpin

Let d_c = Diameter of the crankpin in mm, and

l_c = Length of the crankpin = 0.8 d_c ...(Assume)

Considering the crankpin in bearing, we have

$F_P = d_c . l_c . p_b$

$103 \times 10^3 = d_c \times 0.8\ d_c \times 10 = 8\ (d_c)^2$ ...(Taking p_b = 10 N/mm^2)

$\therefore\ (d_c)^2 = 103 \times 10^3 / 8 = 12\ 875$ or d_c = 113.4 say 115 mm

and $l_c = 0.8\ d_c = 0.8 \times 115 = 92$ mm

Let us now check the induced bending stress in the crankpin.

We know that bending moment at the crankpin,

$$M = \frac{3}{4} F_P \times l_c = \frac{3}{4} \times 103 \times 10^3 \times 92 = 7107 \times 10^3 \text{ N-mm}$$

and section modulus of the crankpin,

$$Z = \frac{\pi}{32}(d_c)^3 = \frac{\pi}{32}(115)^3 = 149 \times 10^3 \text{mm}^3$$

∴ Bending stress induced

$$= \frac{M}{Z} = \frac{7107 \times 10^3}{149 \times 10^3} = 47.7 \text{ N/mm}^2 \text{ or MPa}$$

Since the induced bending stress is within the permissible limits of 60 MPa, therefore, design of crankpin is safe.

(b) Design of bearings

Let d_1 = Diameter of the bearing 1.

Let us take thickness of the crank web,

$$t = 0.6\,d_c = 0.6 \times 115 = 69 \text{ or } 70 \text{ mm}$$

and length of the bearing, $l_1 = 1.7\,d_c = 1.7 \times 115 = 195.5$ say 200 mm

We know that bending moment at the centre of the bearing 1,

$$M = F_P\,(0.75 l_c + t + 0.5\,l_1)$$
$$= 103 \times 10^3\,(0.75 \times 92 + 70 + 0.5 \times 200) = 24.6 \times 10^6 \text{ N-mm}$$

We also know that bending moment (M),

$$24.6 \times 10^6 = \frac{\pi}{32}(d_1)^3 \sigma_b = \frac{\pi}{32}(d_1)^3 60 = 5.9\,(d_1)^3$$

...(Taking σ_b = 60 MPa or N/mm²)

$\therefore$ $(d_1)^3 = 24.6 \times 10^6 / 5.9 = 4.2 \times 10^6$ or $d_1 = 161.3$ mm say 162 mm **Ans.**

(c) Design of crank web

Let w = Width of the crank web in mm.

We know that bending moment on the crank web,

$$M = F_P\,(0.75 l_c + 0.5\,t)$$
$$= 103 \times 10^3\,(0.75 \times 92 + 0.5 \times 70) = 10.7 \times 10^6 \text{ N-mm}$$

and section modulus, $Z = \frac{1}{6} \times w \cdot t^2 = \frac{1}{6} \times w(70)^2 = 817\,w \text{ mm}^3$

$\therefore$ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{10.7 \times 10^6}{817\,w} = \frac{13 \times 10^3}{w}$ N/mm²

and direct compressive stress,

$$\sigma_b = \frac{F_P}{w.t} = \frac{103 \times 10^3}{w \times 70} = \frac{1.47 \times 10^3}{w} \text{ N/mm}^2$$

We know that total stress on the crank web,

$$\sigma_T = \sigma_b + \sigma_d = \frac{13 \times 10^3}{w} + \frac{1.47 \times 10^3}{w} = \frac{14.47 \times 10^3}{w} \text{ N/mm}^2$$

The total stress should not exceed the permissible limit of 60 MPa or N/mm².

$\therefore$ $60 = \frac{14.47 \times 10^3}{w}$ or $w = \frac{14.47 \times 10^3}{60} = 241$ say 245 mm **Ans.**

(d) Design of shaft under the flywheel.

Let d_s = Diameter of shaft under the flywheel.

First of all, let us find the horizontal and vertical reactions at bearings 1 and 2. Assume that the width of flywheel is 250 mm and $l_1 = l_2 = 200$ mm.

