We know that thickness of the vessel,

$$t = \frac{p.d}{4\,\sigma_t\,.\,\eta} = \frac{1.5 \times 3000}{4 \times 90 \times 0.75} = 16.7 \text{ say } 18 \text{ mm } \textbf{Ans.}$$

7.8 Change in Dimensions of a Thin Spherical Shell due to an Internal Pressure

Consider a thin spherical shell subjected to an internal pressure as shown in Fig. 7.5.

Let d = Diameter of the spherical shell,

t = Thickness of the spherical shell,

p = Intensity of internal pressure,

E = Young's modulus for the material of the spherical shell, and

μ = Poisson's ratio.

Increase in diameter of the spherical shell due to an internal pressure is given by,

$$\delta d = \frac{p.d^2}{4\,t.E}(1-\mu) \qquad \text{...}(i)$$

and increase in volume of the spherical shell due to an internal pressure is given by,

$$\delta V = \text{Final volume} - \text{Original volume} = \frac{\pi}{6}(d+\delta d)^3 - \frac{\pi}{6} \times d^3$$

$$= \frac{\pi}{6}(3d^2 \times \delta d) \qquad \text{...(Neglecting higher terms)}$$

Substituting the value of δd from equation (*i*), we have

$$\delta V = \frac{3\,\pi\,d^2}{6}\left[\frac{p.d^2}{4\,t.E}(1-\mu)\right] = \frac{\pi\,p\,d^4}{8\,t.E}(1-\mu)$$

Example 7.6. *A seamless spherical shell, 900 mm in diameter and 10 mm thick is being filled with a fluid under pressure until its volume increases by 150 × 10^3 mm^3. Calculate the pressure exerted by the fluid on the shell, taking modulus of elasticity for the material of the shell as 200 kN/mm^2 and Poisson's ratio as 0.3.*

Solution. Given : d = 900 mm ; t = 10 mm ; δV = 150 × 10^3 mm^3 ; E = 200 kN/mm^2 = 200 × 10^3 N/mm^2 ; μ = 0.3

Let p = Pressure exerted by the fluid on the shell.

We know that the increase in volume of the spherical shell (δV),

$$150 \times 10^3 = \frac{\pi\,p\,d^4}{8\,t\,E}(1-\mu) = \frac{\pi\,p\,(900)^4}{8 \times 10 \times 200 \times 10^3}(1-0.3) = 90\,190\,p$$

$\therefore$ $p = 150 \times 10^3 / 90\,190 = 1.66$ N/mm^2 **Ans.**

7.9 Thick Cylindrical Shells Subjected to an Internal Pressure

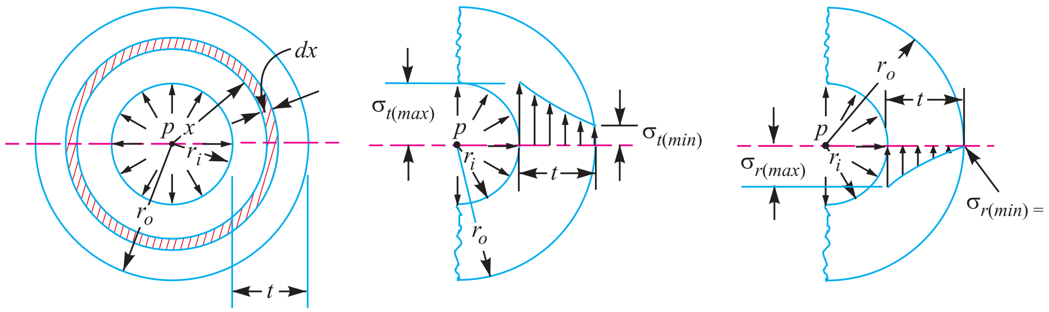
When a cylindrical shell of a pressure vessel, hydraulic cylinder, gunbarrel and a pipe is subjected to a very high internal fluid pressure, then the walls of the cylinder must be made extremely heavy or thick.

