CHAPTER

30

Bevel Gears

30.1 Introduction

The bevel gears are used for transmitting power at a constant velocity ratio between two shafts whose axes intersect at a certain angle. The pitch surfaces for the bevel gear are frustums of cones. The two pairs of cones in contact is shown in Fig. 30.1. The elements of the cones, as shown in Fig. 30.1 (*a*), intersect at the point of intersection of the axis of rotation. Since the radii of both the gears are proportional to their distances from the apex, therefore the cones may roll together without sliding. In Fig. 30.1 (*b*), the elements of both cones do not intersect at the point of shaft intersection. Consequently, there may be pure rolling at only one point of contact and there must be tangential sliding at all other points of contact. Therefore, these cones, cannot be used as pitch surfaces because it is impossible to have positive driving and sliding in the same direction at the same time. We, thus, conclude that the elements of bevel gear pitch cones and shaft axes must intersect at the same point.

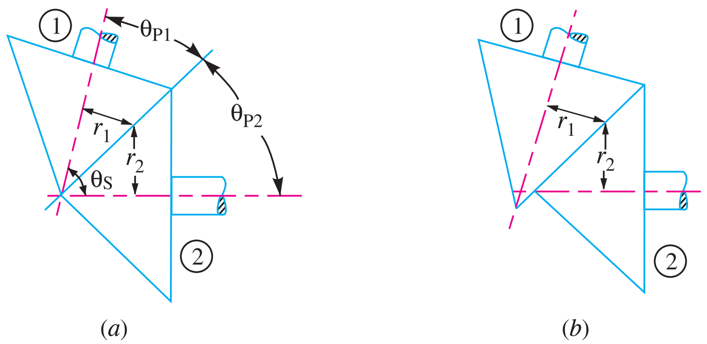


Fig. 30.1. Pitch surface for bevel gears.

The bevel gear is used to change the axis of rotational motion. By using gears of differing numbers of teeth, the speed of rotation can also be changed.

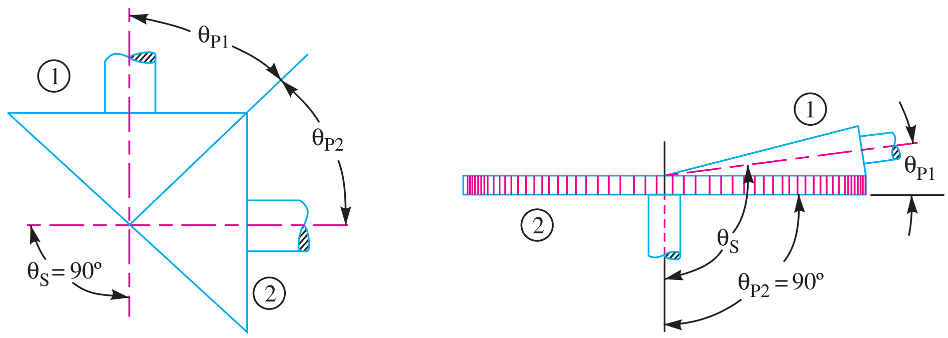
30.2 Classification of Bevel Gears

The bevel gears may be classified into the following types, depending upon the angles between the shafts and the pitch surfaces.

1. *Mitre gears.* When equal bevel gears (having equal teeth and equal pitch angles) connect two shafts whose axes intersect at right angle, as shown in Fig. 30.2 (*a*), then they are known as ***mitre gears.***

2. *Angular bevel gears.* When the bevel gears connect two shafts whose axes intersect at an angle other than a right angle, then they are known as ***angular bevel gears.***

3. *Crown bevel gears.* When the bevel gears connect two shafts whose axes intersect at an angle greater than a right angle and one of the bevel gears has a pitch angle of 90º, then it is known as a crown gear. The crown gear corresponds to a rack in spur gearing, as shown in Fig. 30.2 (*b*).



(*a*) Mitre gears. (*b*) Crown bevel gear.

Fig. 30.2. Classification of bevel gears.

4. *Internal bevel gears.* When the teeth on the bevel gear are cut on the inside of the pitch cone, then they are known as ***internal bevel gears.***

Note : The bevel gears may have straight or spiral teeth. It may be assumed, unless otherwise stated, that the bevel gear has straight teeth and the axes of the shafts intersect at right angle.

