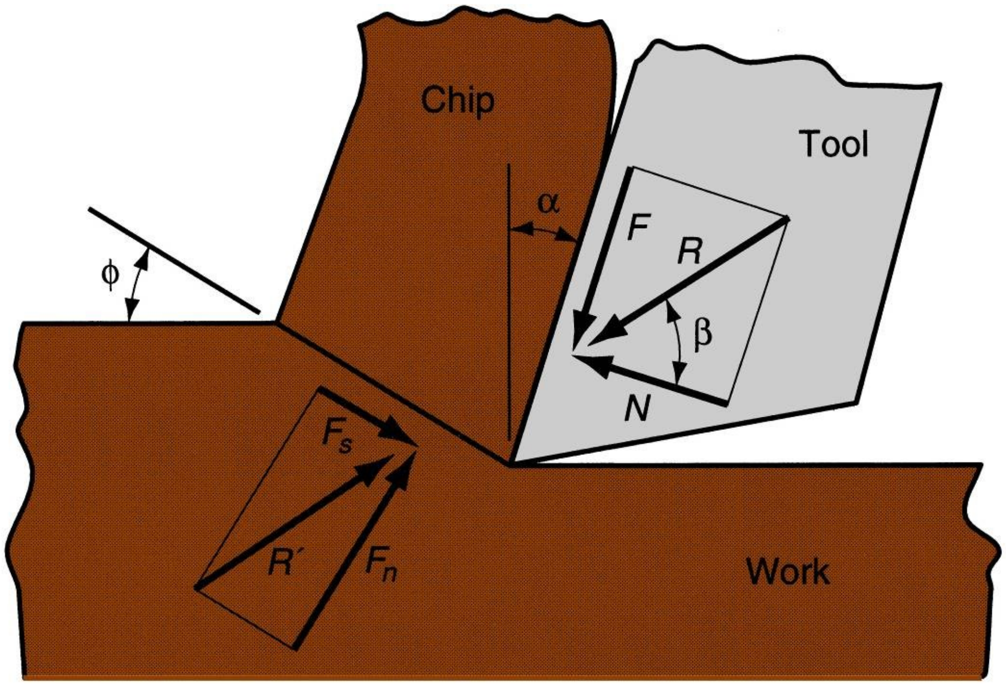
Forces Acting on Chip

- Friction force ***F*** and Normal force to friction ***N***
- Shear force F_s and Normal force to shear F_n

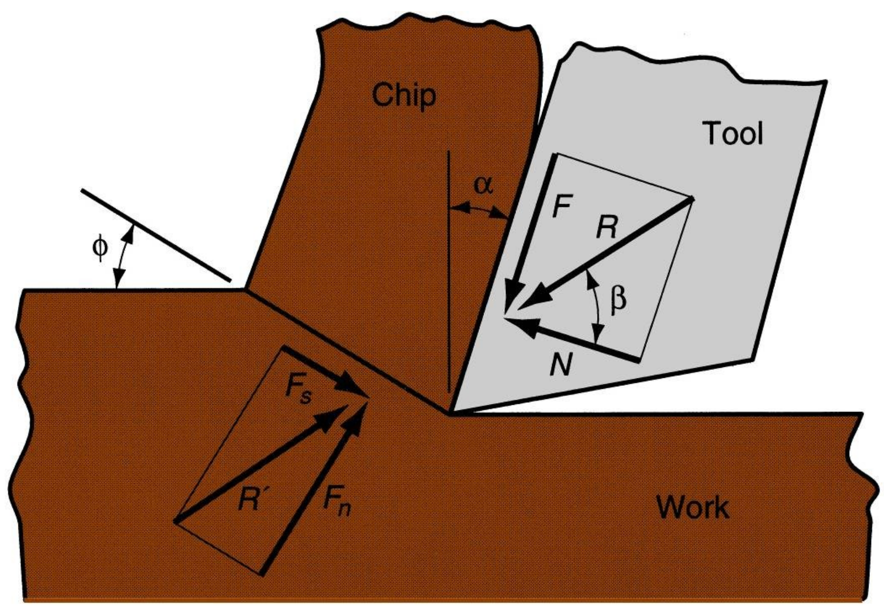
Forces in metal cutting: (a) forces acting on the chip in orthogonal cutting



(a)

Resultant Forces

- Vector addition of *F* and *N* = resultant *R*
- Vector addition of F_s and F_n = resultant *R'*
- Forces acting on the chip must be in balance:
  - *R'* must be equal in magnitude to *R*
  - *R'* must be opposite in direction to *R*
  - *R'* must be collinear with *R*

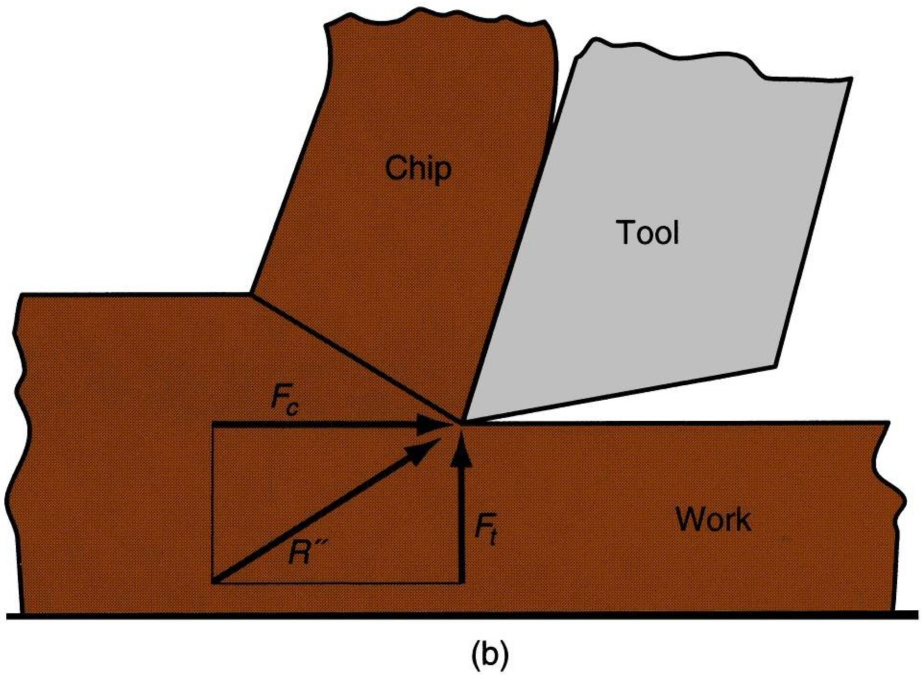


(a)

