Hoop stress

We know that hoop stress,

$$\sigma_{t1} = \frac{p \cdot d}{2\,t} = \frac{2 \times 500}{2 \times 20} = 25 \text{ N/mm}^2 = 25 \text{ MPa } \textbf{Ans.}$$

Longitudinal stress

We know that longitudinal stress,

$$\sigma_{t2} = \frac{p \cdot d}{4\,t} = \frac{2 \times 500}{4 \times 20} = 12.5 \text{ N/mm}^2 = 12.5 \text{ MPa } \textbf{Ans.}$$

Maximum shear stress

We know that according to maximum shear stress theory, the maximum shear stress is one-half the algebraic difference of the maximum and minimum principal stress. Since the maximum principal stress is the hoop stress (σ_{t1}) and minimum principal stress is the longitudinal stress (σ_{t2}), therefore maximum shear stress,

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{25 - 12.5}{2} = 6.25 \text{ N/mm}^2 = 6.25 \text{ MPa } \textbf{Ans.}$$

Example 7.3. *An hydraulic control for a straight line motion, as shown in Fig. 7.4, utilises a spherical pressure tank 'A' connected to a working cylinder B. The pump maintains a pressure of 3 N/mm^2 in the tank.*

1. If the diameter of pressure tank is 800 mm, determine its thickness for 100% efficiency of the joint. Assume the allowable tensile stress as 50 MPa.

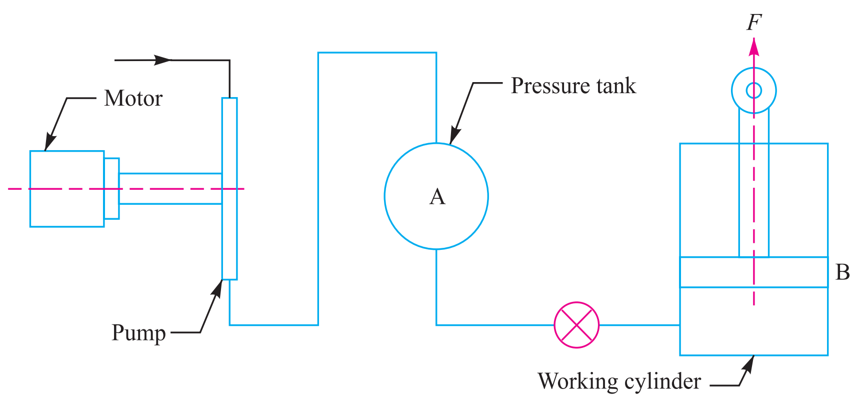

Fig. 7.4

2. Determine the diameter of a cast iron cylinder and its thickness to produce an operating force F = 25 kN. Assume (i) an allowance of 10 per cent of operating force F for friction in the cylinder and packing, and (ii) a pressure drop of 0.2 N/mm^2 between the tank and cylinder. Take safe stress for cast iron as 30 MPa.

3. Determine the power output of the cylinder, if the stroke of the piston is 450 mm and the time required for the working stroke is 5 seconds.

4. Find the power of the motor, if the working cycle repeats after every 30 seconds and the efficiency of the hydraulic control is 80 percent and that of pump 60 percent.

Solution. Given : p = 3 N/mm^2 ; d = 800 mm ; η = 100% = 1 ; σ_{t1} = 50 MPa = 50 N/mm^2 ; F = 25 kN = 25 × 10^3 N ; σ_{tc} = 30 MPa = 30 N/mm^2 : η_H = 80% = 0.8 ; η_P = 60% = 0.6

1. *Thickness of pressure tank*

We know that thickness of pressure tank,

$$t = \frac{p \cdot d}{2\sigma_{t1} \cdot \eta} = \frac{3 \times 800}{2 \times 50 \times 1} = 24 \text{ mm } \textbf{Ans.}$$

2. *Diameter and thickness of cylinder*

Let D = Diameter of cylinder, and

t_1 = Thickness of cylinder.

Since an allowance of 10 per cent of operating force F is provided for friction in the cylinder and packing, therefore total force to be produced by friction,

$$F_1 = F + \frac{10}{100} F = 1.1 F = 1.1 \times 25 \times 10^3 = 27\,500 \text{ N}$$

Jacketed pressure vessel.