Allowing for certain clearance, the distance

$$b = 250 + \frac{l_1}{2} + \frac{l_2}{2} + \text{clearance}$$
$$= 250 + \frac{200}{2} + \frac{200}{2} + 20 = 470 \text{ mm}$$

and $a = 0.75\,l_c + t + 0.5\,l_1$

$$= 0.75 \times 92 + 70 + 0.5 \times 200 = 239 \text{ mm}$$

We know that the horizontal reactions H_1 and H_2 at bearings 1 and 2, due to the piston gas load (F_P) are

$$H_1 = \frac{F_P(a+b)}{b} = \frac{103 \times 10^3 (239 + 470)}{470} = 155.4 \times 10^3 \text{ N}$$

and
$$H_2 = \frac{F_P \times a}{b} = \frac{103 \times 10^3 \times 239}{470} = 52.4 \times 10^3 \text{ N}$$

Assuming $b_1 = b_2 = b/2$, the vertical reactions V_1 and V_2 at bearings 1 and 2 due to the weight of the flywheel are

$$V_1 = \frac{W \cdot b_1}{b} = \frac{W \times b/2}{b} = \frac{W}{2} = \frac{30 \times 10^3}{2} = 15 \times 10^3 \text{ N}$$

and
$$V_2 = \frac{W \cdot b_2}{b} = \frac{W \times b/2}{b} = \frac{W}{2} = \frac{30 \times 10^3}{2} = 15 \times 10^3 \text{ N}$$

Since there is no belt tension, therefore the horizontal reactions due to the belt tension are neglected.

We know that horizontal bending moment at the flywheel location due to piston gas load.

$$M_1 = F_P(a + b_2) - H_1 \cdot b_2$$

$$= 103 \times 10^3 \left(239 + \frac{470}{2}\right) - 155.4 \times 10^3 \times \frac{470}{2} \qquad \ldots\left(\because b_2 = \frac{b}{2}\right)$$

$$= 48.8 \times 10^6 - 36.5 \times 10^6 = 12.3 \times 10^6 \text{ N-mm}$$

Since there is no belt pull, therefore, there will be no horizontal bending moment due to the belt pull, *i.e.* $M_2 = 0$.

∴ Total horizontal bending moment,

$$M_H = M_1 + M_2 = M_1 = 12.3 \times 10^6 \text{ N-mm}$$

We know that vertical bending moment due to the flywheel weight,

$$M_V = \frac{W . b_1 . b_2}{b} = \frac{W \times b \times b}{2 \times 2 \times b} = \frac{W \times b}{4}$$

$$= \frac{30 \times 10^3 \times 470}{4} = 3.525 \times 10^6 \text{ N-mm}$$

Inside view of a car engine

∴ Resultant bending moment,

$$M_R = \sqrt{(M_H)^2 + (M_V)^2} = \sqrt{(12.3 \times 10^6)^2 + (3.525 \times 10^6)^2}$$
$$= 12.8 \times 10^6 \text{ N-mm}$$

We know that bending moment (M_R),

$$12.8 \times 10^6 = \frac{\pi}{32}(d_s)^3 \sigma_b = \frac{\pi}{32}(d_s)^3\ 60 = 5.9(d_s)^3$$

∴ $(d_s)^3 = 12.8 \times 10^6 / 5.9 = 2.17 \times 10^6$ or $d_s = 129$ mm

Actually d_s should be more than d_1. Therefore let us take

$d_s = 200$ mm **Ans.**

2. Design of crankshaft when the crank is at an angtle of maximum twisting moment

We know that piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p' = \frac{\pi}{4}(250)^2\ 0.9 = 44\ 200\ \text{N}$$

In order to find the thrust in the connecting rod (F_Q), we should first find out the angle of inclination of the connecting rod with the line of storke (*i.e.* angle ϕ). We know that

$$\sin\phi = \frac{\sin\theta}{l/r} = \frac{\sin 35°}{4.5} = 0.1275$$

∴ $\phi = \sin^{-1}(0.1275) = 7.32°$

We know that thrust in the connecting rod,

$$F_Q = \frac{F_P}{\cos\phi} = \frac{44\ 200}{\cos 7.32°} = \frac{44\ 200}{0.9918} = 44\ 565\ \text{N}$$