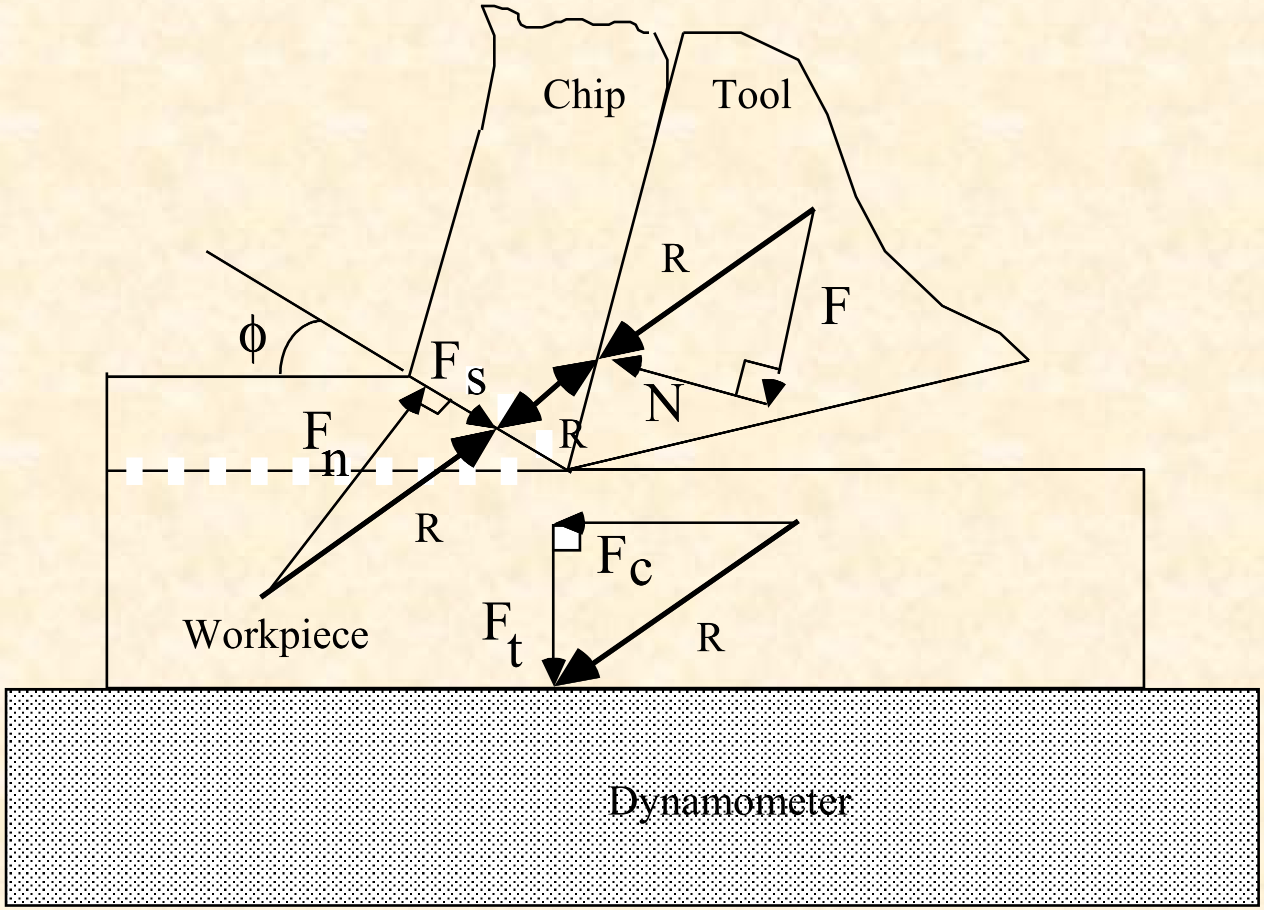
Cutting Force and Thrust Force

- F, N, F_s, and F_n cannot be directly measured
- Forces acting on the tool that can be measured:
  - Cutting force F_c and Thrust force F_t

Figure 21.10 Forces in metal cutting: (b) forces acting on the tool that can be measured



Cutting Forces
(2D Orthogonal Cutting)



Free Body Diagram

Generally we know:
Tool geometry & type
Workpiece material

and we wish to know:
F = Cutting Force
Fc = Thrust Force
Ft = Friction Force
N = Normal Force
Fs = Shear Force
Fn = Force Normal

to Shear

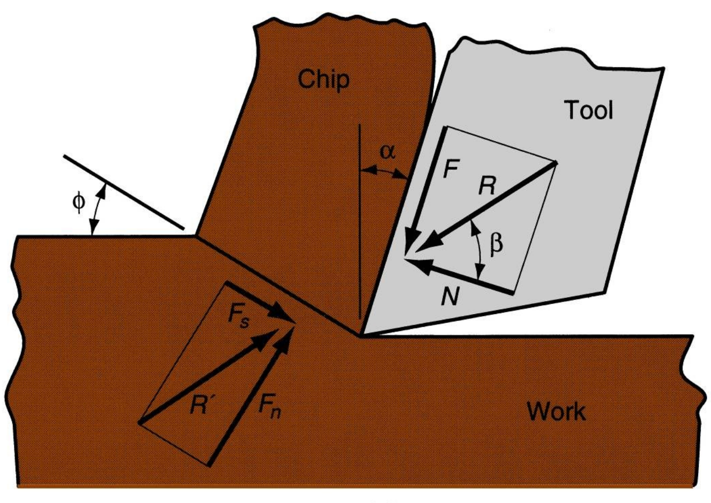
Coefficient of Friction

Coefficient of friction between tool and chip:

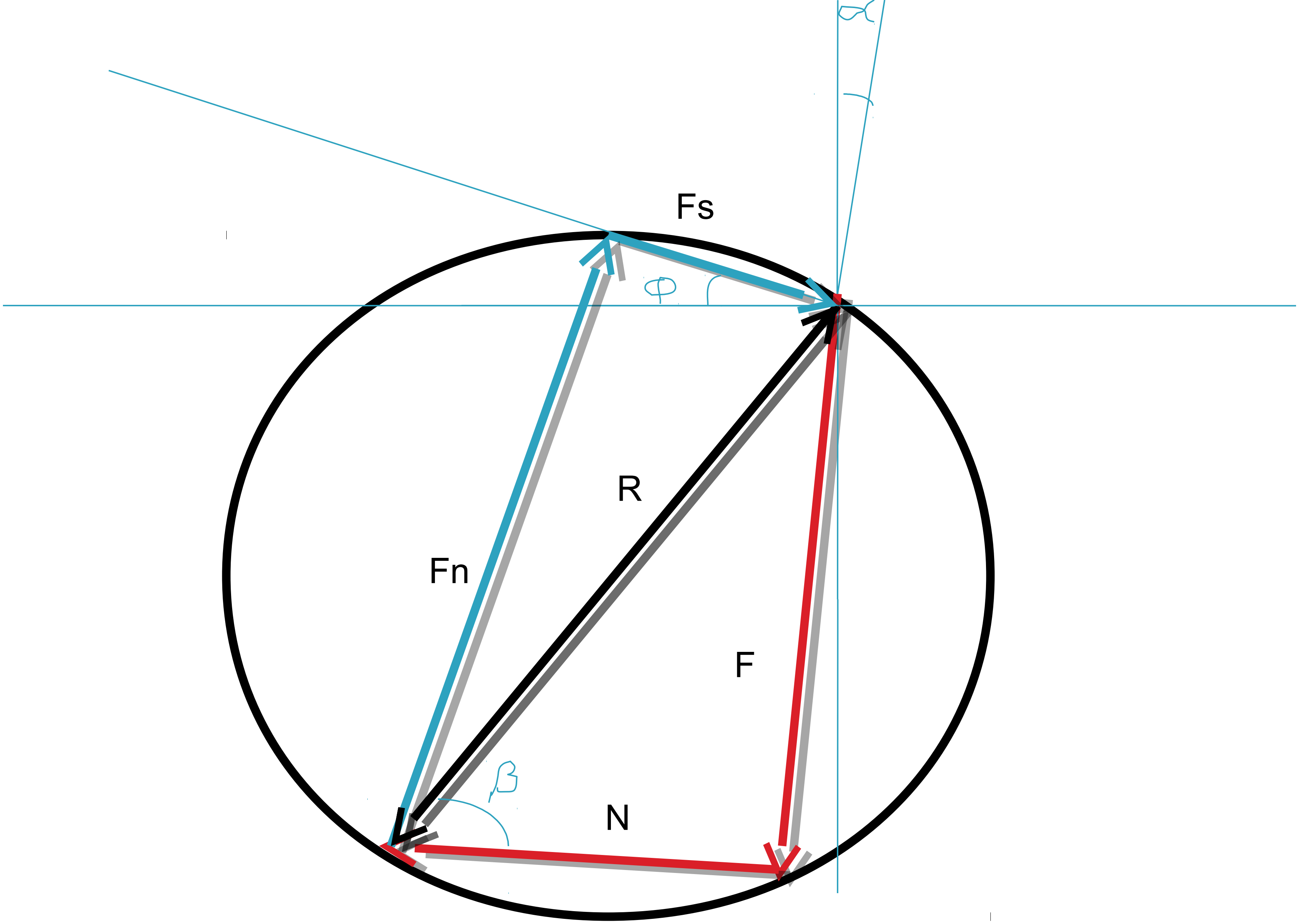
$$\mu = \frac{F}{N}$$

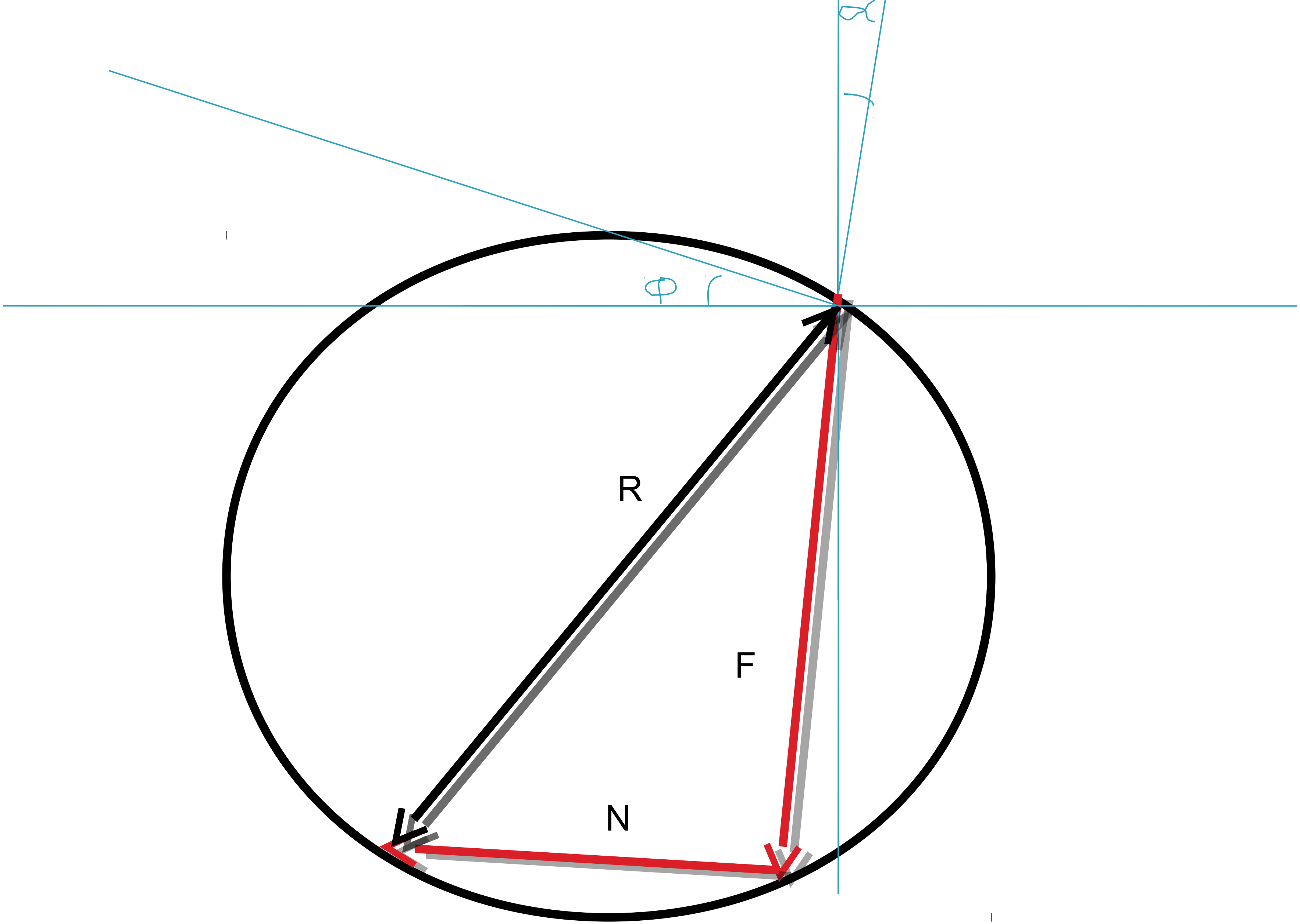
Friction angle related to coefficient of friction as follows:

$$\mu = \tan \beta$$

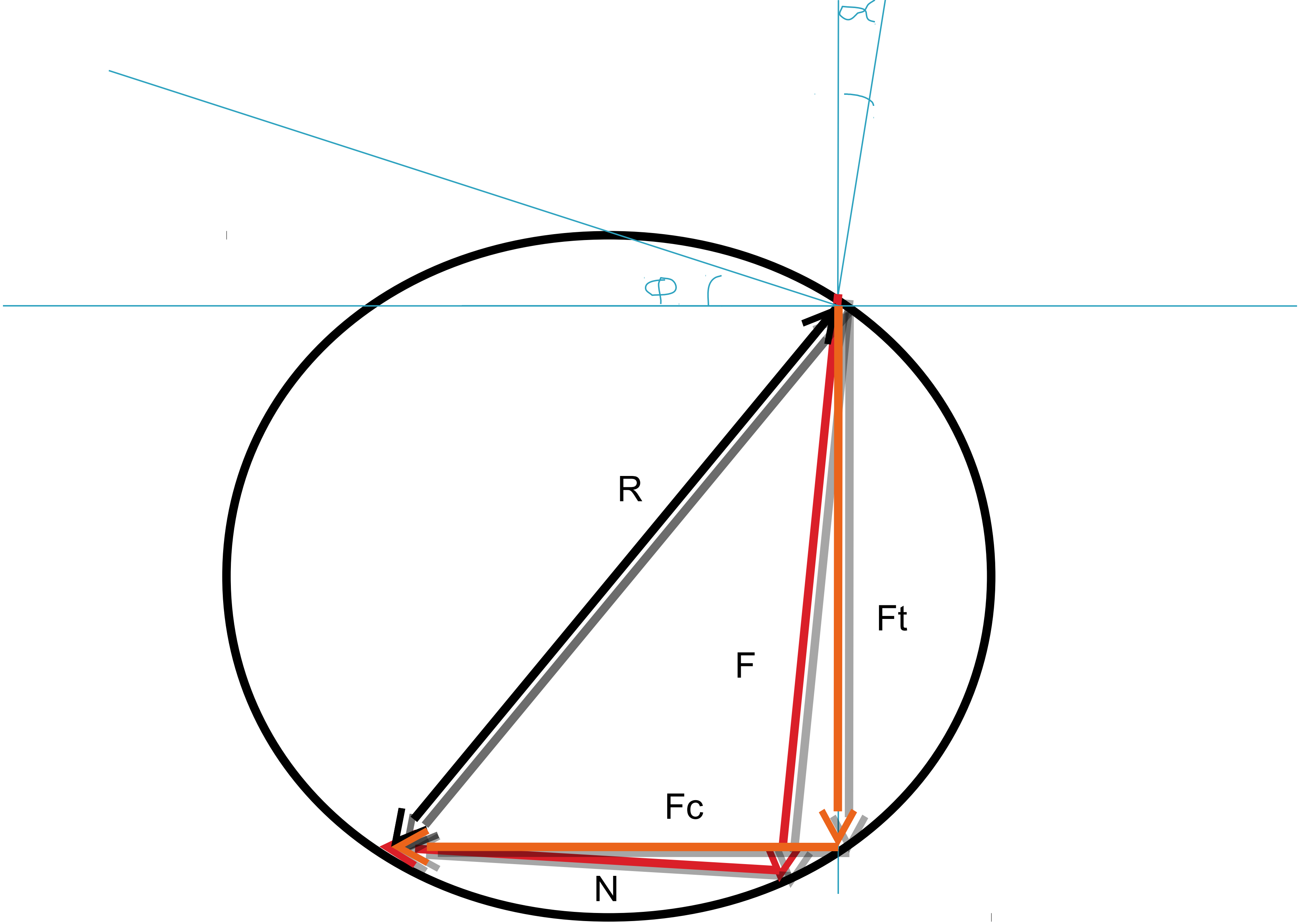


(a)