30.3 Terms used in Bevel Gears

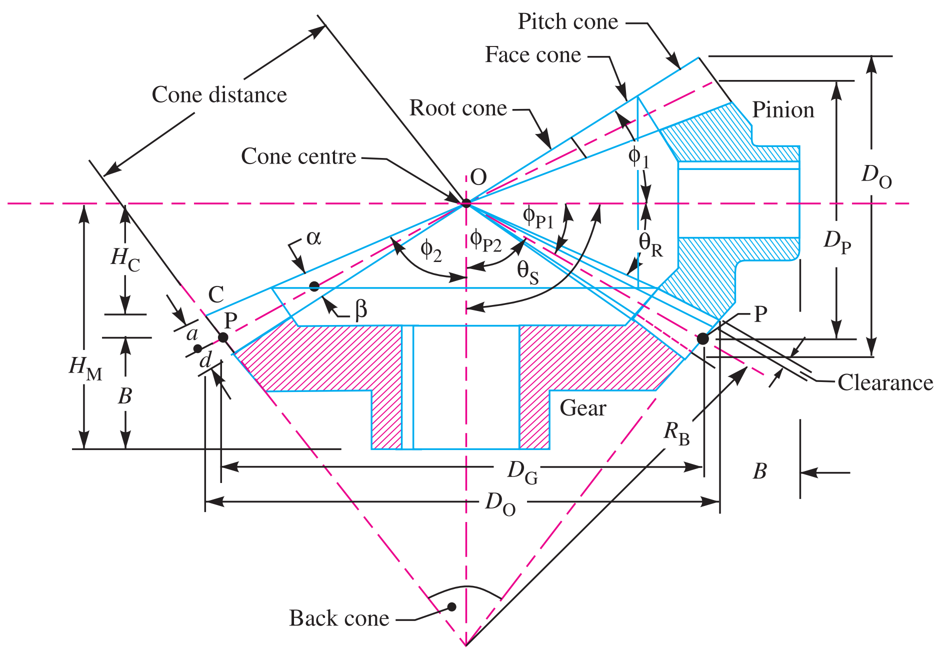


Fig. 30.3. Terms used in bevel gears.

A sectional view of two bevel gears in mesh is shown in Fig. 30.3. The following terms in connection with bevel gears are important from the subject point of view :

1. *Pitch cone.* It is a cone containing the pitch elements of the teeth.

2. *Cone centre.* It is the apex of the pitch cone. It may be defined as that point where the axes of two mating gears intersect each other.

3. *Pitch angle.* It is the angle made by the pitch line with the axis of the shaft. It is denoted by 'θ_P'.

4. *Cone distance.* It is the length of the pitch cone element. It is also called as a ***pitch cone radius***. It is denoted by '*OP*'. Mathematically, cone distance or pitch cone radius,

$$OP = \frac{\text{Pitch radius}}{\sin \theta_P} = \frac{D_P/2}{\sin \theta_{P1}} = \frac{D_G/2}{\sin \theta_{P2}}$$

5. *Addendum angle.* It is the angle subtended by the addendum of the tooth at the cone centre. It is denoted by 'α' Mathematically, addendum angle,

$$\alpha = \tan^{-1}\left(\frac{a}{OP}\right)$$

where a = Addendum, and OP = Cone distance.

6. *Dedendum angle.* It is the angle subtended by the dedendum of the tooth at the cone centre. It is denoted by 'β'. Mathematically, dedendum angle,

$$\beta = \tan^{-1}\left(\frac{d}{OP}\right)$$

where d = Dedendum, and OP = Cone distance.

7. *Face angle.* It is the angle subtended by the face of the tooth at the cone centre. It is denoted by 'ϕ'. The face angle is equal to the pitch angle *plus* addendum angle.

8. *Root angle*. It is the angle subtended by the root of the tooth at the cone centre. It is denoted by 'θ_R'. It is equal to the pitch angle *minus* dedendum angle.

9. *Back (or normal) cone*. It is an imaginary cone, perpendicular to the pitch cone at the end of the tooth.

10. Back cone distance. It is the length of the back cone. It is denoted by 'R_B'. It is also called back cone radius.

11. *Backing.* It is the distance of the pitch point (*P*) from the back of the boss, parallel to the pitch point of the gear. It is denoted by '*B*'.

12. *Crown height.* It is the distance of the crown point (*C*) from the cone centre (*O*), parallel to the axis of the gear. It is denoted by 'H_C'.

