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Pressure Vessels

7.1 Introduction

The pressure vessels (*i.e.* cylinders or tanks) are used to store fluids under pressure. The fluid being stored may undergo a change of state inside the pressure vessel as in case of steam boilers or it may combine with other reagents as in a chemical plant. The pressure vessels are designed with great care because rupture of a pressure vessel means an explosion which may cause loss of life and property. The material of pressure vessels may be brittle such as cast iron, or ductile such as mild steel.

7.2 Classification of Pressure Vessels

The pressure vessels may be classified as follows:

1. *According to the dimensions.* The pressure vessels, according to their dimensions, may be classified as ***thin shell*** or ***thick shell.*** If the wall thickness of the shell (t) is less than 1/10 of the diameter of the shell (d), then it is called a ***thin shell.*** On the other hand, if the wall thickness of the shell is greater than 1/10 of the diameter of the shell, then it is said to be a ***thick shell.*** Thin shells are used in boilers, tanks and pipes, whereas thick shells are used in high pressure cylinders, tanks, gun barrels etc.

Pressure vessels.

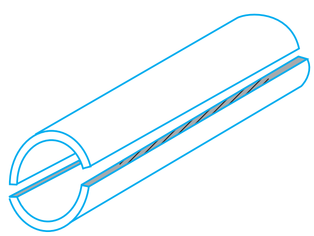
Note: Another criterion to classify the pressure vessels as thin shell or thick shell is the internal fluid pressure (p) and the allowable stress (σ_t). If the internal fluid pressure (p) is less than 1/6 of the allowable stress, then it is called a ***thin shell.*** On the other hand, if the internal fluid pressure is greater than 1/6 of the allowable stress, then it is said to be a ***thick shell.***

2. According to the end construction. The pressure vessels, according to the end construction, may be classified as ***open end*** or ***closed end.*** A simple cylinder with a piston, such as cylinder of a press is an example of an open end vessel, whereas a tank is an example of a closed end vessel. In case of vessels having open ends, the circumferential or hoop stresses are induced by the fluid pressure, whereas in case of closed ends, longitudinal stresses in addition to circumferential stresses are induced.

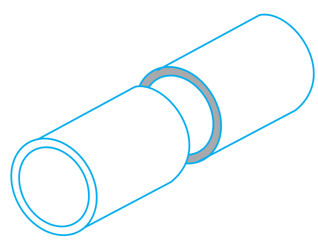
7.3 Stresses in a Thin Cylindrical Shell due to an Internal Pressure

The analysis of stresses induced in a thin cylindrical shell are made on the following assumptions:

1. The effect of curvature of the cylinder wall is neglected.
2. The tensile stresses are uniformly distributed over the section of the walls.
3. The effect of the restraining action of the heads at the end of the pressure vessel is neglected.



(a) Failure of a cylindrical shell along the longitudinal section.



(b) Failure of a cylindrical shell along the transverse section.

Fig. 7.1. Failure of a cylindrical shell.

When a thin cylindrical shell is subjected to an internal pressure, it is likely to fail in the following two ways:

1. It may fail along the longitudinal section (*i.e.* circumferentially) splitting the cylinder into two troughs, as shown in Fig. 7.1 (a).
2. It may fail across the transverse section (*i.e.* longitudinally) splitting the cylinder into two cylindrical shells, as shown in Fig. 7.1 (b).

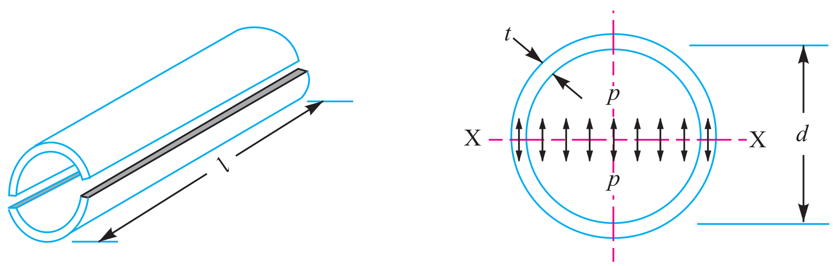
Thus the wall of a cylindrical shell subjected to an internal pressure has to withstand tensile stresses of the following two types:

(*a*) Circumferential or hoop stress, and (*b*) Longitudinal stress.

These stresses are discussed, in detail, in the following articles.

7.4 Circumferential or Hoop Stress

Consider a thin cylindrical shell subjected to an internal pressure as shown in Fig. 7.2 (*a*) and (*b*). A tensile stress acting in a direction tangential to the circumference is called ***circumferential*** or ***hoop stress.*** In other words, it is a tensile stress on *longitudinal section (or on the cylindrical walls).



(*a*) View of shell. (*b*) Cross-section of shell.

Fig. 7.2. Circumferential or hoop stress.

Let p = Intensity of internal pressure,

d = Internal diameter of the cylindrical shell,

l = Length of the cylindrical shell,

t = Thickness of the cylindrical shell, and

σ_{t1} = Circumferential or hoop stress for the material of the cylindrical shell.

