Length of the belt

We know that length of the belt,

$$L = \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi (0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95}$$

$$= 1.02 + 3.9 + 0.054 = 4.974 \text{ m}$$ **Ans.**

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$\sin \alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667$$

$\therefore$ $\alpha = 9.6°$

and $\theta = 180° + 2\alpha = 180 + 2 \times 9.6 = 199.2°$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad}$$ **Ans.**

Power transmitted

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 3.477 = 0.8693$$

$$\log \left(\frac{T_1}{T_2}\right) = \frac{0.8693}{2.3} = 0.378 \quad \text{or} \quad \frac{T_1}{T_2} = 2.387$$ ... (Taking antilog of 0.378)

$$\therefore \quad T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419 \text{ N}$$

We know that the velocity of belt,

$$v = \frac{\pi\, d_1 N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.713 \text{ m/s}$$

$\therefore$ Power transmitted,

$$P = (T_1 - T_2)\, v = (1000 - 419)\, 4.713 = 2738 \text{ W} = 2.738 \text{ kW}$$ **Ans.**

18.20 Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both the tight as well as the slack sides. The tension caused by centrifugal force is called ***centrifugal tension.*** At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

Consider a small portion *PQ* of the belt subtending an angle $d\theta$ at the centre of the pulley, as shown in Fig. 18.18.

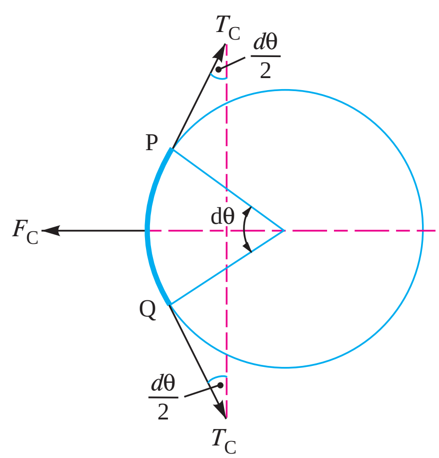

Fig. 18.18. Centrifugal tension.

Let m = Mass of belt per unit length in kg,

v = Linear velocity of belt in m/s,

r = Radius of pulley over which the belt runs in metres, and

T_C = Centrifugal tension acting tangentially at P and Q in newtons.

We know that length of the belt PQ

$$= r.d\theta$$

and mass of the belt PQ $= m.r.d\theta$

∴ Centrifugal force acting on the belt PQ,

$$F_C = m.r.d\theta \times \frac{v^2}{r} = m.d\theta.v^2$$

Belt drive on a lathe

The centrifugal tension T_C acting tangentially at P and Q keeps the belt in equilibrium. Now resolving the forces (*i.e.* centrifugal force and centrifugal tension) horizontally, we have

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C = m.d\theta.v^2 \quad ...(i)$$

Since the angle $d\theta$ is very small, therefore putting $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$ in equation (*i*), we have

$$2T_C\left(\frac{d\theta}{2}\right) = m.d\theta.v^2$$

$$\therefore \quad T_C = m.v^2$$

Notes : 1. When centrifugal tension is taken into account, then total tension in the tight side,

$$T_{t1} = T_1 + T_C$$

and total tension in the slack side,

$$T_{t2} = T_2 + T_C$$

2. Power transmitted,

$$P = (T_{t1} - T_{t2})\,v \qquad \text{...(in watts)}$$
$$= [(T_1 + T_C) - (T_2 + T_C)]v = (T_1 - T_2)\,v \qquad \text{... (same as before)}$$

Thus we see that the centrifugal tension has no effect on the power transmitted.

3. The ratio of driving tensions may also be written as

$$2.3 \log \left(\frac{T_{t1} - T_C}{T_{t2} - T_C}\right) = \mu.\theta$$

where T_{t1} = Maximum or total tension in the belt.

18.21 Maximum Tension in the Belt

A little consideration will show that the maximum tension in the belt (T) is equal to the total tension in the tight side of the belt (T_{t1}).

Let σ = Maximum safe stress,

b = Width of the belt, and

t = Thickness of the belt.