13. *Mounting height.* It is the distance of the back of the boss from the cone centre. It is denoted by 'H_M'.

14. *Pitch diameter*. It is the diameter of the largest pitch circle.

15. *Outside or addendum cone diameter*. It is the maximum diameter of the teeth of the gear. It is equal to the diameter of the blank from which the gear can be cut. Mathematically, outside diameter,

$$D_O = D_P + 2\,a \cos \theta_P$$

where D_P = Pitch circle diameter,

a = Addendum, and

θ_P = Pitch angle.

16. *Inside or dedendum cone diameter*. The inside or the dedendum cone diameter is given by

$$D_d = D_P - 2d \cos \theta_P$$

where D_d = Inside diameter, and

d = Dedendum.

30.4 Determination of Pitch Angle for Bevel Gears

Mitre gears

Consider a pair of bevel gears in mesh, as shown in Fig. 30.3.

Let θ_{P1} = Pitch angle for the pinion,

θ_{P2} = Pitch angle for the gear,

θ_S = Angle between the two shaft axes,

D_P = Pitch diameter of the pinion,

D_G = Pitch diameter of the gear, and

$$V.R. = \text{Velocity ratio} = \frac{D_G}{D_P} = \frac{T_G}{T_P} = \frac{N_P}{N_G}$$

From Fig. 30.3, we find that

$$\theta_S = \theta_{P1} + \theta_{P2} \quad \text{or} \quad \theta_{P2} = \theta_S - \theta_{P1}$$

$$\therefore \quad \sin\theta_{P2} = \sin(\theta_S - \theta_{P1}) = \sin\theta_S \,.\, \cos\theta_{P1} - \cos\theta_S \,.\, \sin\theta_{P1} \quad ...(i)$$

We know that cone distance,

$$OP = \frac{D_P/2}{\sin\theta_{P1}} = \frac{D_G/2}{\sin\theta_{P2}} \quad \text{or} \quad \frac{\sin\theta_{P2}}{\sin\theta_{P1}} = \frac{D_G}{D_P} = V.R.$$

$$\therefore \quad \sin\theta_{P2} = V.R. \times \sin\theta_{P1} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$V.R. \times \sin\theta_{P1} = \sin\theta_S \,.\, \cos\theta_{P1} - \cos\theta_S \,.\, \sin\theta_{P1}$$

Dividing throughout by $\cos\theta_{P1}$ we get

$$V.R. \tan\theta_{P1} = \sin\theta_S - \cos\theta_S \,.\, \tan\theta_{P1}$$

or

$$\tan\theta_{P1} = \frac{\sin\theta_S}{V.R + \cos\theta_S}$$

$$\therefore \quad \theta_{P1} = \tan^{-1}\left(\frac{\sin\theta_S}{V.R + \cos\theta_S}\right) \quad ...(iii)$$

Similarly, we can find that

$$\tan\theta_{P2} = \frac{\sin\theta_S}{\dfrac{1}{V.R} + \cos\theta_S}$$

$$\therefore \quad \theta_{P2} = \tan^{-1}\left(\frac{\sin\theta_S}{\dfrac{1}{V.R} + \cos\theta_S}\right) \quad ...(iv)$$

Note : When the angle between the shaft axes is 90º *i.e.* $\theta_S = 90°$, then equations (*iii*) and (*iv*) may be written as

$$\theta_{P1} = \tan^{-1}\left(\frac{1}{V.R}\right) = \tan^{-1}\left(\frac{D_P}{D_G}\right) = \tan^{-1}\left(\frac{T_P}{T_G}\right) = \tan^{-1}\left(\frac{N_G}{N_P}\right)$$

and

$$\theta_{P2} = \tan^{-1}(V.R.) = \tan^{-1}\left(\frac{D_G}{D_P}\right) = \tan^{-1}\left(\frac{T_G}{T_P}\right) = \tan^{-1}\left(\frac{N_P}{N_G}\right)$$

30.5 Proportions for Bevel Gear

The proportions for the bevel gears may be taken as follows :

1. Addendum, $a = 1\,m$

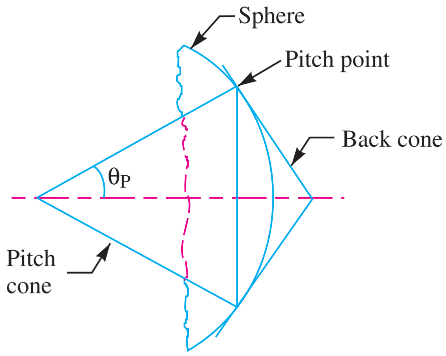
2. Dedendum, $d = 1.2\ m$
3. Clearance $= 0.2\ m$
4. Working depth $= 2\ m$
5. Thickness of tooth $= 1.5708\ m$

where m is the module.