In thin cylindrical shells, we have assumed that the tensile stresses are uniformly distributed over the section of the walls. But in the case of thick wall cylinders as shown in Fig. 7.6 (*a*), the stress over the section of the walls cannot be assumed to be uniformly distributed. They develop both tangential and radial stresses with values which are dependent upon the radius of the element under consideration. The distribution of stress in a thick cylindrical shell is shown in Fig. 7.6 (*b*) and (*c*). We see that the tangential stress is maximum at the inner surface and minimum at the outer surface of the shell. The radial stress is maximum at the inner surface and zero at the outer surface of the shell.

In the design of thick cylindrical shells, the following equations are mostly used:

1. Lame's equation; **2.** Birnie's equation; **3.** Clavarino's equation; and **4.** Barlow's equation.

The use of these equations depends upon the type of material used and the end construction.



(*a*) Thick cylindrical shell. (*b*) Tangential stress distribution. (*c*) Radial stress distribution.

Fig. 7.6. Stress distribution in thick cylindrical shells subjected to internal pressure.

Let
r_o = Outer radius of cylindrical shell,
r_i = Inner radius of cylindrical shell,
t = Thickness of cylindrical shell = $r_o - r_i$,
p = Intensity of internal pressure,
μ = Poisson's ratio,
σ_t = Tangential stress, and
σ_r = Radial stress.

All the above mentioned equations are now discussed, in detail, as below:

1. ***Lame's equation.*** Assuming that the longitudinal fibres of the cylindrical shell are equally strained, Lame has shown that the tangential stress at any radius x is,

$$\sigma_t = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} + \frac{(r_i)^2 (r_o)^2}{x^2} \left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2} \right]$$

While designing a tanker, the pressure added by movement of the vehicle also should be considered.

and radial stress at any radius x,

$$\sigma_r = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} - \frac{(r_i)^2 (r_o)^2}{x^2} \left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2} \right]$$

Since we are concerned with the internal pressure ($p_i = p$) only, therefore substituting the value of external pressure, $p_o = 0$.

∴ Tangential stress at any radius x,

$$\sigma_t = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2} \left[1 + \frac{(r_o)^2}{x^2} \right] \quad \text{...}(i)$$

and radial stress at any radius x,

$$\sigma_r = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2} \left[1 - \frac{(r_o)^2}{x^2} \right] \quad \text{...}(ii)$$

We see that the tangential stress is always a tensile stress whereas the radial stress is a compressive stress. We know that the tangential stress is maximum at the inner surface of the shell (*i.e.* when $x = r_i$) and it is minimum at the outer surface of the shell (*i.e.* when $x = r_o$). Substituting the value of $x = r_i$ and $x = r_o$ in equation **(*i*)**, we find that the *maximum tangential stress at the inner surface of the shell,

$$\sigma_{t(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

and minimum tangential stress at the outer surface of the shell,

$$\sigma_{t(min)} = \frac{2\,p\,(r_i)^2}{(r_o)^2 - (r_i)^2}$$

We also know that the radial stress is maximum at the inner surface of the shell and zero at the outer surface of the shell. Substituting the value of $x = r_i$ and $x = r_o$ in equation (*ii*), we find that maximum radial stress at the inner surface of the shell,

$$\sigma_{r(max)} = -p \text{ (compressive)}$$

and minimum radial stress at the outer surface of the shell,

$$\sigma_{r(min)} = 0$$

In designing a thick cylindrical shell of brittle material (*e.g.* cast iron, hard steel and cast aluminium) with closed or open ends and in accordance with the maximum normal stress theory failure, the tangential stress induced in the cylinder wall,

$$\sigma_t = \sigma_{t(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

Since $r_o = r_i + t$, therefore substituting this value of r_o in the above expression, we get

$$\sigma_t = \frac{p\,[(r_i + t)^2 + (r_i)^2]}{(r_i + t)^2 - (r_i)^2}$$

$$\sigma_t (r_i + t)^2 - \sigma_t (r_i)^2 = p\,(r_i + t)^2 + p\,(r_i)^2$$

$$(r_i + t)^2 (\sigma_t - p) = (r_i)^2 (\sigma_t + p)$$

$$\frac{(r_i + t)^2}{(r_i)^2} = \frac{\sigma_t + p}{\sigma_t - p}$$

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* The maximum tangential stress is always greater than the internal pressure acting on the shell.

