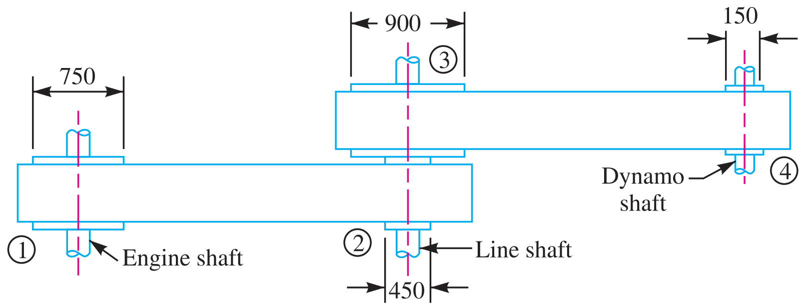
Let N_4 = Speed of the dynamo shaft.

1. *When there is no slip*

We know that

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{or} \quad \frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} = 10$$

$\therefore$ N_4 = 150 × 10 = 1500 r.p.m. **Ans.**



All dimensions in mm.

Fig. 18.12

2. *When there is a slip of 2% at each drive*

We know that

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

or

$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right) = 9.6$$

$\therefore$ N_4 = 150 × 9.6 = 1440 r.p.m. **Ans.**

18.16 Length of an Open Belt Drive

We have discussed in Art. 18.12, that in an open belt drive, both the pulleys rotate in the same direction as shown in Fig. 18.13.

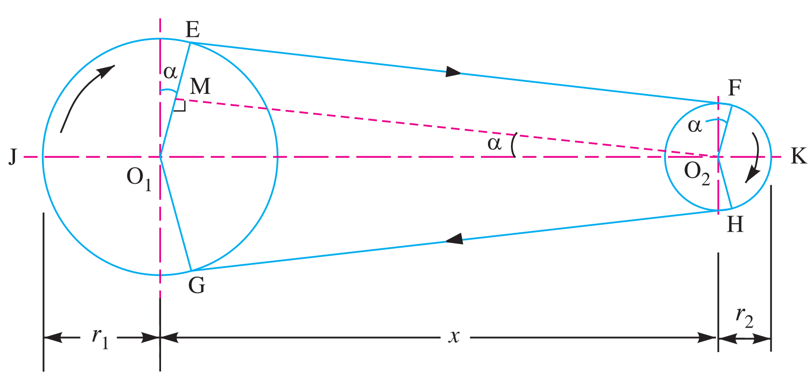


Fig. 18.13. Open belt drive.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* O_1O_2), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. 18.13. Through O_2 draw O_2M parallel to FE.

From the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the angle $MO_2O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \qquad \text{...}(i)$$

From the geometry of the figure, we also find that

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x}$$

Since the angle α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x} \qquad \text{...}(ii)$$

$$\therefore \qquad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \qquad \text{...}(iii)$$

Similarly,

$$\text{arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right) \qquad \text{...}(iv)$$

and

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem, we have

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \ldots\right] = x - \frac{(r_1 - r_2)^2}{2x} \qquad \text{...}(v)$$

Substituting the values of arc JE from equation ***(iii)***, arc FK from equation ***(iv)*** and EF from equation ***(v)*** in equation ***(i)***, we get

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \frac{\pi}{2} - r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha\,(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi\,(r_1 + r_2) + 2\alpha\,(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

Substituting the value of $\alpha = \dfrac{(r_1 - r_2)}{x}$ from equation ***(ii)***, we get

$$L = \pi\,(r_1 + r_2) + 2 \times \frac{(r_1 - r_2)}{x}\,(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi\,(r_1 + r_2) + \frac{2\,(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \quad \text{... (in terms of pulley radii)}$$

$$= \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \quad \text{... (in terms of pulley diameters)}$$

18.17 Length of a Cross Belt Drive

We have discussed in Art. 18.12 that in a cross belt drive, both the pulleys rotate in the opposite directions as shown in Fig. 18.14.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* O_1O_2), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. 18.14.

Through O_2 draw O_2M parallel to FE.