Tangential force acting on the crankshaft,

$$F_T = F_Q \sin(\theta + \phi) = 44\ 565 \sin(35° + 7.32°) = 30 \times 10^3 \text{ N}$$

and radial force, $F_R = F_Q \cos(\theta + \phi) = 44\ 565 \cos(35° + 7.32°) = 33 \times 10^3$ N

Due to the tangential force (F_T), there will be two reactions at the bearings 1 and 2, such that

$$H_{T1} = \frac{F_T(a+b)}{b} = \frac{30 \times 10^3(239 + 470)}{470} = 45 \times 10^3\ \text{N}$$

and $$H_{T2} = \frac{F_T \times a}{b} = \frac{30 \times 10^3 \times 239}{470} = 15.3 \times 10^3\ \text{N}$$

Due to the radial force (F_R), there will be two reactions at the bearings 1 and 2, such that

$$H_{R1} = \frac{F_R(a+b)}{b} = \frac{33 \times 10^3 \times (239 + 470)}{470} = 49.8 \times 10^3\ \text{N}$$

and $$H_{R2} = \frac{F_R \times a}{b} = \frac{33 \times 10^3 \times 239}{470} = 16.8 \times 10^3\ \text{N}$$

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crank web

We know that bending moment due to the tangential force,

$$M_{bT} = F_T\left(r - \frac{d_1}{2}\right) = 30 \times 10^3\left(150 - \frac{180}{2}\right) = 1.8 \times 10^6\ \text{N-mm}$$

∴ Bending stress due to the tangential force,

$$\sigma_{bT} = \frac{M_{bT}}{Z} = \frac{6M_{bT}}{t.w^2} = \frac{6 \times 1.8 \times 10^6}{70\,(245)^2} \qquad ...(\because Z = \frac{1}{6} \times t.w^2)$$