$$\frac{r_i + t}{r_i} = \sqrt{\frac{\sigma_t + p}{\sigma_t - p}} \quad \text{or} \quad 1 + \frac{t}{r_i} = \sqrt{\frac{\sigma_t + p}{\sigma_t - p}}$$

$$\therefore \quad \frac{t}{r_i} = \sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \quad \text{or} \quad t = r_i \left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \right] \qquad ...(iii)$$

The value of σ_t for brittle materials may be taken as 0.125 times the ultimate tensile strength (σ_u).

We have discussed above the design of a thick cylindrical shell of brittle materials. In case of cylinders made of ductile material, Lame's equation is modified according to maximum shear stress theory.

According to this theory, the maximum shear stress at any point in a strained body is equal to one-half the algebraic difference of the maximum and minimum principal stresses at that point. We know that for a thick cylindrical shell,

Maximum principal stress at the inner surface,

$$\sigma_{t\,(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

and minimum principal stress at the outer surface,

$$\sigma_{t(min)} = -\,p$$

$\therefore$ Maximum shear stress,

$$\tau = \tau_{max} = \frac{\sigma_{t(max)} - \sigma_{t(min)}}{2} = \frac{\dfrac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2} - (-\,p)}{2}$$

$$= \frac{p\,[(r_o)^2 + (r_i)^2] + p\,[(r_o)^2 - (r_i)^2]}{2[(r_o)^2 - (r_i)^2]} = \frac{2\,p\,(r_o)^2}{2[(r_o)^2 - (r_i)^2]}$$

$$= \frac{p\,(r_i + t)^2}{(r_i + t)^2 - (r_i)^2} \qquad ...(\because r_o = r_i + t)$$

or

$$\tau(r_i + t)^2 - \tau(r_i)^2 = p(r_i + t)^2$$

$$(r_i + t)^2\,(\tau - p) = \tau(r_i)^2$$

$$\frac{(r_i + t)^2}{(r_i)^2} = \frac{\tau}{\tau - p}$$

$$\frac{r_i + t}{r_i} = \sqrt{\frac{\tau}{\tau - p}} \quad \text{or} \quad 1 + \frac{t}{r_i} = \sqrt{\frac{\tau}{\tau - p}}$$

$$\therefore \quad \frac{t}{r_i} = \sqrt{\frac{\tau}{\tau - p}} - 1 \quad \text{or} \quad t = r_i \left[\sqrt{\frac{\tau}{\tau - p}} - 1 \right] \qquad ...(iv)$$

The value of shear stress (τ) is usually taken as one-half the tensile stress (σ_t). Therefore the above expression may be written as

$$t = r_i \left[\sqrt{\frac{\sigma_t}{\sigma_t - 2\,p}} - 1 \right] \qquad ...(v)$$

From the above expression, we see that if the internal pressure (p) is equal to or greater than the allowable working stress (σ_t or τ), then no thickness of the cylinder wall will prevent failure. Thus, it is impossible to design a cylinder to withstand fluid pressure greater than the allowable working stress for a given material. This difficulty is overcome by using compound cylinders (See Art. 7.10).