From the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the angle $MO_2O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + FE + \text{Arc } FK) \quad \text{...}(i)$$

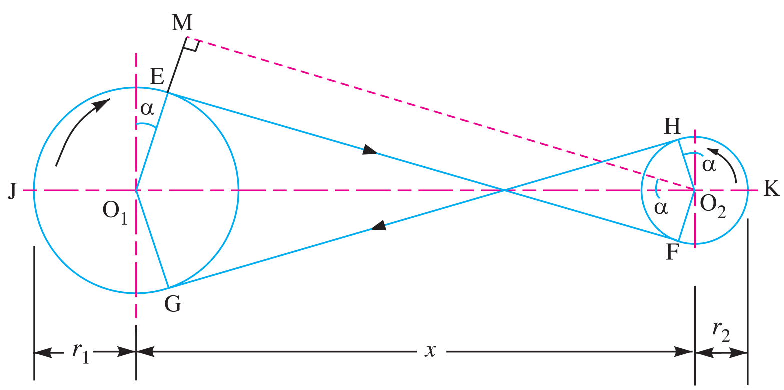


Fig. 18.14. Crossed belt drive.

From the geometry of the figure, we find that

$$\sin \alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2} = \frac{r_1 + r_2}{x}$$

Since the angle α is very small, therefore putting

$$\sin \alpha = \alpha \text{ (in radians)} = \frac{r_1 + r_2}{x} \quad \text{...}(ii)$$

$$\therefore \quad \text{Arc } JE = r_1 \left(\frac{\pi}{2} + \alpha \right) \quad \text{...}(iii)$$

Similarly, $$\text{arc } FK = r_2 \left(\frac{\pi}{2} + \alpha \right) \quad \text{...}(iv)$$

and
$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$
$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem, we have
$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + ...\right] = x - \frac{(r_1 + r_2)^2}{2x} \qquad ...(v)$$

In the above conveyor belt is used to transport material as well as to drive the rollers

Substituting the values of arc *JE* from equation ***(iii)***, arc *FK* from equation ***(iv)*** and *EF* from equation ***(v)*** in equation ***(i)***, we get,
$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$
$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \times \frac{\pi}{2} + r_2.\alpha\right]$$
$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha\,(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$
$$= \pi\,(r_1 + r_2) + 2\alpha\,(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{(r_1 + r_2)}{x}$ from equation ***(ii)***, we get
$$L = \pi\,(r_1 + r_2) + 2 \times \frac{(r_1 + r_2)}{x}(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$
$$= \pi\,(r_1 + r_2) + \frac{2\,(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \qquad \text{... (in terms of pulley radii)}$$

$$= \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \qquad \text{... (in terms of pulley diameters)}$$

It may be noted that the above expression is a function of $(r_1 + r_2)$. It is thus obvious, that if sum of the radii of the two pulleys be constant, length of the belt required will also remain constant, provided the distance between centres of the pulleys remain unchanged.

18.18 Power Transmitted by a Belt

Fig. 18.15 shows the driving pulley (or driver) *A* and the driven pulley (or follower) *B*. As already discussed, the driving pulley pulls the belt from one side and delivers it to the other side. It is thus obvious that the tension on the former side (*i.e.* tight side) will be greater than the latter side (*i.e.* slack side) as shown in Fig. 18.15.

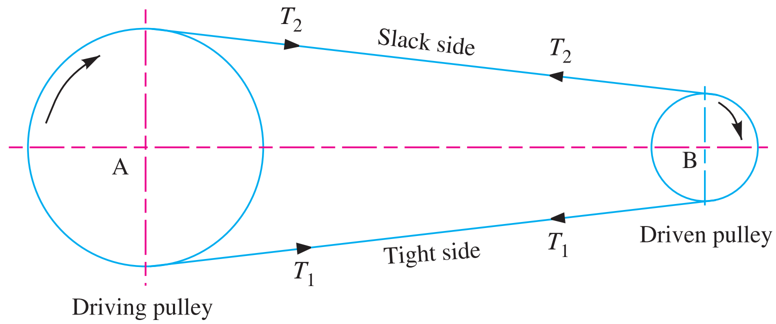


Fig. 18.15. Power transmitted by a belt.

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively in newtons,

r_1 and r_2 = Radii of the driving and driven pulleys respectively in metres,

and v = Velocity of the belt in m/s.

The effective turning (driving) force at the circumference of the driven pulley or follower is the difference between the two tensions (*i.e.* $T_1 - T_2$).

This massive shaft-like pulley drives the conveyor belt.