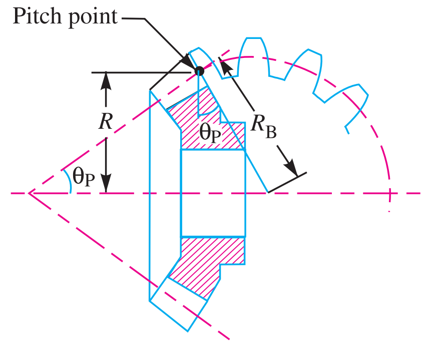
Note : Since the bevel gears are not interchangeable, therefore these are designed in pairs.

30.6 Formative or Equivalent Number of Teeth for Bevel Gears – Tredgold's Approximation

We have already discussed that the involute teeth for a spur gear may be generated by the edge of a plane as it rolls on a base cylinder. A similar analysis for a bevel gear will show that a true section of the resulting involute lies on the surface of a sphere. But it is not possible to represent on a plane surface the exact profile of a bevel gear tooth lying on the surface of a sphere. Therefore, it is important to approximate the bevel gear tooth profiles as accurately as possible. The approximation (known as ***Tredgold's approximation***) is based upon the fact that a cone tangent to the sphere at the pitch point will closely approximate the surface of the sphere for a short distance either side of the pitch point, as shown in Fig. 30.4 (*a*). The cone (known as back cone) may be developed as a plane surface and spur gear teeth corresponding to the pitch and pressure angle of the bevel gear and the radius of the developed cone can be drawn. This procedure is shown in Fig. 30.4 (*b*).



(*a*) Back cone.



(*b*) Development of back cone.

Fig. 30.4

Let θ_P = Pitch angle or half of the cone angle,

R = Pitch circle radius of the bevel pinion or gear, and

R_B = Back cone distance or equivalent pitch circle radius of spur pinion or gear.

Now from Fig. 30.4 (*b*), we find that

$$R_B = R \sec \theta_P$$

We know that the equivalent (or formative) number of teeth,

$$T_E = \frac{2\ R_B}{m} \quad \ldots \left(\because \text{Number of teeth} = \frac{\text{Pitch circle diameter}}{\text{Module}}\right)$$

$$= \frac{2\ R \sec \theta_P}{m} = T \sec \theta_P$$

where T = Actual number of teeth on the gear.

Notes : 1. The action of bevel gears will be same as that of equivalent spur gears.

2. Since the equivalent number of teeth is always greater than the actual number of teeth, therefore a given pair of bevel gears will have a larger contact ratio. Thus, they will run more smoothly than a pair of spur gears with the same number of teeth.

30.7 Strength of Bevel Gears

The strength of a bevel gear tooth is obtained in a similar way as discussed in the previous articles. The modified form of the Lewis equation for the tangential tooth load is given as follows:

$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y' \left(\frac{L-b}{L}\right)$$