2. *Birnie's equation.* In case of open-end cylinders (such as pump cylinders, rams, gun barrels etc.) made of ductile material (*i.e.* low carbon steel, brass, bronze, and aluminium alloys), the allowable stresses cannot be determined by means of maximum-stress theory of failure. In such cases, the maximum-strain theory is used. According to this theory, the failure occurs when the strain reaches a limiting value and Birnie's equation for the wall thickness of a cylinder is

$$t = r_i \left[\sqrt{\frac{\sigma_t + (1 - \mu)\, p}{\sigma_t - (1 + \mu)\, p}} - 1 \right]$$

The value of σ_t may be taken as 0.8 times the yield point stress (σ_y).

3. *Clavarino's equation.* This equation is also based on the maximum-strain theory of failure, but it is applied to closed-end cylinders (or cylinders fitted with heads) made of ductile material. According to this equation, the thickness of a cylinder,

Oil is frequently transported by ships called tankers. The larger tankers, such as this Acrco Alaska oil transporter, are known as supertankers. They can be hundreds of metres long.

$$t = r_i \left[\sqrt{\frac{\sigma_t + (1 - 2\mu)\, p}{\sigma_t - (1 + \mu)\, p}} - 1 \right]$$

In this case also, the value of σ_t may be taken as 0.8 σ_y.

4. *Barlow's equation.* This equation is generally used for high pressure oil and gas pipes. According to this equation, the thickness of a cylinder,

$$t = p.r_o / \sigma_t$$

For ductile materials, $\sigma_t = 0.8\ \sigma_y$ and for brittle materials $\sigma_t = 0.125\ \sigma_u$, where σ_u is the ultimate stress.

Example 7.7. *A cast iron cylinder of internal diameter 200 mm and thickness 50 mm is subjected to a pressure of 5 N/mm². Calculate the tangential and radial stresses at the inner, middle (radius = 125 mm) and outer surfaces.*

Solution. Given : d_i = 200 mm or r_i = 100 mm ; t = 50 mm ; p = 5 N/mm²

We know that outer radius of the cylinder,

$$r_o = r_i + t = 100 + 50 = 150 \text{ mm}$$

Tangential stresses at the inner, middle and outer surfaces

We know that the tangential stress at any radius x,

$$\sigma_t = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_o)^2}{x^2}\right]$$

$\therefore$ Tangential stress at the inner surface (*i.e.* when $x = r_i = 100$ mm),

$$\sigma_{t(inner)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2} = \frac{5\,[(150)^2 + (100)^2]}{(150)^2 - (100)^2} = 13 \text{ N/mm}^2 = 13 \text{ MPa } \textbf{Ans.}$$

Tangential stress at the middle surface (*i.e.* when $x = 125$ mm),

$$\sigma_{t(middle)} = \frac{5\,(100)^2}{(150)^2 - (100)^2}\left[1 + \frac{(150)^2}{(125)^2}\right] = 9.76 \text{ N/mm}^2 = 9.76 \text{ MPa } \textbf{Ans.}$$

and tangential stress at the outer surface (*i.e.* when $x = r_o = 150$ mm),

$$\sigma_{t(outer)} = \frac{2\,p\,(r_i)^2}{(r_o)^2 - (r_i)^2} = \frac{2 \times 5\,(100)^2}{(150)^2 - (100)^2} = 8 \text{ N/mm}^2 = 8 \text{ MPa } \textbf{Ans.}$$

Radial stresses at the inner, middle and outer surfaces

We know that the radial stress at any radius x,

$$\sigma_r = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_o)^2}{x^2}\right]$$

$\therefore$ Radial stress at the inner surface (*i.e.* when $x = r_i = 100$ mm),

$$\sigma_{r(inner)} = -p = -5 \text{ N/mm}^2 = 5 \text{ MPa (compressive) } \textbf{Ans.}$$

