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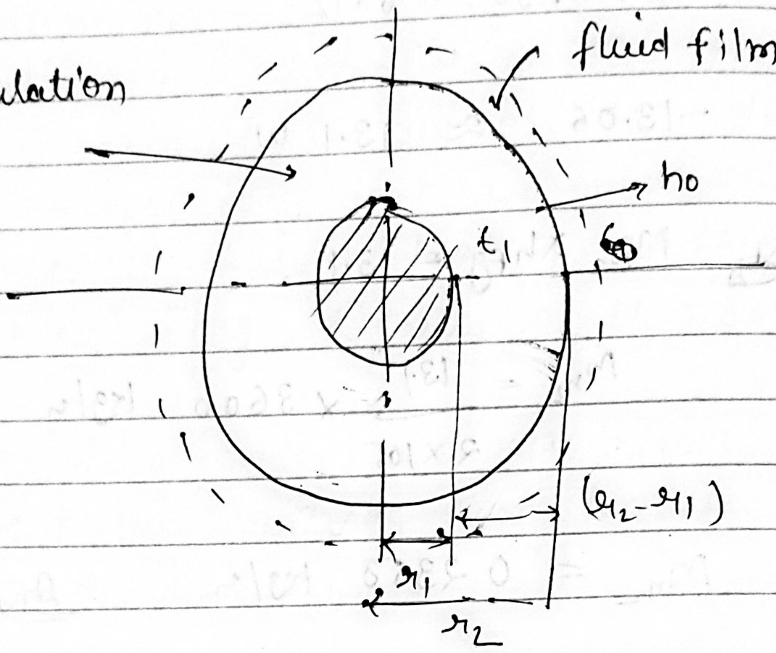
* Critical thickness of insulation for cylinder:



The addition of insulation always increases the thermal conductive resistance but when the total thermal resistance is made of conductive thermal resistance & convective thermal resistance, the addition of insulation in some cases may reduce the convective thermal resistance due to increase in surface area as in the case of cylinder & sphere & the total thermal resistance may actually decrease resulting in increased heat flow.

Critical radius :- The thickness upto which heat flow increases & after which heat flow decreases is called critical radius.

Derivation :→

Consider a solid cyliender of readius r_1 insulated with any insulation of thickness $(r_2 - r_1)$.

Let L = Length of Cyliender
t_1 = Surface temp. of Cyliender
t_{air} = temp. of air
h_o = heat transfer coefficient at the outer surface of the insulation.
K = thermal conductivity of insulating material

Than the rate of heat transfer from the surface of solid cyliender to the surrounding is given by

$$Q = \frac{2\pi L (t_1 - t_{air})}{\frac{\ln(r_2/r_1)}{K} + \frac{1}{h_o r_2}} \quad \text{---} (1)$$

from eqn (1) it is evident that as the r_2 increases the factor $\frac{\ln(r_2/r_1)}{k}$ increases but the factor $\left(\frac{1}{h_o r_2}\right)$ decreases. Thus Q becomes maximum when the denominator $\left[\frac{\ln r_2/r_1}{k_1} + \frac{1}{h_o r_2}\right]$ will becomes minimum. The required condition is,

$$\Rightarrow \frac{d}{dr_2}\left[\frac{\ln(r_2/r_1)}{K} + \frac{1}{h_o r_2}\right]$$

$$\Rightarrow \frac{1}{K} - \frac{1}{h_0 r_2} = 0$$

$$\Rightarrow h_0 r_2 = K$$

$$\Rightarrow r_2 = \frac{K}{h_0}$$

$$\boxed{r_2 = r_c = K/h_0}$$

$r_2 = r_c$ = critical radius at which the heat transfer is maximum.

The above relation represents the conditions for minimum resistance & consequently maximum heat flow rate.

The term $\frac{\ln(r_2/r_1)}{K}$ is the conduction thermal resistance which increases with increasing r_2 & $\frac{1}{h_0 r_2}$ is conductive thermal resistance which decreases with increasing r_2.

Following conclusions may be derived :-

1. For cylindrical bodies with $r_1 < r_c$ than the heat transfer increases by adding insulation thickness is further increased the rate of heat loss will decreases from its peak value. But until a certain

amount of insulation denoted by r_2 at P added this happens when r_1 is small & r_c is large i.e. the thermal conductivity of insulation a is high & h_0 is low.

Example: A practicle application would be the insulation of electric cables which should be a good insulation for current but poor for heat.

2. For cylindrical bodies $r_1 > r_c$ than the heat transfer decreases by adding insulation. This happens when r_1 is large & r_c is small i.e. a good insulating material is used with low k and h_0 is high.

Application :- In steam and refrigeration pipe heat insulation is main objectives for insulation to be properly effective in restricting heat transmission the outer radius must be greater than or equal to critical radius.