where

σ_o = Allowable static stress,

C_v = Velocity factor,

$= \dfrac{3}{3+v}$, for teeth cut by form cutters,

$= \dfrac{6}{6+v}$, for teeth generated with precision machines,

v = Peripheral speed in m / s,

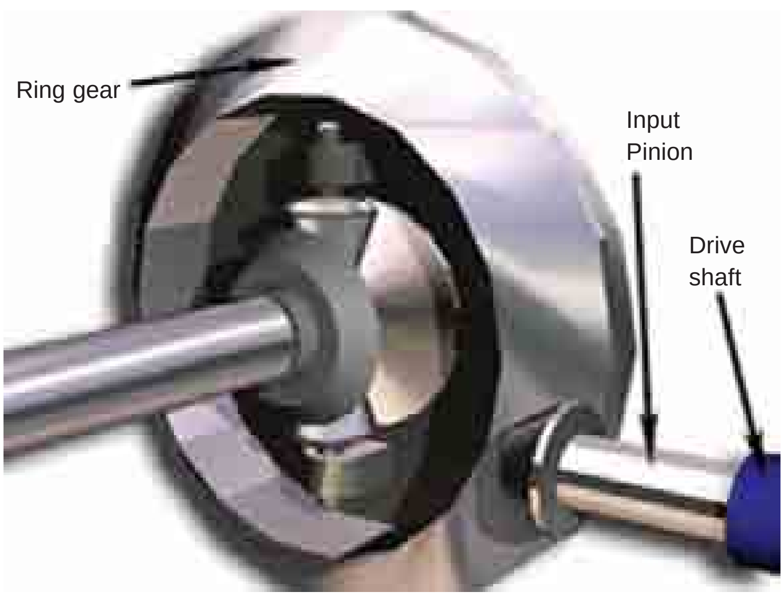
b = Face width,

m = Module,

y' = Tooth form factor (or Lewis factor) for the equivalent number of teeth,

L = Slant height of pitch cone (or cone distance),

$$= \sqrt{\left(\frac{D_G}{2}\right)^2 + \left(\frac{D_P}{2}\right)^2}$$



Hypoid bevel gears in a car differential

D_G = Pitch diameter of the gear, and

D_P = Pitch diameter of the pinion.

Notes : 1. The factor $\left(\frac{L-b}{L}\right)$ may be called as ***bevel factor.***

2. For satisfactory operation of the bevel gears, the face width should be from 6.3 *m* to 9.5 *m*, where *m* is the module. Also the ratio *L* / *b* should not exceed 3. For this, the number of teeth in the pinion must not less than $\frac{48}{\sqrt{1+(V.R.)^2}}$, where *V.R.* is the required velocity ratio.

3. The dynamic load for bevel gears may be obtained in the similar manner as discussed for spur gears.

4. The static tooth load or endurance strength of the tooth for bevel gears is given by

$$W_S = \sigma_e.b.\pi\, m.y' \left(\frac{L-b}{L}\right)$$

The value of flexural endurance limit (σ_e) may be taken from Table 28.8, in spur gears.

5. The maximum or limiting load for wear for bevel gears is given by

$$W_w = \frac{D_P.b.Q.K}{\cos\theta_{P1}}$$

where D_P, *b*, *Q* and *K* have usual meanings as discussed in spur gears except that *Q* is based on formative or equivalent number of teeth, such that

$$Q = \frac{2\,T_{EG}}{T_{EG}+T_{EP}}$$

30.8 Forces Acting on a Bevel Gear

Consider a bevel gear and pinion in mesh as shown in Fig. 30.5. The normal force (W_N) on the tooth is perpendicular to the tooth profile and thus makes an angle equal to the pressure angle (ϕ) to the pitch circle. Thus normal force can be resolved into two components, one is the tangential component (W_T) and the other is the radial component (W_R). The tangential component (*i.e.* the tangential tooth load) produces the bearing reactions while the radial component produces end thrust in the shafts. The magnitude of the tangential and radial components is as follows :

$$W_T = W_N\cos\phi,\ \text{and}\ W_R = W_N\sin\phi = W_T\tan\phi \qquad ...(i)$$

These forces are considered to act at the mean radius (R_m). From the geometry of the Fig. 30.5, we find that

$$R_m = \left(L - \frac{b}{2}\right) \sin \theta_{P1} = \left(L - \frac{b}{2}\right) \frac{D_P}{2L} \qquad \ldots \left(\because \sin \theta_{P1} = \frac{D_P/2}{L}\right)$$

Now the radial force (W_R) acting at the mean radius may be further resolved into two components, W_{RH} and W_{RV}, in the axial and radial directions as shown in Fig. 30.5. Therefore the axial force acting on the pinion shaft,

$$W_{RH} = W_R \sin \theta_{P1} = W_T \tan \phi \,.\, \sin \theta_{P1} \qquad \ldots\text{[From equation (i)]}$$