Radial stress at the middle surface (*i.e.* when $x = 125$ mm)

$$\sigma_{r(middle)} = \frac{5\,(100)^2}{(150)^2 - (100)^2}\left[1 - \frac{(150)^2}{(125)^2}\right] = -1.76 \text{ N/mm}^2 = -1.76 \text{ MPa}$$

$$= 1.76 \text{ MPa (compressive) } \textbf{Ans.}$$

and radial stress at the outer surface (*i.e.* when $x = r_o = 150$ mm),

$$\sigma_{r(outer)} = 0 \ \textbf{Ans.}$$

Example 7.8. *A hydraulic press has a maximum capacity of 1000 kN. The piston diameter is 250 mm. Calculate the wall thickness if the cylinder is made of material for which the permissible strength may be taken as 80 MPa. This material may be assumed as a brittle material.*

Solution. Given : $W = 1000$ kN $= 1000 \times 10^3$ N ; $d = 250$ mm ; $\sigma_t = 80$ MPa $= 80$ N/mm^2

First of all, let us find the pressure inside the cylinder (p). We know that load on the hydraulic press (W),

Hydraulic Press

$$1000 \times 10^3 = \frac{\pi}{4} \times d^2 \times p = \frac{\pi}{4}\,(250)^2\,p = 49.1 \times 10^3 p$$

$$\therefore \quad p = 1000 \times 10^3 / 49.1 \times 10^3 = 20.37 \text{ N/mm}^2$$

Let r_i = Inside radius of the cylinder = $d/2 = 125$ mm

We know that wall thickness of the cylinder,

$$t = r_i \left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \right] = 125 \left[\sqrt{\frac{80 + 20.37}{80 - 20.37}} - 1 \right] \text{mm}$$

$$= 125\,(1.297 - 1) = 37 \text{ mm} \quad \textbf{Ans.}$$

Example 7.9. *A closed-ended cast iron cylinder of 200 mm inside diameter is to carry an internal pressure of 10 N/mm² with a permissible stress of 18 MPa. Determine the wall thickness by means of Lame's and the maximum shear stress equations. What result would you use? Give reason for your conclusion.*

Solution. Given : d_i = 200 mm or r_i = 100 mm ; p = 10 N/mm² ; σ_t = 18 MPa = 18 N/mm²

According to Lame's equation, wall thickness of a cylinder,

$$t = r_i \left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \right] = 100 \left[\sqrt{\frac{80 + 10}{80 - 10}} - 1 \right] = 87 \text{ mm}$$

According to maximum shear stress equation, wall thickness of a cylinder,

$$t = r_i \left[\sqrt{\frac{\tau}{\tau - p}} - 1 \right]$$

We have discussed in Art. 7.9 [equation (*iv*)], that the shear stress (τ) is usually taken one-half the tensile stress (σ_t). In the present case, $\tau = \sigma_t / 2 = 18/2 = 9$ N/mm². Since τ is less than the internal pressure (p = 10 N/mm²), therefore the expression under the square root will be negative. Thus no thickness can prevent failure of the cylinder. Hence it is impossible to design a cylinder to withstand fluid pressure greater than the allowable working stress for the given material. This difficulty is overcome by using compound cylinders as discussed in Art. 7.10.

Thus, we shall use a cylinder of wall thickness, t = 87 mm **Ans.**

Example 7.10. *The cylinder of a portable hydraulic riveter is 220 mm in diameter. The pressure of the fluid is 14 N/mm² by gauge. Determine suitable thickness of the cylinder wall assuming that the maximum permissible tensile stress is not to exceed 105 MPa.*

Solution. Given : d_i = 220 mm or r_i = 110 mm ; p = 14 N/mm² ; σ_t = 105 MPa = 105 N/mm²

Since the pressure of the fluid is high, therefore thick cylinder equation is used.