$\therefore$ Work done per second $= (T_1 - T_2)\, v$ N-m/s

and power transmitted $= (T_1 - T_2)\, v$ W ... ($\because$ 1 N-m/s = 1W)

A little consideration will show that torque exerted on the driving pulley is $(T_1 - T_2)\, r_1$. Similarly, the torque exerted on the driven pulley is $(T_1 - T_2)\, r_2$.

18.19 Ratio of Driving Tensions for Flat Belt Drive

Consider a driven pulley rotating in the clockwise direction as shown in Fig. 18.16.

Let T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtended by the arc *AB*, along which the belt touches the pulley, at the centre).

Now consider a small portion of the belt *PQ*, subtending an angle $\delta\theta$ at the centre of the pulley as shown in Fig. 18.16. The belt *PQ* is in equilibrium under the following forces:

1. Tension *T* in the belt at *P*,
2. Tension $(T + \delta T)$ in the belt at *Q*,
3. Normal reaction R_N, and
4. Frictional force $F = \mu \times R_N$, where μ is the coefficient of friction between the belt and pulley.

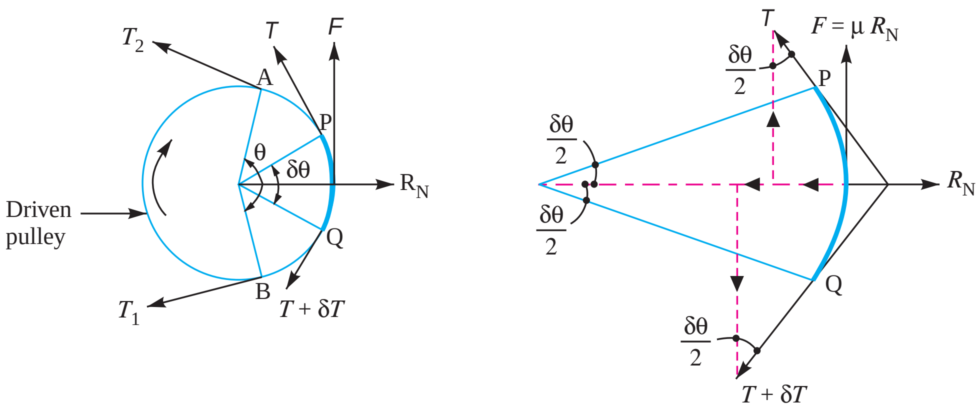


Fig. 18.16. Ratio of driving tensions for flat belt.

Resolving all the forces horizontally, we have

$$R_N = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \qquad \text{...}(i)$$

Since the angle $\delta\theta$ is very small, therefore putting $\sin \delta\theta/2 = \delta\theta/2$ in equation ***(i)***, we have

$$R_N = (T + \delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} = \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2}$$

$$= T.\delta\theta \qquad \text{...}\left(\text{Neglecting } \frac{\delta T.\delta\theta}{2}\right) \qquad \text{...}(ii)$$

Now resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \qquad \text{...}(iii)$$

Since the angle $\delta\theta$ is very small, therefore putting $\cos \delta\theta/2 = 1$ in equation ***(iii)***, we have

$$\mu \times R_N = T + \delta T - T = \delta T \quad \text{or} \quad R_N = \frac{\delta T}{\mu} \qquad \text{...}(iv)$$

Equating the values of R_N from equations **(ii)** and **(iv)**, we get

$$T.\delta\theta = \frac{\delta T}{\mu} \quad \text{or} \quad \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating the above equation between the limits T_2 and T_1 and from 0 to θ, we have

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta$$

$$\therefore \qquad \log_e \left(\frac{T_1}{T_2}\right) = \mu.\theta \quad \text{or} \quad \frac{T_1}{T_2} = e^{\mu.\theta} \qquad \text{...(v)}$$

The equation **(v)** can be expressed in terms of corresponding logarithm to the base 10, *i.e.*

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta$$

The above expression gives the relation between the tight side and slack side tensions, in terms of coefficient of friction and the angle of contact.