$$= 2.6 \text{ N/mm}^2 \text{ or MPa}$$

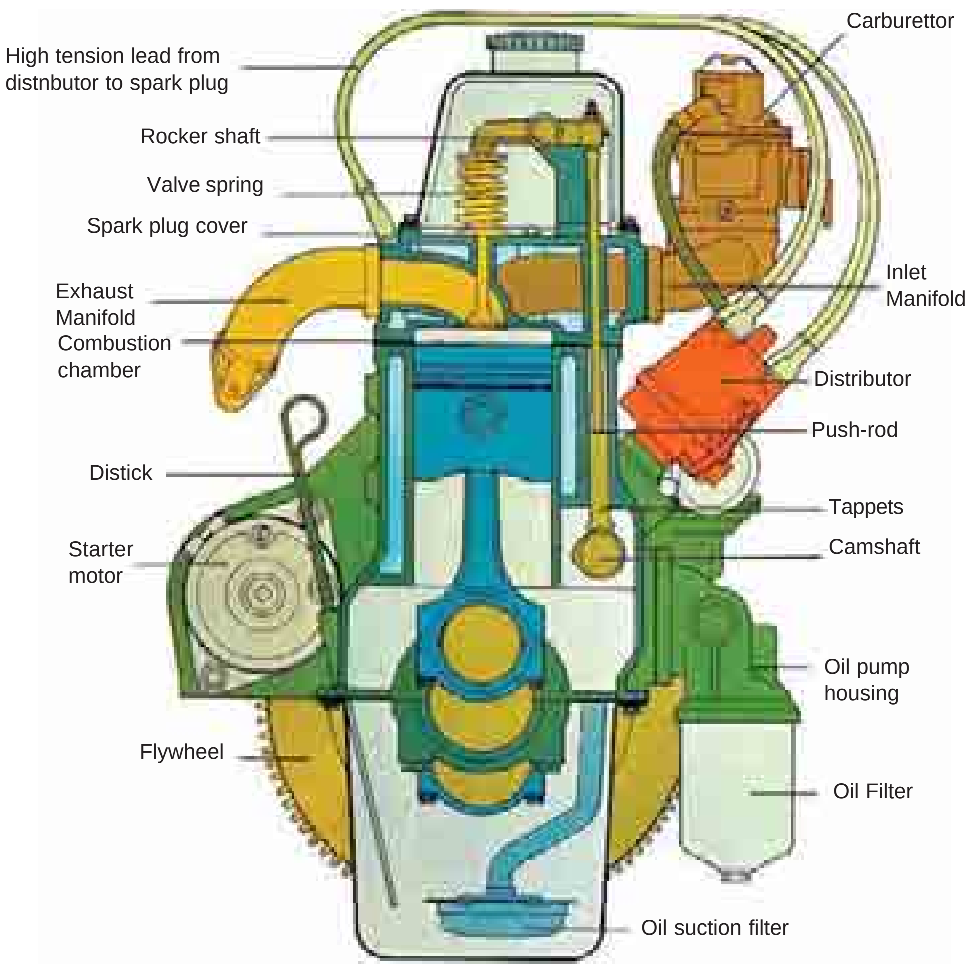
Bending moment due to the radial force,

$$M_{bR} = F_R\,(0.75\,l_c + 0.5\,t)$$
$$= 33 \times 10^3\,(0.75 \times 92 + 0.5 \times 70) = 3.43 \times 10^6 \text{ N-mm}$$

∴ Bending stress due to the radial force,

$$\sigma_{bR} = \frac{M_{bR}}{Z} = \frac{6\,M_{bR}}{w.t^2} \qquad ...(\because Z = \frac{1}{6} \times w.t^2)$$

$$= \frac{6 \times 3.43 \times 10^6}{245\,(70)^2} = 17.1 \text{ N/mm}^2 \text{ or MPa}$$



Schematic of a 4 cylinder IC engine

We know that direct compressive stress,

$$\sigma_d = \frac{F_R}{w \cdot t} = \frac{33 \times 10^3}{245 \times 70} = 1.9 \text{ N/mm}^2 \text{ or MPa}$$

∴ Total compressive stress,

$$\sigma_c = \sigma_{bT} + \sigma_{bR} + \sigma_d = 2.6 + 17.1 + 1.9 = 21.6 \text{ MPa}$$

We know that twisting moment due to the tangential force,

$$T = F_T (0.75\, l_c + 0.5\, t)$$

$$= 30 \times 10^3 (0.75 \times 92 + 0.5 \times 70) = 3.12 \times 10^6 \text{ N-mm}$$

∴ Shear stress, $$\tau = \frac{T}{Z_P} = \frac{4.5\, T}{w \cdot t^2} = \frac{4.5 \times 3.12 \times 10^6}{245\,(70)^2} \qquad \ldots\left[\because Z_P = \frac{w \cdot t^2}{4.5}\right]$$

$$= 11.7 \text{ N/mm}^2 \text{ or MPa}$$

We know that total or maximum stress,

$$\sigma_{max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2} = \frac{21.6}{2} + \frac{1}{2}\sqrt{(21.6)^2 + 4(11.7)^2}$$

$$= 10.8 + 15.9 = 26.7 \text{ MPa}$$

Since this stress is less than the permissible value of 60 MPa, therefore, the design is safe.

(*b*) *Design of shaft at the junction of crank*

Let d_{s1} = Diameter of shaft at the junction of crank.

We know that bending moment at the junction of crank,

$$M = F_Q\,(0.75 l_c + t) = 44\,565\,(0.75 \times 92 + 70) = 6.2 \times 10^6 \text{ N-mm}$$

and twisting moment, $$T = F_T \times r = 30 \times 10^3 \times 150 = 4.5 \times 10^6 \text{ N-mm}$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(6.2 \times 10^6)^2 + (4.5 \times 10^6)^2} = 7.66 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$7.66 \times 10^6 = \frac{\pi}{16}(d_{s1})^3 \tau = \frac{\pi}{16}(180)^3 \tau = 1.14 \times 10^6 \tau \qquad \ldots(\text{Taking } d_{s1} = d_1)$$

∴ $$\tau = 7.66 \times 10^6 / 1.14 \times 10^6 = 6.72 \text{ N/mm}^2 \text{ or MPa}$$

Since the induced shear stress is less than the permissible limit of 30 to 40 MPa, therefore, the design is safe.

(*c*) *Design of shaft under the flywheel*

Let d_s = Diameter of shaft under the flywheel.