and the radial force acting on the pinion shaft,

$$W_{RV} = W_R \cos \theta_{P1} = W_T \tan \phi . \cos \theta_{P1}$$

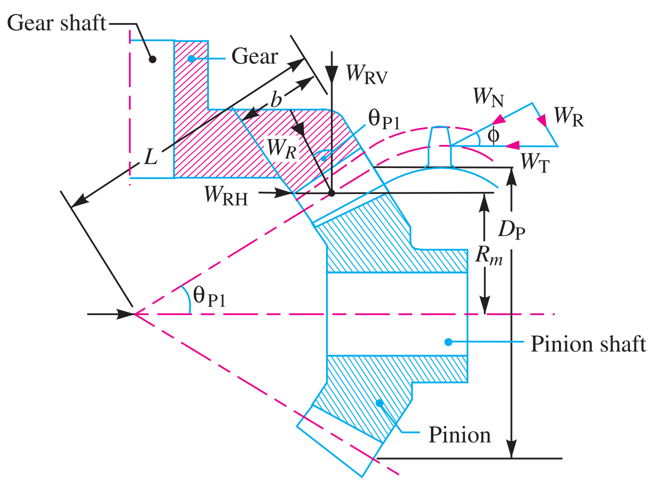


Fig. 30.5. Forces acting on a bevel gear.

A little consideration will show that the axial force on the pinion shaft is equal to the radial force on the gear shaft but their directions are opposite. Similarly, the radial force on the pinion shaft is equal to the axial force on the gear shaft, but act in opposite directions.

30.9 Design of a Shaft for Bevel Gears

In designing a pinion shaft, the following procedure may be adopted :

1. First of all, find the torque acting on the pinion. It is given by

$$T = \frac{P \times 60}{2 \pi N_P} \text{ N-m}$$

where P = Power transmitted in watts, and

N_P = Speed of the pinion in r.p.m.

2. Find the tangential force (W_T) acting at the mean radius (R_m) of the pinion. We know that

$$W_T = T / R_m$$

3. Now find the axial and radial forces (*i.e.* W_{RH} and W_{RV}) acting on the pinion shaft as discussed above.

4. Find resultant bending moment on the pinion shaft as follows :

The bending moment due to W_{RH} and W_{RV} is given by

$$M_1 = W_{RV} \times \text{Overhang} - W_{RH} \times R_m$$

and bending moment due to W_T,

$$M_2 = W_T \times \text{Overhang}$$

∴ Resultant bending moment,

$$M = \sqrt{(M_1)^2 + (M_2)^2}$$

5. Since the shaft is subjected to twisting moment (T) and resultant bending moment (M), therefore equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2}$$

6. Now the diameter of the pinion shaft may be obtained by using the torsion equation. We know that

$$T_e = \frac{\pi}{16} \times \tau\,(d_P)^3$$

where d_P = Diameter of the pinion shaft, and

τ = Shear stress for the material of the pinion shaft.

7. The same procedure may be adopted to find the diameter of the gear shaft.

Example 30.1. *A 35 kW motor running at 1200 r.p.m. drives a compressor at 780 r.p.m. through a 90° bevel gearing arrangement. The pinion has 30 teeth. The pressure angle of teeth is $14\,^1/_2°$. The wheels are capable of withstanding a dynamic stress,*

$$\sigma_w = 140\left(\frac{280}{280 + v}\right) \text{ MPa, where } v \text{ is the pitch line speed in m / min.}$$

The form factor for teeth may be taken as $0.124 - \dfrac{0.686}{T_E}$, where T_E is the number of teeth equivalent of a spur gear.

The face width may be taken as $\dfrac{1}{4}$ of the slant height of pitch cone. Determine for the pinion, the module pitch, face width, addendum, dedendum, outside diameter and slant height.

High performance 2- and 3 -way bevel gear boxes

Solution : Given : $P = 35$ kW $= 35 \times 10^3$ W ; $N_P = 1200$ r.p.m. ; $N_G = 780$ r.p.m. ; $\theta_S = 90°$; $T_P = 30$; $\phi = 14\,^1/_2°$; $b = L / 4$

Module and face width for the pinion

Let m = Module in mm,

b = Face width in mm

$= L / 4$, and ...(Given)

D_P = Pitch circle diameter of the pinion.

We know that velocity ratio,

$$V.R. = \frac{N_P}{N_G} = \frac{1200}{780} = 1.538$$

∴ Number of teeth on the gear,

$$T_G = V.R. \times T_P = 1.538 \times 30 = 46$$

Since the shafts are at right angles, therefore pitch angle for the pinion,

$$\theta_{P1} = \tan^{-1}\left(\frac{1}{V.R.}\right) = \tan^{-1}\left(\frac{1}{1.538}\right) = \tan^{-1}(0.65) = 33°$$

and pitch angle for the gear,

$$\theta_{P2} = 90° - 33° = 57°$$