We know that there is a pressure drop of 0.2 N/mm^2 between the tank and cylinder, therefore pressure in the cylinder,

$$p_1 = \text{Pressure in tank} - \text{Pressure drop} = 3 - 0.2 = 2.8 \text{ N/mm}^2$$

and total force produced by friction (F_1),

$$27\,500 = \frac{\pi}{4} \times D^2 \times p_1 = 0.7854 \times D^2 \times 2.8 = 2.2\, D^2$$

$$\therefore \quad D^2 = 27\,500 / 2.2 = 12\,500 \quad \text{or} \quad D = 112 \text{ mm } \textbf{Ans.}$$

We know that thickness of cylinder,

$$t_1 = \frac{p_1 \cdot D}{2\,\sigma_{tc}} = \frac{2.8 \times 112}{2 \times 30} = 5.2 \text{ mm } \textbf{Ans.}$$

3. *Power output of the cylinder*

We know that stroke of the piston

= 450 mm = 0.45 m ...(Given)

and time required for working stroke

= 5 s ...(Given)

∴ Distance moved by the piston per second

$$= \frac{0.45}{5} = 0.09 \text{ m}$$

We know that work done per second

$$= \text{Force} \times \text{Distance moved per second}$$

$$= 27\,500 \times 0.09 = 2475 \text{ N-m}$$

∴ Power output of the cylinder

$$= 2475 \text{ W} = 2.475 \text{ kW} \quad \textbf{Ans.} \qquad ...(\because 1 \text{ N-m/s} = 1 \text{ W})$$

4. Power of the motor

Since the working cycle repeats after every 30 seconds, therefore the power which is to be produced by the cylinder in 5 seconds is to be provided by the motor in 30 seconds.

∴ Power of the motor

$$= \frac{\text{Power of the cylinder}}{\eta_H \times \eta_P} \times \frac{5}{30} = \frac{2.475}{0.8 \times 0.6} \times \frac{5}{30} = 0.86 \text{ kW} \quad \textbf{Ans.}$$

7.6 Change in Dimensions of a Thin Cylindrical Shell due to an Internal Pressure

When a thin cylindrical shell is subjected to an internal pressure, there will be an increase in the diameter as well as the length of the shell.

Let l = Length of the cylindrical shell,

d = Diameter of the cylindrical shell,

t = Thickness of the cylindrical shell,

p = Intensity of internal pressure,

E = Young's modulus for the material of the cylindrical shell, and

μ = Poisson's ratio.

The increase in diameter of the shell due to an internal pressure is given by,

$$\delta d = \frac{p.d^2}{2\,t.E}\left(1 - \frac{\mu}{2}\right)$$

The increase in length of the shell due to an internal pressure is given by,

$$\delta l = \frac{p.d.l}{2\,t.E}\left(\frac{1}{2} - \mu\right)$$

It may be noted that the increase in diameter and length of the shell will also increase its volume. The increase in volume of the shell due to an internal pressure is given by

$$\delta V = \text{Final volume} - \text{Original volume} = \frac{\pi}{4}(d + \delta d)^2 (l + \delta l) - \frac{\pi}{4} \times d^2.l$$

$$= \frac{\pi}{4}(d^2.\delta l + 2\,d.l.\delta d) \qquad \text{...(Neglecting small quantities)}$$

Example 7.4. *Find the thickness for a tube of internal diameter 100 mm subjected to an internal pressure which is 5/8 of the value of the maximum permissible circumferential stress. Also find the increase in internal diameter of such a tube when the internal pressure is 90 N/mm². Take E = 205 kN/mm² and μ = 0.29. Neglect longitudinal strain.*

Solution. Given : $p = 5/8 \times \sigma_{t1} = 0.625\,\sigma_{t1}$; $d = 100$ mm ; $p_1 = 90$ N/mm² ; $E = 205$ kN/mm² $= 205 \times 10^3$ N/mm² ; $\mu = 0.29$

Thickness of a tube

We know that thickness of a tube,

$$t = \frac{p\,.\,d}{2\,\sigma_{t1}} = \frac{0.625\,\sigma_{t1} \times 100}{2\,\sigma_{t1}} = 31.25 \text{ mm} \quad \textbf{Ans.}$$

Increase in diameter of a tube

We know that increase in diameter of a tube,

$$\delta d = \frac{p_1 \, d^2}{2\, t.E}\left(1 - \frac{\mu}{2}\right) = \frac{90\,(100)^2}{2 \times 31.25 \times 205 \times 10^3}\left[1 - \frac{0.29}{2}\right] \text{mm}$$

$$= 0.07\,(1 - 0.145) = 0.06 \text{ mm } \textbf{Ans.}$$

7.7 Thin Spherical Shells Subjected to an Internal Pressure

Consider a thin spherical shell subjected to an internal pressure as shown in Fig. 7.5.

Let V = Storage capacity of the shell,

p = Intensity of internal pressure,

d = Diameter of the shell,

t = Thickness of the shell,

σ_t = Permissible tensile stress for the shell material.

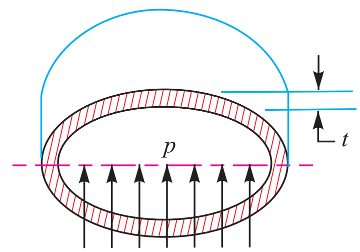

Fig. 7.5. Thin spherical shell.

In designing thin spherical shells, we have to determine 1. Diameter of the shell, and 2. Thickness of the shell.