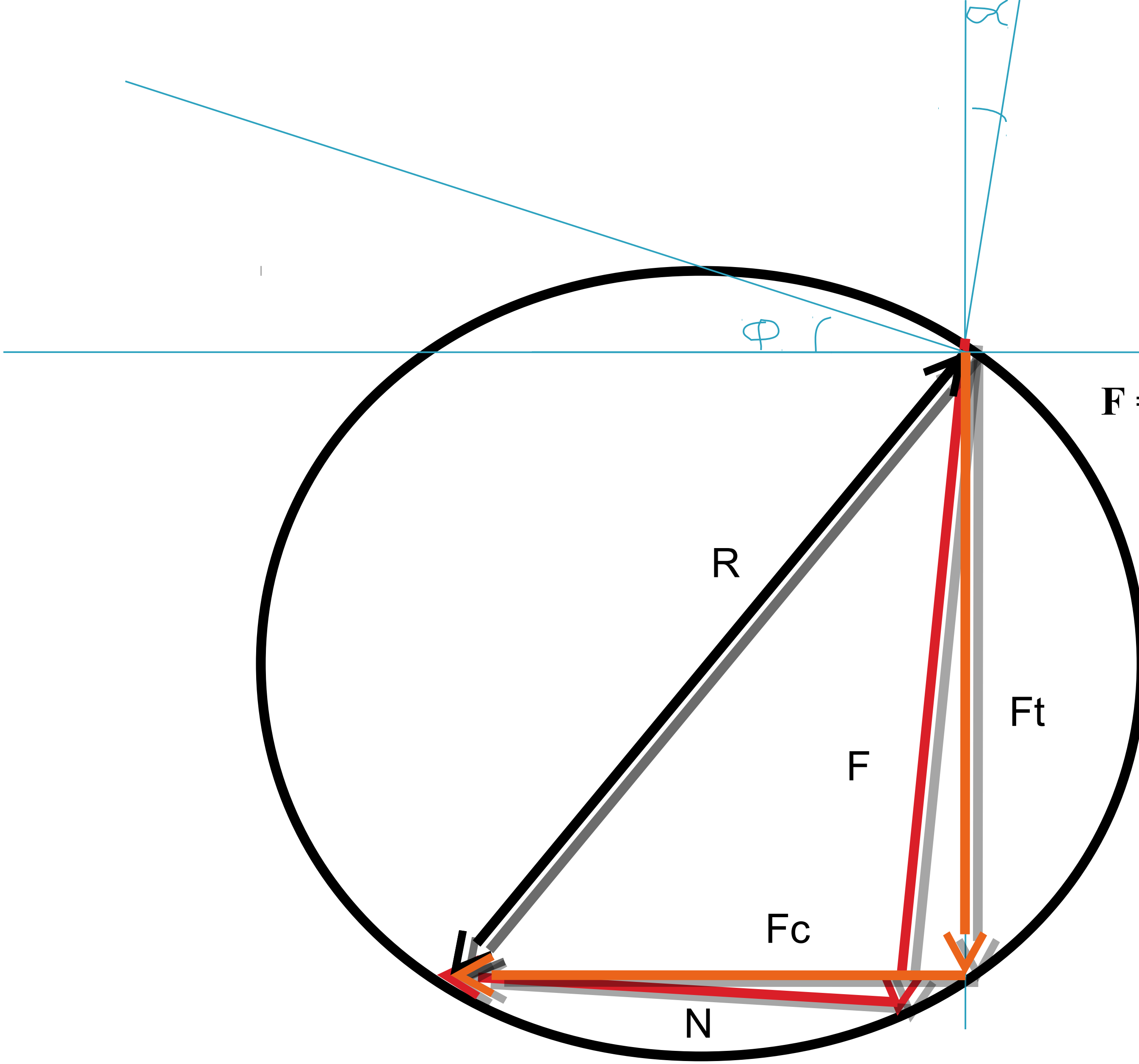

Fs
φ
R
Fn
F
β
N



φ
R
F
N

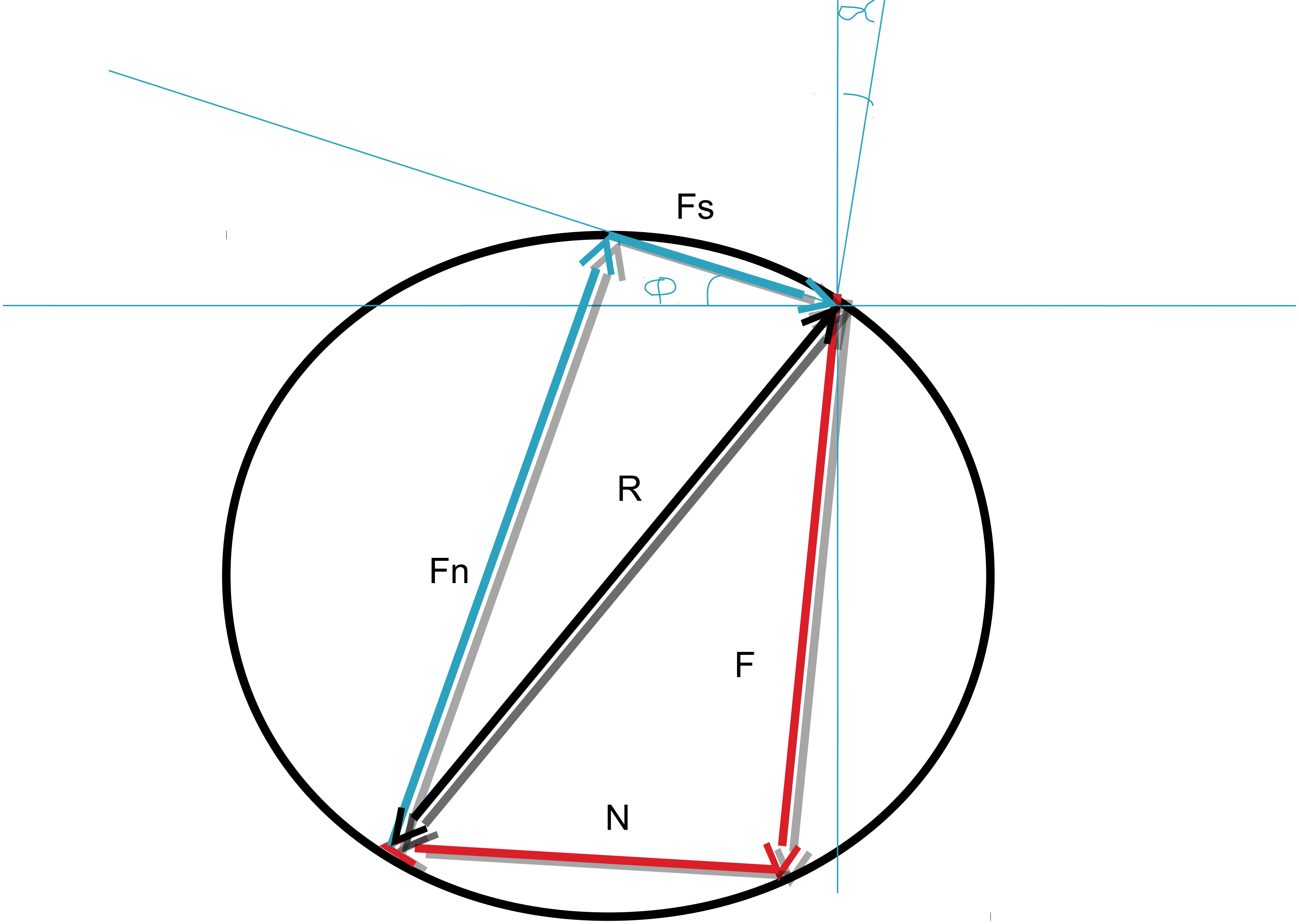


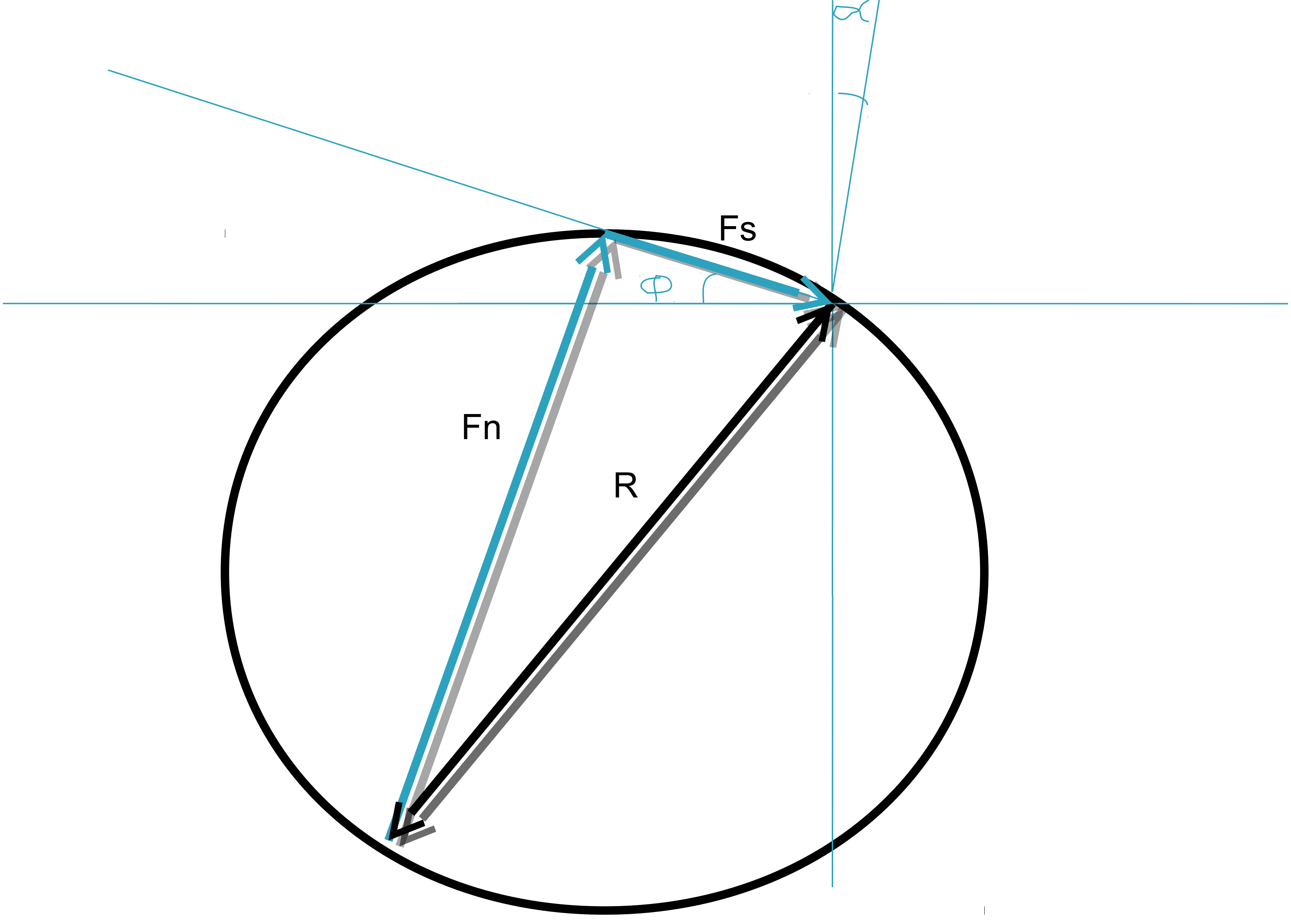
R
Ft
F
Fc
N

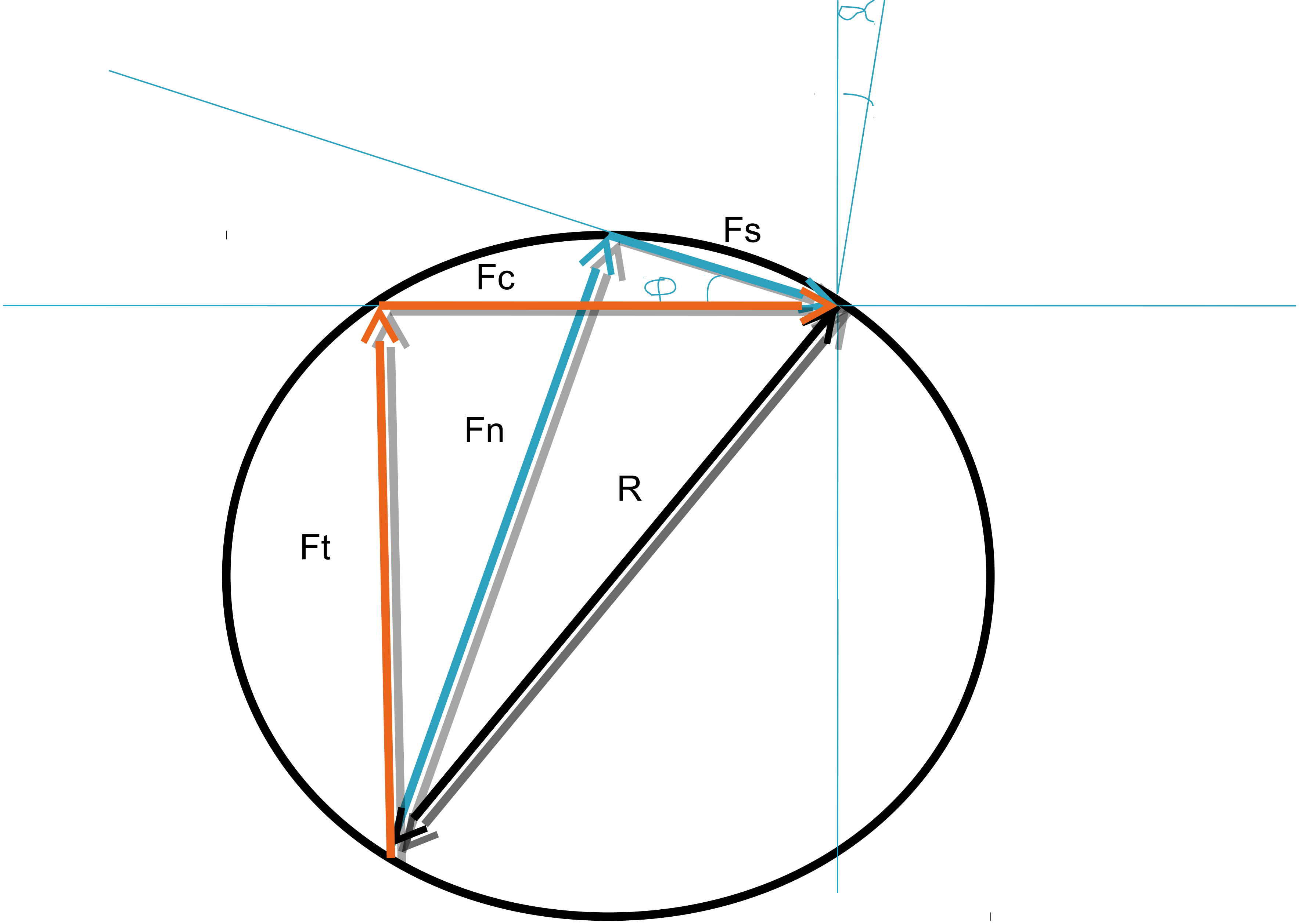


$$F = F_c \sin \alpha + F_t \cos \alpha$$