Notes : 1. While determining the angle of contact, it must be remembered that it is the angle of contact at the smaller pulley, if both the pulleys are of the same material. We know that

$$\sin\alpha = \frac{r_1 - r_2}{x} \qquad \text{... (for open belt drive)}$$

$$= \frac{r_1 + r_2}{x} \qquad \text{... (for cross-belt drive)}$$

∴ Angle of contact or lap,

$$\theta = (180° - 2\alpha)\frac{\pi}{180} \text{ rad} \qquad \text{...(for open belt drive)}$$

$$= (180° + 2\alpha)\frac{\pi}{180} \text{ rad} \qquad \text{... (for cross-belt drive)}$$

2. When the pulleys are made of different material (*i.e.* when the coefficient of friction of the pulleys or the angle of contact are different), then the design will refer to the pulley for which μ.θ is small.

Example 18.2. *Two pulleys, one 450 mm diameter and the other 200 mm diameter, on parallel shafts 1.95 m apart are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley.*

What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25?

Solution. Given : d_1 = 450 mm = 0.45 m or r_1 = 0.225 m ; d_2 = 200 mm = 0.2 m or r_2 = 0.1 m ; x = 1.95 m ; N_1 = 200 r.p.m. ; T_1 = 1 kN = 1000 N ; μ = 0.25

The arrangement of crossed belt drive is shown in Fig. 18.17.

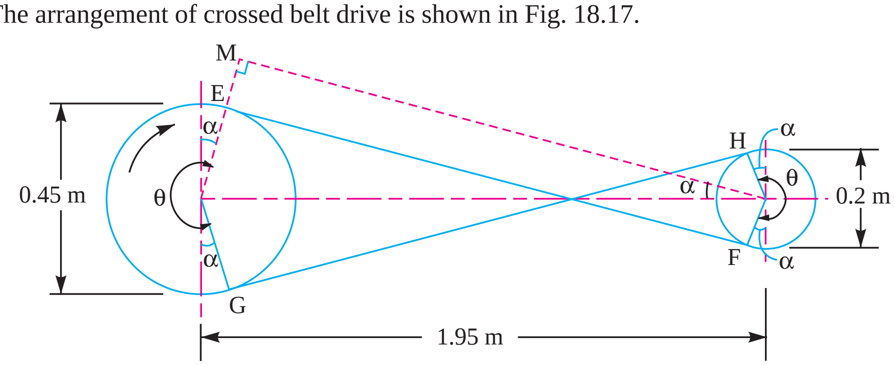


Fig. 18.17

Length of the belt

We know that length of the belt,

$$L = \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi (0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95}$$

$$= 1.02 + 3.9 + 0.054 = 4.974 \text{ m}$$ **Ans.**

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$\sin \alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667$$

$\therefore$ $\alpha = 9.6°$

and $\theta = 180° + 2\alpha = 180 + 2 \times 9.6 = 199.2°$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad}$$ **Ans.**

Power transmitted

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 3.477 = 0.8693$$

$$\log \left(\frac{T_1}{T_2}\right) = \frac{0.8693}{2.3} = 0.378 \quad \text{or} \quad \frac{T_1}{T_2} = 2.387 \qquad \text{... (Taking antilog of 0.378)}$$

$$\therefore \quad T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419 \text{ N}$$

We know that the velocity of belt,

$$v = \frac{\pi\, d_1 N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.713 \text{ m/s}$$

$\therefore$ Power transmitted,

$$P = (T_1 - T_2)\, v = (1000 - 419)\, 4.713 = 2738 \text{ W} = 2.738 \text{ kW}$$ **Ans.**

18.20 Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both the tight as well as the slack sides. The tension caused by centrifugal force is called ***centrifugal tension.*** At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

Consider a small portion PQ of the belt subtending an angle $d\theta$ at the centre of the pulley, as shown in Fig. 18.18.

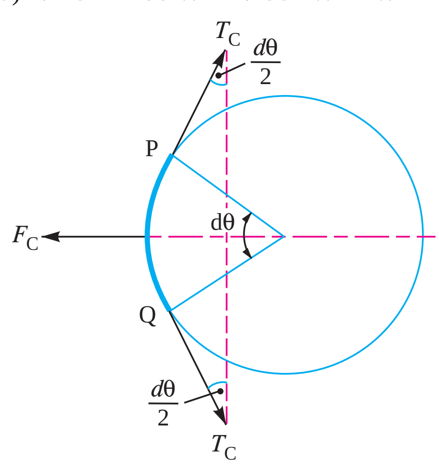


Fig. 18.18. Centrifugal tension.