We know that the maximum tension in the belt,

$$T = \text{Maximum stress} \times \text{Cross-sectional area of belt} = \sigma.b.t$$

When centrifugal tension is neglected, then

$$T \text{ (or } T_{t1}) = T_1, \textit{ i.e.} \text{ Tension in the tight side of the belt.}$$

When centrifugal tension is considered, then

$$T \text{ (or } T_{t1}) = T_1 + T_C$$

18.22 Condition for the Transmission of Maximum Power

We know that the power transmitted by a belt,

$$P = (T_1 - T_2)\,v \qquad \text{...}(i)$$

where T_1 = Tension in the tight side in newtons,

T_2 = Tension in the slack side in newtons, and

v = Velocity of the belt in m/s.

From Art. 18.19, ratio of driving tensions is

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{or} \quad T_2 = \frac{T_1}{e^{\mu\theta}} \qquad \text{...}(ii)$$

Substituting the value of T_2 in equation (*i*), we have

$$P = \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right) v = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right) v = T_1.v.C \qquad \text{...}(iii)$$

where $C = \left(1 - \dfrac{1}{e^{\mu\theta}}\right)$

We know that

$$T_1 = T - T_C$$

where T = Maximum tension to which the belt can be subjected in newtons, and

T_C = Centrifugal tension in newtons.

Substituting the value of T_1 in equation (*iii*), we have

$$P = (T - T_C)\,v \times C$$
$$= (T - mv^2)\,v \times C = (T.v - m.v^3)\,C \qquad \text{... (Substituting } T_C = m.v^2)$$

For maximum power, differentiate the above expression with respect to v and equate to zero, *i.e.*

$$\frac{dP}{dv} = 0 \quad \text{or} \quad \frac{d}{dv}(T.v - m.v^3)\, C = 0$$

or $$T - 3\, m.v^2 = 0 \qquad \text{...(iv)}$$

$\therefore$ $$T - 3\, T_C = 0 \quad \text{or} \quad T = 3T_C \qquad \text{... } (\because\ m.v^2 = T_C)$$

It shows that when the power transmitted is maximum, 1/3rd of the maximum tension is absorbed as centrifugal tension.

Notes : 1. We know that $T_1 = T - T_C$ and for maximum power, $T_C = \dfrac{T}{3}$.

$\therefore$ $$T_1 = T - \frac{T}{3} = \frac{2T}{3}$$

2. From equation **(*iv*)**, we find that the velocity of the belt for maximum power,

$$v = \sqrt{\frac{T}{3m}}$$

Example 18.3. *A leather belt 9 mm × 250 mm is used to drive a cast iron pulley 900 mm in diameter at 336 r.p.m. If the active arc on the smaller pulley is 120° and the stress in tight side is 2 MPa, find the power capacity of the belt. The density of leather may be taken as 980 kg/m³, and the coefficient of friction of leather on cast iron is 0.35.*

Solution. Given: t = 9 mm = 0.009 m ; b = 250 mm = 0.25 m; d = 900 mm = 0.9 m ; N = 336 r.p.m ; $\theta = 120° = 120 \times \dfrac{\pi}{180} = 2.1$ rad ; σ = 2 MPa = 2 N/mm^2 ; ρ = 980 kg/m^3 ; μ = 0.35

We know that the velocity of the belt,

$$v = \frac{\pi\, d.N}{60} = \frac{\pi \times 0.9 \times 336}{60} = 15.8 \text{ m/s}$$

and cross-sectional area of the belt,

$$a = b.t = 9 \times 250 = 2250 \text{ mm}^2$$

$\therefore$ Maximum or total tension in the tight side of the belt,

$$T = T_{t1} = \sigma.a = 2 \times 2250 = 4500 \text{ N}$$

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.t.l.\rho = 0.25 \times 0.009 \times 1 \times 980 \text{ kg/m}$$
$$= 2.2 \text{ kg/m}$$

$\therefore$ Centrifugal tension,

$${}^*T_C = m.v^2 = 2.2\,(15.8)^2 = 550 \text{ N}$$

and tension in the tight side of the belt,

$$T_1 = T - T_C = 4500 - 550 = 3950 \text{ N}$$

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.35 \times 2.1 = 0.735$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.735}{2.3} = 0.3196 \quad \text{or} \quad \frac{T_1}{T_2} = 2.085 \qquad \text{... (Taking antilog of 0.3196)}$$