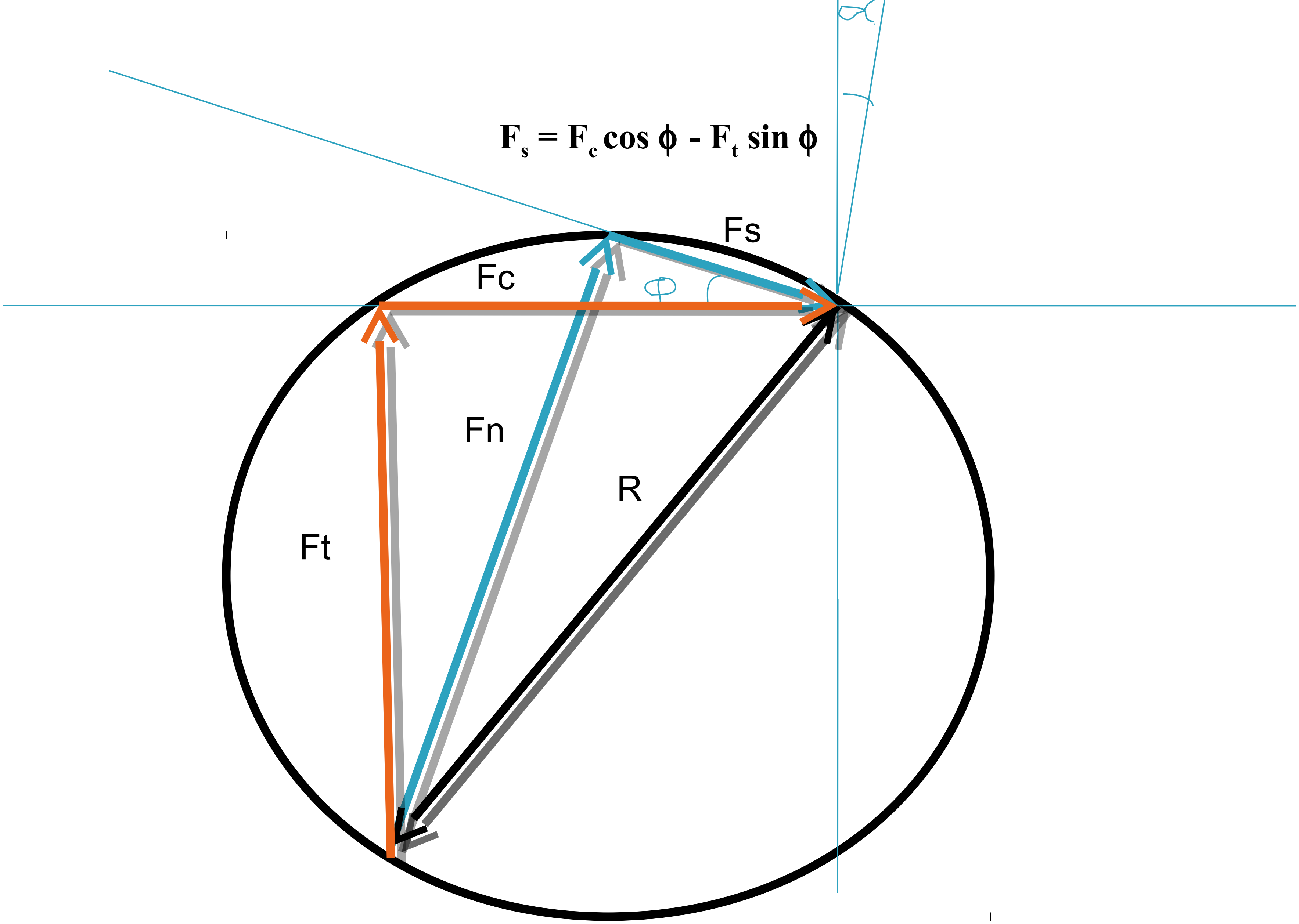
$$N = F_c \cos \alpha - F_t \sin \alpha$$


Fs
φ
R
Fn
F
N


Fs
Fn
R


Fs
Fc
φ
Fn
R
Ft

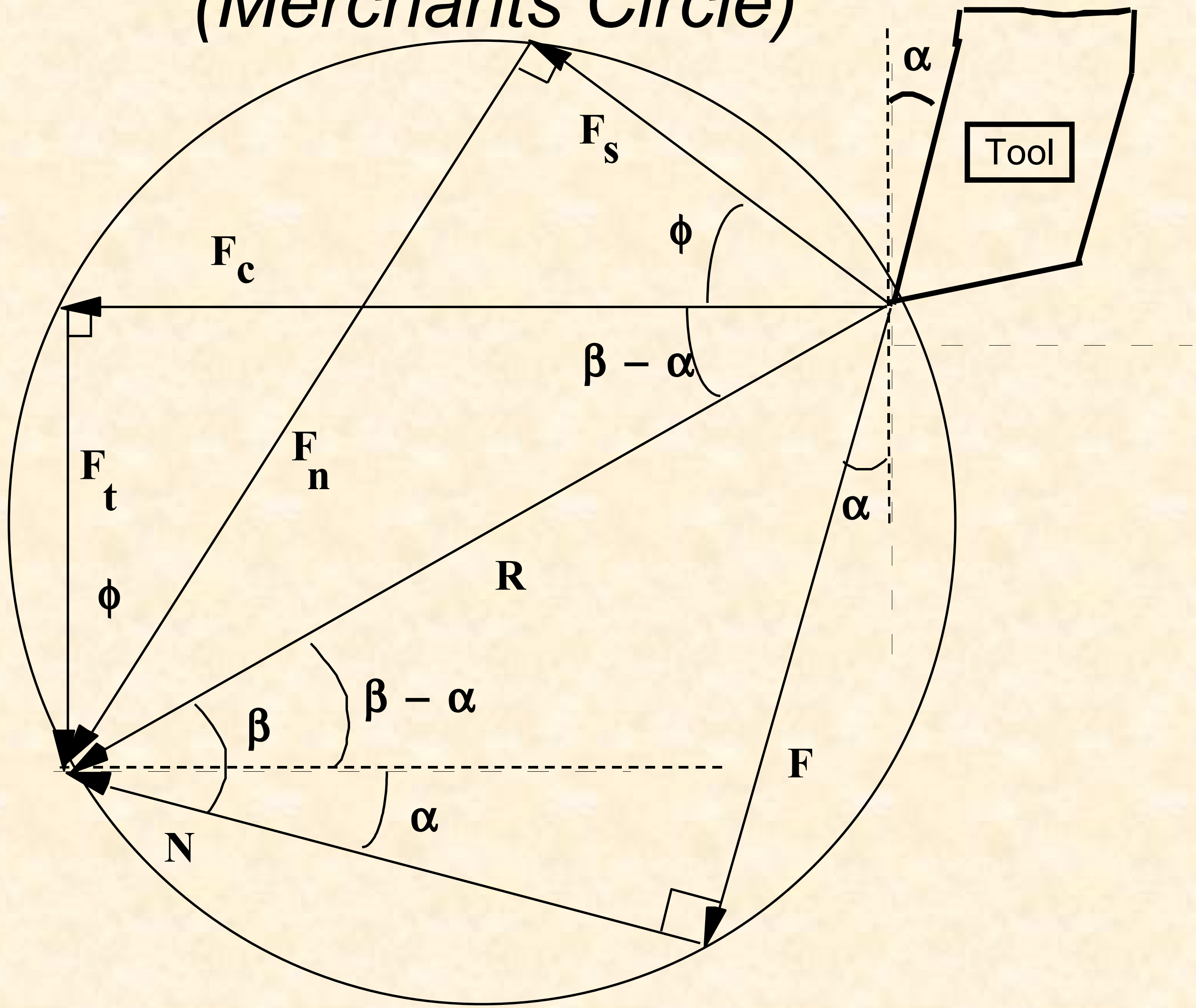
$$F_s = F_c \cos\phi - F_t \sin\phi$$



$$F_n = F_c \sin\phi + F_t \cos\phi$$

Force Circle Diagram

(Merchants Circle)



Results from Force Circle Diagram

(Merchant's Circle)

Friction Force $F = F_c \sin\alpha + F_t \cos\alpha$

Normal Force $N = F_c \cos\alpha - F_t \sin\alpha$

$\mu = F/N$ **and** $\mu = \tan\beta$ (**typically** 0.5 - 2.0)