Assuming the material of the cylinder as steel, the thickness of the cylinder wall (t) may be obtained by using Birnie's equation. We know that

$$t = r_i \left[\sqrt{\frac{\sigma_t + (1 - \mu)\,p}{\sigma_t - (1 + \mu)\,p}} - 1 \right]$$

$$= 110 \left[\sqrt{\frac{105 + (1 - 0.3)\,14}{105 - (1 + 0.3)\,14}} - 1 \right] = 16.5 \text{ mm} \quad \textbf{Ans.}$$

...(Taking Poisson's ratio for steel, μ = 0.3)

Example 7.11. *The hydraulic cylinder 400 mm bore operates at a maximum pressure of 5 N/mm². The piston rod is connected to the load and the cylinder to the frame through hinged joints. Design: 1. cylinder, 2. piston rod, 3. hinge pin, and 4. flat end cover.*

The allowable tensile stress for cast steel cylinder and end cover is 80 MPa and for piston rod is 60 MPa.

Draw the hydraulic cylinder with piston, piston rod, end cover and O-ring.

Solution. Given : d_i = 400 mm or r_i = 200 mm ; p = 5 N/mm² ; σ_t = 80 MPa = 80 N/mm² ; σ_{tp} = 60 MPa = 60 N/mm²

1. *Design of cylinder*

Let d_o = Outer diameter of the cylinder.

We know that thickness of cylinder,

$$t = r_i \left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \right] = 200 \left[\sqrt{\frac{80 + 5}{80 - 5}} - 1 \right] \text{mm}$$

$= 200\,(1.06 - 1) = 12$ mm **Ans.**

∴ Outer diameter of the cylinder,

$$d_o = d_i + 2t = 400 + 2 \times 12 = 424 \text{ mm} \quad \textbf{Ans.}$$

2. *Design of piston rod*

Let d_p = Diameter of the piston rod.

We know that the force acting on the piston rod,

$$F = \frac{\pi}{4}(d_i)^2\, p = \frac{\pi}{4}(400)^2\, 5 = 628\,400 \text{ N} \qquad ...(i)$$

We also know that the force acting on the piston rod,

$$F = \frac{\pi}{4}(d_i)^2\, \sigma_{tp} = \frac{\pi}{4}(d_p)^2\, 60 = 47.13\,(d_p)^2 \text{ N} \qquad ...(ii)$$

From equations ***(i)*** and ***(ii)***, we have

$$(d_p)^2 = 628\,400/47.13 = 13\,333.33 \quad \text{or} \quad d_p = 115.5 \text{ say } 116 \text{ mm} \quad \textbf{Ans.}$$

3. *Design of the hinge pin*

Let d_h = Diameter of the hinge pin of the piston rod.

Since the load on the pin is equal to the force acting on the piston rod, and the hinge pin is in double shear, therefore

$$F = 2 \times \frac{\pi}{4}(d_h)^2\, \tau$$

$$628\,400 = 2 \times \frac{\pi}{4}(d_h)^2\, 45 = 70.7\,(d_h)^2 \qquad ...(\text{Taking } \tau = 45 \text{ N/mm}^2)$$

$$\therefore \quad (d_h)^2 = 628\,400 / 70.7 = 8888.3 \quad \text{or} \quad d_h = 94.3 \text{ say } 95 \text{ mm} \quad \textbf{Ans.}$$

When the cover is hinged to the cylinder, we can use two hinge pins only diametrically opposite to each other. Thus the diameter of the hinge pins for cover,