* $T_C = m.v^2 = \dfrac{\text{kg}}{\text{m}} \times \dfrac{\text{m}^2}{\text{s}^2} = \text{kg-m/s}^2$ or N ...($\because$ 1 N = 1 kg-m/s^2)

and $$T_2 = \frac{T_1}{2.085} = \frac{3950}{2.085} = 1895 \text{ N}$$

We know that the power capacity of the belt,

$$P = (T_1 - T_2)\, v = (3950 - 1895)\, 15.8 = 32\,470 \text{ W} = 32.47 \text{ kW}$$ **Ans.**

Notes : The power capacity of the belt, when centrifugal tension is taken into account, may also be obtained as discussed below :

1. We know that the maximum tension in the tight side of the belt,

$$T_{t1} = T = 4500 \text{ N}$$

Centrifugal tension, $$T_C = 550 \text{ N}$$

and tension in the slack side of the belt,

$$T_2 = 1895 \text{ N}$$

∴ Total tension in the slack side of the belt,

$$T_{t2} = T_2 + T_C = 1895 + 550 = 2445 \text{ N}$$

We know that the power capacity of the belt,

$$P = (T_{t1} - T_{t2})\, v = (4500 - 2445)\, 15.8 = 32\,470 \text{ W} = 32.47 \text{ kW}$$ **Ans.**

2. The value of total tension in the slack side of the belt (T_{t2}) may also be obtained by using the relation as discussed in Art. 18.20, *i.e.*

$$2.3 \log \left(\frac{T_{t1} - T_C}{T_{t2} - T_C} \right) = \mu.\theta$$

Example 18.4. *A flat belt is required to transmit 30 kW from a pulley of 1.5 m effective diameter running at 300 r.p.m. The angle of contact is spread over $\frac{11}{24}$ of the circumference. The coefficient of friction between the belt and pulley surface is 0.3. Determine, taking centrifugal tension into account, width of the belt required. It is given that the belt thickness is 9.5 mm, density of its material is 1100 kg / m^3 and the related permissible working stress is 2.5 MPa.*

Solution. Given : $P = 30$ kW $= 30 \times 10^3$ W ; $d = 1.5$ m ; $N = 300$ r.p.m. ; $\theta = \frac{11}{24} \times 360 = 165°$ $= 165 \times \pi / 180 = 2.88$ rad ; $\mu = 0.3$; $t = 9.5$ mm $= 0.0095$ m ; $\rho = 1100$ kg/m^3 ; $\sigma = 2.5$ MPa $= 2.5 \times 10^6$ N/m^2

Let T_1 = Tension in the tight side of the belt in newtons, and

T_2 = Tension in the slack side of the belt in newtons.

We know that the velocity of the belt,

$$v = \frac{\pi\, d\, N}{60} = \frac{\pi \times 1.5 \times 300}{60} = 23.57 \text{ m/s}$$

and power transmitted (P),

$$30 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 23.57$$

$$\therefore \quad T_1 - T_2 = 30 \times 10^3 / 23.57 = 1273 \text{ N} \qquad \text{...}(i)$$

We know that

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu.\theta = 0.3 \times 2.88 = 0.864$$

$$\therefore \quad \log \left(\frac{T_1}{T_2} \right) = \frac{0.864}{2.3} = 0.3756 \quad \text{or} \quad \frac{T_1}{T_2} = 2.375 \qquad \text{...}(ii)$$

... (Taking antilog of 0.3756)

From equations (*i*) and (*ii*), we find that

$$T_1 = 2199 \text{ N} ; \quad \text{and} \quad T_2 = 926 \text{ N}$$

Let b = Width of the belt required in metres.

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$
$$= b \times 0.0095 \times 1 \times 1100 = 10.45\, b \text{ kg/m}$$

and centrifugal tension, $T_C = m.v^2 = 10.45\, b\, (23.57)^2 = 5805\, b$ N

We know that maximum tension in the belt,

$$T = T_1 + T_C = \text{Stress} \times \text{Area} = \sigma.b.t$$

or $2199 + 5805\, b = 2.5 \times 10^6 \times b \times 0.0095 = 23\,750\, b$

$\therefore$ $23\,750\, b - 5805\, b = 2199$ or $b = 0.122$ m or 122 mm

The standard width of the belt is 125 mm. **Ans.**

Example 18.5. *An electric motor drives an exhaust fan. Following data are provided :*

	Motor pulley	*Fan pulley*
Diameter	*400 mm*	*1600 mm*
Angle of warp	*2.5 radians*	*3.78 radians*
Coefficient of friction	*0.3*	*0.25*
Speed	*700 r.p.m.*	—
Power transmitted	*22.5 kW*	—

Calculate the width of 5 mm thick flat belt. Take permissible stress for the belt material as 2.3 MPa.