We know that the total force acting on a longitudinal section (*i.e.* along the diameter *X-X*) of the shell

$$= \text{Intensity of pressure} \times \text{Projected area} = p \times d \times l \qquad ...(i)$$

and the total resisting force acting on the cylinder walls

$$= \sigma_{t1} \times 2t \times l \qquad ...(\because \text{ of two sections}) \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\sigma_{t1} \times 2t \times l = p \times d \times l \quad \text{or} \quad \sigma_{t1} = \frac{p \times d}{2t} \quad \text{or} \quad t = \frac{p \times d}{2\,\sigma_{t1}} \qquad ...(iii)$$

The following points may be noted:

1. In the design of engine cylinders, a value of 6 mm to 12 mm is added in equation (*iii*) to permit reboring after wear has taken place. Therefore

$$t = \frac{p \times d}{2\,\sigma_{t1}} + 6 \text{ to } 12 \text{ mm}$$

2. In constructing large pressure vessels like steam boilers, riveted joints or welded joints are used in joining together the ends of steel plates. In case of riveted joints, the wall thickness of the cylinder,

$$t = \frac{p \times d}{2\sigma_{t1} \times \eta_l}$$

where η_l = Efficiency of the longitudinal riveted joint.

* A section cut from a cylinder by a plane that contains the axis is called longitudinal section.

3. In case of cylinders of ductile material, the value of circumferential stress (σ_{t1}) may be taken 0.8 times the yield point stress (σ_y) and for brittle materials, σ_{t1} may be taken as 0.125 times the ultimate tensile stress (σ_u).
4. In designing steam boilers, the wall thickness calculated by the above equation may be compared with the minimum plate thickness as provided in boiler code as given in the following table.

Table 7.1. Minimum plate thickness for steam boilers.

Boiler diameter	*Minimum plate thickness (t)*
0.9 m or less	6 mm
Above 0.9 m and upto 1.35 m	7.5 mm
Above 1.35 m and upto 1.8 m	9 mm
Over 1.8 m	12 mm

Note: If the calculated value of *t* is less than the code requirement, then the latter should be taken, otherwise the calculated value may be used.

The boiler code also provides that the factor of safety shall be at least 5 and the steel of the plates and rivets shall have as a minimum the following ultimate stresses.

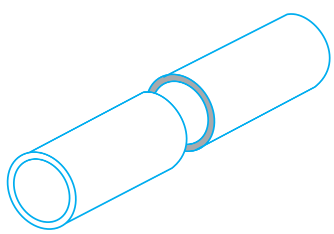
Tensile stress, $\sigma_t = 385$ MPa

Compressive stress, $\sigma_c = 665$ MPa

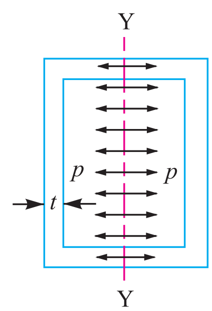
Shear stress, $\tau = 308$ MPa

7.5 Longitudinal Stress

Consider a closed thin cylindrical shell subjected to an internal pressure as shown in Fig. 7.3 (*a*) and (*b*). A tensile stress acting in the direction of the axis is called ***longitudinal stress***. In other words, it is a tensile stress acting on the *transverse or circumferential section *Y-Y* (or on the ends of the vessel).



(*a*) View of shell.



(*b*) Cross-section of shell.

Fig. 7.3. Longitudinal stress.

Let σ_{t2} = Longitudinal stress.

In this case, the total force acting on the transverse section (*i.e.* along *Y-Y*)

$$= \text{Intensity of pressure} \times \text{Cross-sectional area}$$

$$= p \times \frac{\pi}{4}(d)^2 \quad ...(i)$$

and total resisting force $= \sigma_{t2} \times \pi\, d.t \quad ...(ii)$

* A section cut from a cylinder by a plane at right angles to the axis of the cylinder is called transverse section.

From equations ***(i)*** and ***(ii)***, we have

$$\sigma_{t2} \times \pi\, d.t = p \times \frac{\pi}{4} (d)^2$$

$$\therefore \qquad \sigma_{t2} = \frac{p \times d}{4\, t} \qquad \text{or} \qquad t = \frac{p \times d}{4\, \sigma_{t2}}$$

If η_c is the efficiency of the circumferential joint, then

$$t = \frac{p \times d}{4 \sigma_{t2} \times \eta_c}$$

From above we see that the longitudinal stress is half of the circumferential or hoop stress. Therefore, the design of a pressure vessel must be based on the maximum stress *i.e*. hoop stress.

Example 7.1. *A thin cylindrical pressure vessel of 1.2 m diameter generates steam at a pressure of 1.75 N/mm^2. Find the minimum wall thickness, if (a) the longitudinal stress does not exceed 28 MPa; and (b) the circumferential stress does not exceed 42 MPa.*

Cylinders and tanks are used to store fluids under pressure.

Solution. Given : d = 1.2 m = 1200 mm ; p = 1.75 N/mm^2 ; σ_{t2} = 28 MPa = 28 N/mm^2 ; σ_{t1} = 42 MPa = 42 N/mm^2

(a) When longitudinal stress (σ_{t2}) does not exceed 28 MPa

We know that minimum wall thickness,

$$t = \frac{p\,.\,d}{4\,\sigma_{t2}} = \frac{1.75 \times 1200}{4 \times 28} = 18.75 \text{ say } 20 \text{ mm} \quad \textbf{Ans.}$$

(b) When circumferential stress (σ_{t1}) does not exceed 42 MPa

We know that minimum wall thickness,

$$t = \frac{p\,.\,d}{2\,\sigma_{t1}} = \frac{1.75 \times 1200}{2 \times 42} = 25 \text{ mm} \quad \textbf{Ans.}$$

Example 7.2. *A thin cylindrical pressure vessel of 500 mm diameter is subjected to an internal pressure of 2 N/mm^2. If the thickness of the vessel is 20 mm, find the hoop stress, longitudinal stress and the maximum shear stress.*

Solution. Given : d = 500 mm ; p = 2 N/mm^2 ; t = 20 mm