We know that horizontal bending moment acting on the shaft due to piston gas load,

$$M_H = F_P\,(a + b_2) - \left[\sqrt{(H_{R1})^2 + (H_{T1})^2}\right] b_2$$

$$= 44\,200\left(239 + \frac{470}{2}\right) - \left[\sqrt{(49.8 \times 10^3)^2 + (45 \times 10^3)^2}\right]\frac{470}{2}$$

$$= 20.95 \times 10^6 - 15.77 \times 10^6 = 5.18 \times 10^6 \text{ N-mm}$$

and bending moment due to the flywheel weight

$$M_V = \frac{W.b_1.b_2}{b} = \frac{30 \times 10^3 \times 235 \times 235}{470} = 3.53 \times 10^6 \text{ N-mm}$$

...($b_1 = b_2 = b / 2 = 470 / 2 = 235$ mm)

∴ Resultant bending moment,

$$M_R = \sqrt{(M_H)^2 + (M_V)^2} = \sqrt{(5.18 \times 10^6)^2 + (3.53 \times 10^6)^2}$$
$$= 6.27 \times 10^6 \text{ N-mm}$$

We know that twisting moment on the shaft,

$$T = F_T \times r = 30 \times 10^3 \times 150 = 4.5 \times 10^6 \text{ N-mm}$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{(M_R)^2 + T^2} = \sqrt{(6.27 \times 10^6)^2 + (4.5 \times 10^6)^2}$$
$$= 7.72 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$7.72 \times 10^6 = \frac{\pi}{16} (d_s)^3 \tau = \frac{\pi}{16} (d_s)^3 \, 30 = 5.9 \, (d_s)^3$$

...(Taking τ = 30 MPa)

∴ $(d_s)^3 = 7.72 \times 10^6 / 5.9 = 1.31 \times 10^6$ or d_s = 109 mm

Actually, d_s should be more than d_1. Therefore let us take

d_s = 200 mm **Ans.**

32.22 Valve Gear Mechanism

The valve gear mechanism of an I.C. engine consists of those parts which actuate the inlet and exhaust valves at the required time with respect to the position of piston and crankshaft.Fig .32.20 (*a*) shows the valve gear arrangement for vertical engines. The main components of the mechanism are valves, rocker arm, * valve springs, **push rod, ***cam and camshaft.

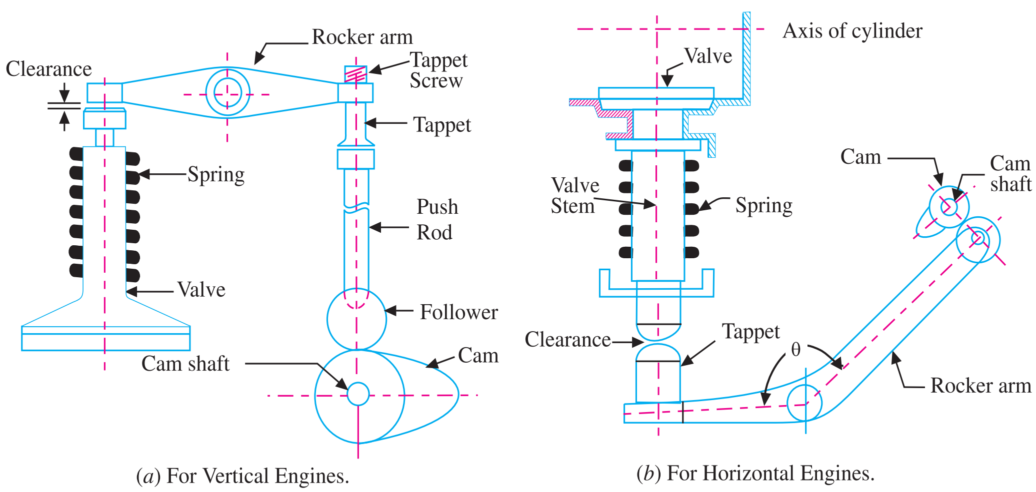


Fig. 32.20. Valve gear mechanism.

* For the design of springs, refer Chapter 23.

** For the design of push rod, refer Chapter 16 (Art. 16.14).

*** For the design of cams, refer to Authors' popular book on **'Theory of Machines'.**