Solution. Given : d_1 = 400 mm or r_1 = 200 mm ; d_2 = 1600 mm or r_2 = 800 mm ; θ_1 = 2.5 rad ; θ_2 = 3.78 rad ; μ_1 = 0.3; μ_2 = 0.25 ; N_1 = 700 r.p.m. ; P = 22.5 kW = 22.5×10^3 W ; t = 5 mm = 0.005 m ; σ = 2.3 MPa = 2.3×10^6 N/m^2

Fig. 18.19 shows a system of flat belt drive. Suffix 1 refers to motor pulley and suffix 2 refers to fan pulley.

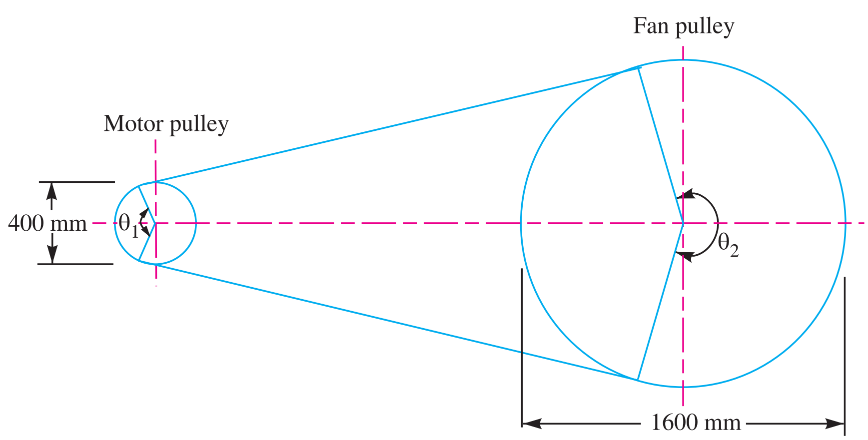

Fig. 18.19

We have discussed in Art. 18.19 (Note 2) that when the pulleys are made of different material [*i.e.* when the pulleys have different coefficient of friction (μ) or different angle of contact (θ), then the design will refer to a pulley for which $\mu.\theta$ is small.

$\therefore$ For motor pulley, $\mu_1.\theta_1 = 0.3 \times 2.5 = 0.75$

and for fan pulley, $\mu_2.\theta_2 = 0.25 \times 3.78 = 0.945$

Since $\mu_1.\theta_1$ for the motor pulley is small, therefore the design is based on the motor pulley.

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the velocity of the belt,

$$v = \frac{\pi\, d_1.N_1}{60} = \frac{\pi \times 0.4 \times 700}{60} = 14.7 \text{ m/s} \qquad \text{... (}d_1 \text{ is taken in metres)}$$

and the power transmitted (P),

$$22.5 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 14.7$$

$$\therefore \quad T_1 - T_2 = 22.5 \times 10^3 / 14.7 = 1530 \text{ N} \qquad ...(i)$$

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu_1.\theta_1 = 0.3 \times 2.5 = 0.75$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{0.75}{2.3} = 0.3261 \quad \text{or} \quad \frac{T_1}{T_2} = 2.12 \qquad ...(ii)$$

... (Taking antilog of 0.3261)

From equations ***(i)*** and ***(ii)***, we find that

$$T_1 = 2896 \text{ N} ; \quad \text{and} \quad T_2 = 1366 \text{ N}$$

Let b = Width of the belt in metres.

Since the velocity of the belt is more than 10 m/s, therefore centrifugal tension must be taken into consideration. Assuming a leather belt for which the density may be taken as 1000 kg / m^3.