$$d_{hc} = \frac{d_h}{2} = \frac{95}{2} = 47.5 \text{ mm} \quad \textbf{Ans.}$$

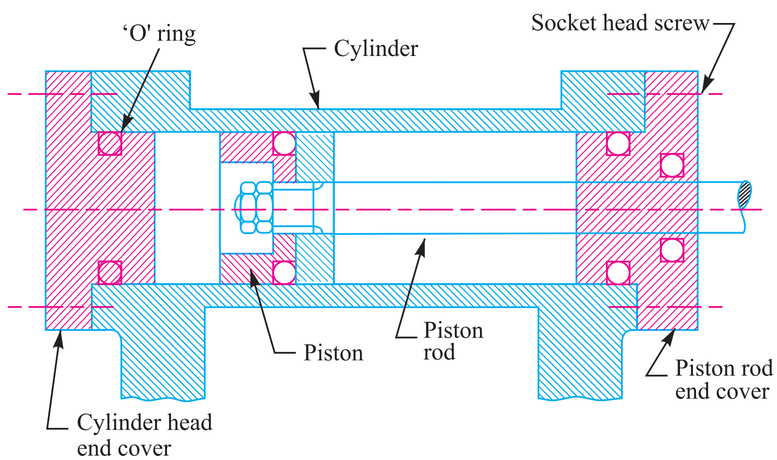

Fig. 7.7

4. *Design of the flat end cover*

Let t_c = Thickness of the end cover.

We know that force on the end cover,

$$F = d_i \times t_c \times \sigma_t$$

$$628\ 400 = 400 \times t_c \times 80 = 32 \times 10^3\ t_c$$

$$\therefore \quad t_c = 628\ 400 / 32 \times 10^3 = 19.64 \text{ say } 20 \text{ mm}$$ **Ans.**

The hydraulic cylinder with piston, piston rod, end cover and *O*-ring is shown in Fig. 7.7.

7.10 Compound Cylindrical Shells

According to Lame's equation, the thickness of a cylindrical shell is given by

$$t = r_i\left(\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1\right)$$

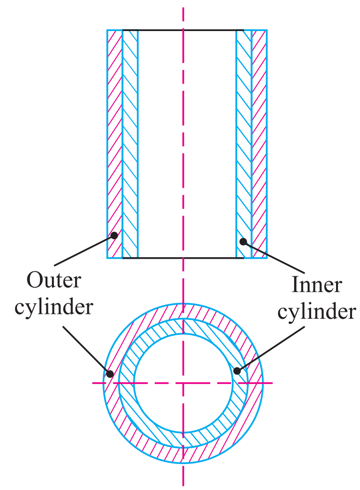

Fig. 7.8. Compound cylindrical shell.

From this equation, we see that if the internal pressure (p) acting on the shell is equal to or greater than the allowable working stress (σ_t) for the material of the shell, then no thickness of the shell will prevent failure. Thus it is impossible to design a cylinder to withstand internal pressure equal to or greater than the allowable working stress.

This difficulty is overcome by inducing an initial compressive stress on the wall of the cylindrical shell. This may be done by the following two methods:

1. By using compound cylindrical shells, and
2. By using the theory of plasticity.

In a compound cylindrical shell, as shown in Fig. 7.8, the outer cylinder (having inside diameter smaller than the outside diameter of the inner cylinder) is shrunk fit over the inner cylinder by heating and cooling. On cooling, the contact pressure is developed at the junction of the two cylinders, which induces compressive tangential stress in the material of the inner cylinder and tensile tangential stress in the material of the outer cylinder. When the cylinder is loaded, the compressive stresses are first relieved and then tensile stresses are induced. Thus, a compound cylinder is effective in resisting higher internal pressure than a single cylinder with the same overall dimensions. The principle of compound cylinder is used in the design of gun tubes.

In the theory of plasticity, a temporary high internal pressure is applied till the plastic stage is reached near the inside of the cylinder wall. This results in a residual compressive stress upon the removal of the internal pressure, thereby making the cylinder more effective to withstand a higher internal pressure.

7.11 Stresses in Compound Cylindrical Shells

Fig. 7.9 (*a*) shows a compound cylindrical shell assembled with a shrink fit. We have discussed in the previous article that when the outer cylinder is shrunk fit over the inner cylinder, a contact pressure (p) is developed at junction of the two cylinders (*i.e.* at radius r_2) as shown in Fig. 7.9 (*b*) and (*c*). The stresses resulting from this pressure may be easily determined by using Lame's equation.

According to this equation (See Art. 7.9), the tangential stress at any radius x is

$$\sigma_t = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} + \frac{(r_i)^2 (r_o)^2}{x^2}\left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2}\right] \quad \text{...}(i)$$