1. *Diameter of the shell*

We know that the storage capacity of the shell,

$$V = \frac{4}{3} \times \pi\, r^3 = \frac{\pi}{6} \times d^3 \quad \text{or} \quad d = \left(\frac{6\,V}{\pi}\right)^{1/3}$$

2. *Thickness of the shell*

As a result of the internal pressure, the shell is likely to rupture along the centre of the sphere. Therefore force tending to rupture the shell along the centre of the sphere or bursting force,

$$= \text{Pressure} \times \text{Area} = p \times \frac{\pi}{4} \times d^2 \qquad \ldots(i)$$

and resisting force of the shell

$$= \text{Stress} \times \text{Resisting area} = \sigma_t \times \pi\, d.t \qquad \ldots(ii)$$

Equating equations (*i*) and (*ii*), we have

$$p \times \frac{\pi}{4} \times d^2 = \sigma_t \times \pi\, d.t$$

or

$$t = \frac{p.d}{4\,\sigma_t}$$

If η is the efficiency of the circumferential joints of the spherical shell, then

$$t = \frac{p.d}{4\,\sigma_t.\eta}$$

The Trans-Alaska Pipeline carries crude oil 1, 284 kilometres through Alaska. The pipeline is 1.2 metres in diameter and can transport 318 million litres of crude oil a day.

Example 7.5. *A spherical vessel 3 metre diameter is subjected to an internal pressure of 1.5 N/mm². Find the thickness of the vessel required if the maximum stress is not to exceed 90 MPa. Take efficiency of the joint as 75%.*

Solution. Given: d = 3 m = 3000 mm ; p = 1.5 N/mm^2 ; σ_t = 90 MPa = 90 N/mm^2 ; η = 75% = 0.75

We know that thickness of the vessel,

$$t = \frac{p.d}{4\,\sigma_t\,.\,\eta} = \frac{1.5 \times 3000}{4 \times 90 \times 0.75} = 16.7 \text{ say } 18 \text{ mm } \textbf{Ans.}$$

7.8 Change in Dimensions of a Thin Spherical Shell due to an Internal Pressure

Consider a thin spherical shell subjected to an internal pressure as shown in Fig. 7.5.

Let d = Diameter of the spherical shell,

t = Thickness of the spherical shell,

p = Intensity of internal pressure,

E = Young's modulus for the material of the spherical shell, and

μ = Poisson's ratio.

Increase in diameter of the spherical shell due to an internal pressure is given by,

$$\delta d = \frac{p.d^2}{4\,t.E}(1-\mu) \qquad \ldots(i)$$

and increase in volume of the spherical shell due to an internal pressure is given by,

$$\delta V = \text{Final volume} - \text{Original volume} = \frac{\pi}{6}(d+\delta d)^3 - \frac{\pi}{6} \times d^3$$

$$= \frac{\pi}{6}(3d^2 \times \delta d) \qquad \ldots\text{(Neglecting higher terms)}$$

Substituting the value of δd from equation ***(i)***, we have

$$\delta V = \frac{3\,\pi\,d^2}{6}\left[\frac{p.d^2}{4\,t.E}(1-\mu)\right] = \frac{\pi\,p\,d^4}{8\,t.E}(1-\mu)$$

Example 7.6. *A seamless spherical shell, 900 mm in diameter and 10 mm thick is being filled with a fluid under pressure until its volume increases by 150 × 10^3 mm^3. Calculate the pressure exerted by the fluid on the shell, taking modulus of elasticity for the material of the shell as 200 kN/mm^2 and Poisson's ratio as 0.3.*

Solution. Given : d = 900 mm ; t = 10 mm ; δV = 150 × 10^3 mm^3 ; E = 200 kN/mm^2 = 200 × 10^3 N/mm^2 ; μ = 0.3

Let p = Pressure exerted by the fluid on the shell.

We know that the increase in volume of the spherical shell (δV),

$$150 \times 10^3 = \frac{\pi\,p\,d^4}{8\,t\,E}(1-\mu) = \frac{\pi\,p\,(900)^4}{8 \times 10 \times 200 \times 10^3}(1-0.3) = 90\,190\,p$$

$\therefore$ p = 150 × 10^3/90 190 = 1.66 N/mm^2 **Ans.**

7.9 Thick Cylindrical Shells Subjected to an Internal Pressure

When a cylindrical shell of a pressure vessel, hydraulic cylinder, gunbarrel and a pipe is subjected to a very high internal fluid pressure, then the walls of the cylinder must be made extremely heavy or thick.

In thin cylindrical shells, we have assumed that the tensile stresses are uniformly distributed over the section of the walls. But in the case of thick wall cylinders as shown in Fig. 7.6 (*a*), the stress over the section of the walls cannot be assumed to be uniformly distributed. They develop both tangential and radial stresses with values which are dependent upon the radius of the element under consideration. The distribution of stress in a thick cylindrical shell is shown in Fig. 7.6 (*b*) and (*c*). We see that the tangential stress is maximum at the inner surface and minimum at the outer surface of the shell. The radial stress is maximum at the inner surface and zero at the outer surface of the shell.