∴ Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$

$$= b \times 0.005 \times 1 \times 1000 = 5\, b \text{ kg/m}$$

and centrifugal tension, $T_C = m.v^2 = 5\, b\, (14.7)^2 = 1080\, b$ N

We know that the maximum (or total) tension in the belt,

$$T = T_1 + T_C = \text{Stress} \times \text{Area} = \sigma.b.t$$

or $$2896 + 1080\, b = 2.3 \times 10^6\, b \times 0.005 = 11\,500\, b$$

$$\therefore \quad 11\,500\, b - 1080\, b = 2896 \quad \text{or} \quad b = 0.278 \text{ say } 0.28 \text{ m or } 280 \text{ mm}$$ **Ans.**

Example 18.6. *Design a rubber belt to drive a dynamo generating 20 kW at 2250 r.p.m. and fitted with a pulley 200 mm diameter. Assume dynamo efficiency to be 85%.*

Allowable stress for belt	= *2.1 MPa*
Density of rubber	= *1000 kg / m^3*
Angle of contact for dynamo pulley	= *165°*
Coefficient of friction between belt and pulley	= *0.3*

Solution. Given : P = 20 kW = 20 × 10^3 W ; N = 2250 r.p.m. ; d = 200 mm = 0.2 m ; η_d = 85% = 0.85 ; σ = 2.1 MPa = 2.1 × 10^6 N/m^2 ; ρ = 1000 kg/m^3 ; θ = 165° = 165 × π/180 = 2.88 rad ; μ = 0.3

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that velocity of the belt,

$$v = \frac{\pi\, d.N}{60} = \frac{\pi \times 0.2 \times 2250}{60} = 23.6 \text{ m/s}$$

and power transmitted (P),

$$20 \times 10^3 = (T_1 - T_2)\, v.\eta_d = (T_1 - T_2)\, 23.6 \times 0.85 = 20.1\,(T_1 - T_2)$$

$$\therefore \quad T_1 - T_2 = 20 \times 10^3 / 20.1 = 995 \text{ N} \quad ...(i)$$

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.88 = 0.864$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{0.864}{2.3} = 0.3756$$

or

$$\frac{T_1}{T_2} = 2.375 \quad ...(ii)$$

... (Taking antilog of 0.3756)

Rubber belt

From equations **(i)** and **(ii)**, we find that

$T_1 = 1719$ N ; and $T_2 = 724$ N

Let b = Width of the belt in metres, and

t = Thickness of the belt in metres.

Assuming thickness of the belt, t = 10 mm = 0.01 m, we have

Cross-sectional area of the belt

$$= b \times t = b \times 0.01 = 0.01\, b \text{ m}^2$$

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = 0.01\, b \times 1 \times 1000 = 10\, b \text{ kg / m}$$

$\therefore$ Centrifugal tension,

$$T_C = m.v^2 = 10\, b\, (23.6)^2 = 5570\, b \text{ N}$$

We know that maximum tension in the belt,

$$T = \sigma.b.t = 2.1 \times 10^6 \times b \times 0.01 = 21\,000\, b \text{ N}$$

and tension in the tight side of belt (T_1),

$$1719 = T - T_C = 21\,000\, b - 5570\, b = 15\,430\, b$$

$$\therefore \quad b = 1719 / 15\,430 = 0.1114 \text{ m} = 111.4 \text{ mm}$$

The standard width of the belt (b) is 112 mm. **Ans.**

Example 18.7. *Design a belt drive to transmit 110 kW for a system consisting of two pulleys of diameters 0.9 m and 1.2 m, centre distance of 3.6 m, a belt speed 20 m / s, coefficient of friction 0.3, a slip of 1.2% at each pulley and 5% friction loss at each shaft, 20% over load.*

Solution. Given : $P = 110$ kW $= 110 \times 10^3$ W ; $d_1 = 0.9$ m or $r_1 = 0.45$ m ; $d_2 = 1.2$ m or $r_2 = 0.6$ m ; $x = 3.6$ m ; $v = 20$ m/s ; $\mu = 0.3$; $s_1 = s_2 = 1.2\%$

Fig 18.20 shows a system of flat belt drive consisting of two pulleys.

Let N_1 = Speed of the smaller or driving pulley in r.p.m., and

and N_2 = Speed of the larger or driven pulley in r.p.m.