Shear Force $F_s = F_c \cos\phi - F_t \sin\phi$

Force Normal to Shear plane $F_n = F_c \sin\phi + F_t \cos\phi$

$$R = \sqrt{F_c^2 + F_t^2} = \sqrt{F_s^2 + F_n^2} = \sqrt{F^2 + N^2}$$

Forces on the Cutting Tool and the workpiece

- Importance: Stiffness of tool holder, stiffness of machine, and stiffness of workpiece must be sufficient to avoid significant deflections (dimensional accuracy and surface finish)
- Primary cause: Friction force of chip up rake face + Shearing force along shear plane
- Cutting speed does not effect tool forces much (friction forces decrease slightly as velocity increases; static friction is the greatest)
- The greater the depth of cut the greater the forces on the tool
- Using a coolant reduces the forces slightly but greatly increases tool life

Stresses

On the Shear plane:

$$\text{Normal Stress} = \sigma_s = \text{Normal Force / Area} = \frac{F_n}{AB\,w} = \frac{F_n \sin\phi}{t_o w}$$

$$\text{Shear Stress} = \tau_s = \text{Shear Force / Area} = \frac{F_s}{AB\,w} = \frac{F_s \sin\phi}{t_o w}$$

Note: $\tau_s = \tau_y$ **= yield strength of the material in shear**

On the tool rake face:

$$\sigma = \text{Normal Force / Area} = \frac{N}{t_c\,w}$$ **(often assume** t_c **= contact length)**

$$\tau = \text{Shear Force / Area} = \frac{F}{t_c\,w}$$

Cutting forces given shear strength

Letting S = shear strength, we can derive the following equations for the cutting and thrust forces*:

$$F_s = S\,A_s$$

$$F_c = F_s \cos(\beta - \alpha)/[\cos(\phi + \beta - \alpha)]$$

$$F_t = F_s \sin(\beta - \alpha)/[\cos(\phi + \beta - \alpha)]$$

*** The other forces can be determined from the equations on the previous slide.**

Pow
er

- Power (or energy consumed per unit time) is the product of force and velocity. Power at the cutting spindle:

$$\textbf{Cutting Power} \quad P_c = F_c V$$

- Power is dissipated mainly in the shear zone and on the rake face:

$$\textbf{Power for Shearing} \quad P_s = F_s V_s$$

$$\textbf{Friction Power} \quad P_f = F V_c$$

- Actual Motor Power requirements will depend on machine efficiency E (%):

$$\textbf{Motor Power Required} = \frac{P_c}{E} \times 100$$

Material Removal Rate (MRR)

$$\text{Material Removal Rate (MRR)} = \frac{\text{Volume Removed}}{\text{Time}}$$

$$\text{Volume Removed} = Lwt_o$$

$$\text{Time to move a distance L} = L/V$$

$$\text{Therefore, MRR} = \frac{Lwt_o}{L/V} = Vwt_o$$

MRR = Cutting velocity x width of cut x depth of cut

Specific Cutting Energy
(or Unit Power)

Energy required to remove a unit volume of material (often quoted as a function of workpiece material, tool and process:

$$U_t = \frac{\text{Energy}}{\text{Volume Removed}} = \frac{\text{Energy per unit time}}{\text{Volume Removed per unit time}}$$

$$U_t = \frac{\text{Cutting Power } (P_c)}{\text{Material Removal Rate (MRR)}} = \frac{F_c V}{V w t_o} = \frac{F_c}{w t_o}$$

Specific Energy for shearing $U_s = \frac{F_s V_s}{V w t_o}$

Specific Energy for friction $U_f = \frac{F V_c}{V w t_o} = \frac{F r}{w t_o}$

Specific Cutting Energy

Decomposition

1. Shear Energy/unit volume (Us)
   (required for deformation in shear zone)

2. Friction Energy/unit volume (Uf)
   (expended as chip slides along rake face)

3. Chip curl energy/unit volume (Uc)
   (expended in curling the chip)

4. Kinetic Energy/unit volume (Um)
   (required to accelerate chip)

$$\mathbf{U_t = U_s + U_f + U_c + U_m}$$

Specific Cutting Energy

Relationship to Shear strength of Material

SHEAR ENERGY / UNIT VOLUME

Specific Energy for shearing $U_s = \frac{F_s V_s}{V w t_o}$

$$U_s = \frac{\tau_s \cos\alpha}{\sin\phi \cos(\phi - \alpha)} = \tau_s . \gamma$$

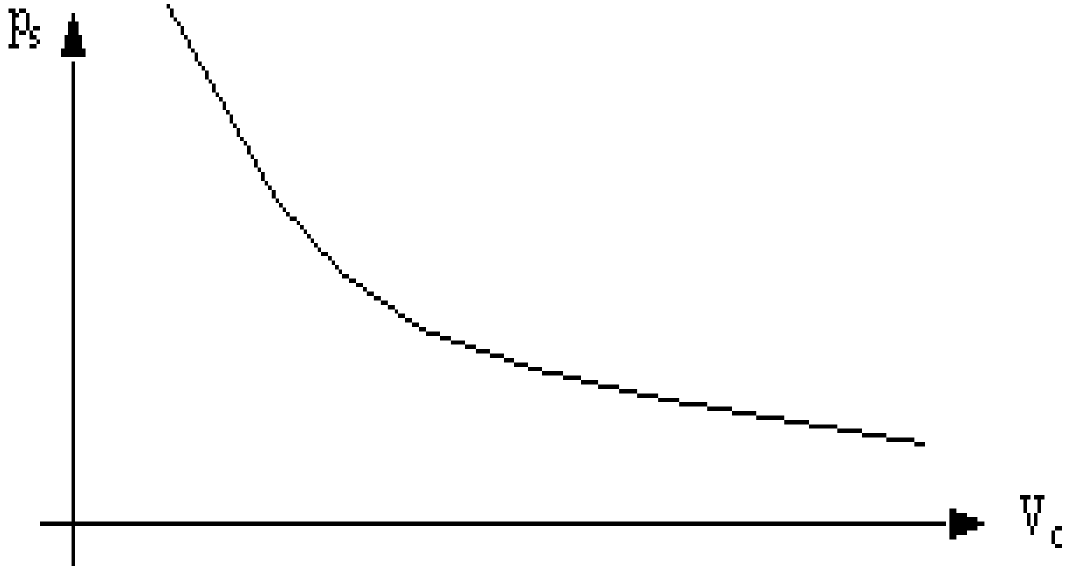
FRICTION ENERGY / UNIT VOLUME

Specific Energy for friction $U_f = \frac{F V_c}{V w t_o} = \frac{F r}{w t_o} = \frac{F}{w t_c} = \tau$

APPROXIMATE TOTAL SPECIFIC CUTTING ENERGY

$$U_t = U_s + U_f = \tau_s \gamma + \tau = \tau_y (1 + \gamma)$$

Relation between Pressure and Cutting velocity



This curve turns downward for two reasons,

1. The tool experiences edge forces that are more significant at lower cutting speeds.
2. As the velocity increases, the temperature increases, and less energy is required to shear the metal.

Effect of Rake angle on Cutting Force