We know that speed of the belt (v),

$$20 = \frac{\pi\, d_1.N_1}{60}\left(1 - \frac{s_1}{100}\right) = \frac{\pi \times 0.9\, N_1}{60}\left(1 - \frac{1.2}{100}\right) = 0.0466\, N_1$$

$$\therefore \quad N_1 = 20 / 0.0466 = 430 \text{ r.p.m.}$$

and peripheral velocity of the driven pulley,

$$\frac{\pi\, d_2.N_2}{60} = \text{Belt speed in m/s}\left(1-\frac{s_2}{100}\right) = v\left(1-\frac{s_2}{100}\right)$$

or
$$\frac{\pi \times 1.2 \times N_2}{60} = 20\left(1-\frac{1.2}{100}\right) = 19.76$$

$$\therefore \quad N_2 = \frac{19.76 \times 60}{\pi \times 1.2} = 315 \text{ r.p.m.}$$

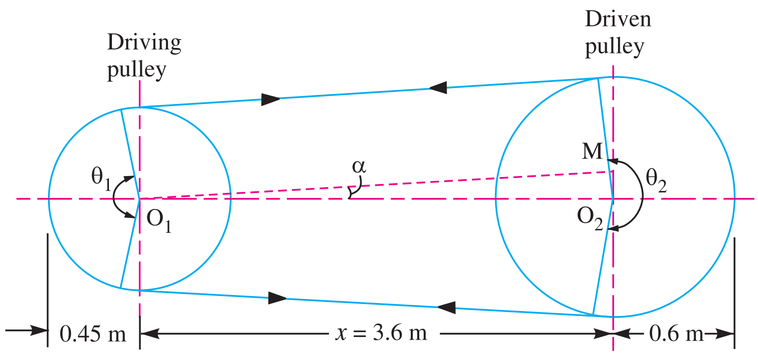

Fig. 18.20

We know that the torque acting on the driven shaft

$$= \frac{\text{Power transmitted} \times 60}{2\,\pi\, N_2} = \frac{110 \times 10^3 \times 60}{2\,\pi \times 315} = 3334 \text{ N-m}$$

Since there is a 5% friction loss at each shaft, therefore torque acting on the belt

$$= 1.05 \times 3334 = 3500 \text{ N-m}$$

Since the belt is to be designed for 20% overload, therefore design torque

$$= 1.2 \times 3500 = 4200 \text{ N-m}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the torque exerted on the driven pulley

$$= (T_1 - T_2)\, r_2 = (T_1 - T_2)\, 0.6 = 0.6\,(T_1 - T_2) \text{ N-m}$$

Equating this to the design torque, we have

$$0.6\,(T_1 - T_2) = 4200 \quad \text{or} \quad T_1 - T_2 = 4200 / 0.6 = 7000 \text{ N} \qquad ...(i)$$

Now let us find out the angle of contact (θ_1) of the belt on the smaller or driving pulley.

From the geometry of the Fig. 18.20, we find that

$$\sin\alpha = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{0.6 - 0.45}{3.6} = 0.0417 \quad \text{or} \quad \alpha = 2.4°$$

$$\therefore \quad \theta_1 = 180° - 2\alpha = 180 - 2 \times 2.4 = 175.2° = 175.2 \times \frac{\pi}{180} = 3.06 \text{ rad}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta_1 = 0.3 \times 3.06 = 0.918$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.918}{2.3} = 0.3991 \quad \text{or} \quad \frac{T_1}{T_2} = 2.51 \text{ ... (Taking antilog of 0.3991)} \qquad ...(ii)$$

From equations **(*i*)** and **(*ii*)**, we find that

$$T_1 = 11\ 636 \text{ N} \text{ ; and } T_2 = 4636 \text{ N}$$

Let σ = Safe stress for the belt = 2.5 MPa = 2.5×10^6 N/m^2 ...(Assume)

t = Thickness of the belt = 15 mm = 0.015 m, and ...(Assume)

b = Width of the belt in metres.

Since the belt speed is more than 10 m/s, therefore centrifugal tension must be taken into consideration. Assuming a leather belt for which the density may be taken as 1000 kg / m^3.

∴ Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$

$$= b \times 0.015 \times 1 \times 1000 = 15\, b \text{ kg/m}$$

and centrifugal tension,

$$T_C = m.v^2 = 15\, b\, (20)^2 = 6000\, b \text{ N}$$

We know that maximum tension in the belt,

$$T = T_1 + T_C = \sigma.b.t$$

or $$11\ 636 + 6000\, b = 2.5 \times 10^6 \times b \times 0.015 = 37\ 500\, b$$

∴ $$37\ 500\, b - 6000\, b = 11\ 636 \quad \text{or} \quad b = 0.37 \text{ m or } 370 \text{ mm}$$

The standard width of the belt (b) is 400 mm. **Ans.**

We know that length of the belt,

$$L = \pi\,(r_2 + r_1) + 2\,x + \frac{(r_2 - r_1)^2}{x}$$

$$= \pi\,(0.6 + 0.45) + 2 \times 3.6 + \frac{(0.6 - 0.45)^2}{3.6}$$

$$= 3.3 + 7.2 + 0.006 = 10.506 \text{ m}$$ **Ans.**

Example 18.8. *A belt 100 mm wide and 10 mm thick is transmitting power at 1000 metres/min. The net driving tension is 1.8 times the tension on the slack side. If the safe permissible stress on the belt section in 1.6 MPa, calculate the maximum power, that can be transmitted at this speed. Assume density of the leather as 1000 kg/m^3.*

Calculate the absolute maximum power that can be transmitted by this belt and the speed at which this can be transmitted.

Solution. Given : b = 100 mm = 0.1 m ; t = 10 mm = 0.01 m ; v = 1000 m/min = 16.67 m/s ; $T_1 - T_2 = 1.8\, T_2$; σ = 1.6 MPa = 1.6 N/mm^2 ; ρ = 1000 kg/m^3

Power transmitted

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the maximum tension in the belt,

$$T = \sigma.b.t = 1.6 \times 100 \times 10 = 1600 \text{ N}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$

$$= 0.1 \times 0.01 \times 1 \times 1000 = 1 \text{ kg/m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 1\,(16.67)^2 = 278 \text{ N}$$

We know that

$$T_1 = T - T_C = 1600 - 278 = 1322 \text{ N}$$

and $$T_1 - T_2 = 1.8\,T_2 \qquad \text{...(Given)}$$

$$\therefore \qquad T_2 = \frac{T_1}{2.8} = \frac{1322}{2.8} = 472 \text{ N}$$

We know that the power transmitted.

$$P = (T_1 - T_2)\,v = (1322 - 472)\,16.67 = 14\,170 \text{ W} = 14.17 \text{ kW} \quad \textbf{Ans.}$$

Speed at which absolute maximum power can be transmitted

We know that the speed of the belt for maximum power,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{1600}{3 \times 1}} = 23.1 \text{ m/s} \quad \textbf{Ans.}$$

Absolute maximum power

We know that for absolute maximum power, the centrifugal tension,

$$T_C = T/3 = 1600/3 = 533 \text{ N}$$

$\therefore$ Tension in the tight side,

$$T_1 = T - T_C = 1600 - 533 = 1067 \text{ N}$$

and tension in the slack side,

$$T_2 = \frac{T_1}{2.8} = \frac{1067}{2.8} = 381 \text{ N}$$

$\therefore$ Absolute maximum power transmitted,

$$P = (T_1 - T_2)\,v = (1067 - 381)\,23.1 = 15\,850 \text{ W} = 15.85 \text{ kW} \quad \textbf{Ans.}$$

18.23 Initial Tension in the Belt

When a belt is wound round the two pulleys (*i.e.* driver and follower), its two ends are joined together, so that the belt may continuously move over the pulleys, since the motion of the belt (from the driver) and the follower (from the belt) is governed by a firm grip due to friction between the belt and the pulleys. In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tension, called ***initial tension***.

When the driver starts rotating, it pulls the belt from one side (increasing tension in the belt on this side) and delivers to the other side (decreasing tension in the belt on that side). The increased tension in one side of the belt is called tension in tight side and the decreased tension in the other side of the belt is called tension in the slack side.

Let T_0 = Initial tension in the belt,

T_1 = Tension in the tight side of the belt,

T_2 = Tension in the slack side of the belt, and

α = Coefficient of increase of the belt length per unit force.

A little consideration will show that the increase of tension in the tight side

$$= T_1 - T_0$$

and increase in the length of the belt on the tight side

$$= \alpha\,(T_1 - T_0) \qquad \text{...}(i)$$

Similarly, decrease in tension in the slack side

$$= T_0 - T_2$$

and decrease in the length of the belt on the slack side

$$= \alpha\,(T_0 - T_2) \qquad \text{...}(ii)$$

Assuming that the belt material is perfectly elastic such that the length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to decrease in the length on the slack side. Thus, equating equations (*i*) and (*ii*), we have

$$\alpha\,(T_1 - T_0) = \alpha\,(T_0 - T_2)$$