But, $$C_{fx} = \frac{0.654}{Re_x} = \frac{0.654}{\sqrt{\frac{Ux}{\nu}}} = \frac{0.654}{\sqrt{\frac{4 \times 0.25}{0.15 \times 10^{-4}}}} = 0.002533$$

∴ $$(\tau_0)_{x = 0.25\text{m}} = 0.002533 \times \frac{1.24 \times 4^2}{2} = \mathbf{0.025\ N/m^2\ (Ans.)}$$

(*iii*) Drag force on one side of the plate, F_D :

$$F_D = \bar{C}_f \times \frac{1}{2}\rho A U^2$$

where, $$\bar{C}_f = \frac{1.31}{\sqrt{Re_L}} = \frac{1.31}{\sqrt{1.33 \times 10^5}} = 0.003592$$

and A = area of the plate = $L \times B = 0.5 \times 0.6 = 0.3\ m^2$

∴ $$F_D = 0.003592 \times \frac{1}{2} \times 1.24 \times 0.3 \times 4^2 = \mathbf{0.01069\ N\ (Ans.)}$$

7.1.5. THERMAL BOUNDARY LAYER

Whenever a flow of fluid takes place past a heated or cold surface, a temperature field is set up in the field next to the surface. If the surface of the plate is hotter than fluid, the temperature distribution will be as shown in the Fig. 7.6. *The zone or this layer wherein the temperature field exists is called the* **thermal boundary layer**. Due to the exchange of heat between the plate and the fluid, temperature gradient occurs/results.

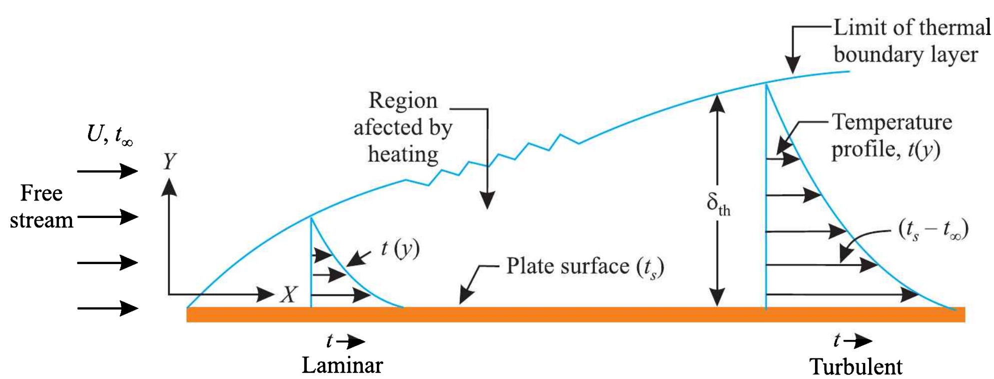


Fig. 7.6. Thermal boundary layer formed during flow of cool fluid over a warm plate.

The thermal boundary layer thickness (δ_{th}) is arbitrarily defined as the distance y from the plate surface at which

$$\frac{t_s - t}{t_s - t_\infty} = 0.99 \qquad \text{...(7.40)}$$

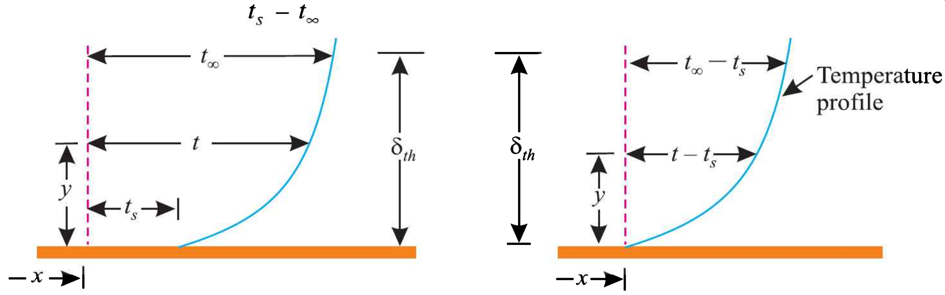
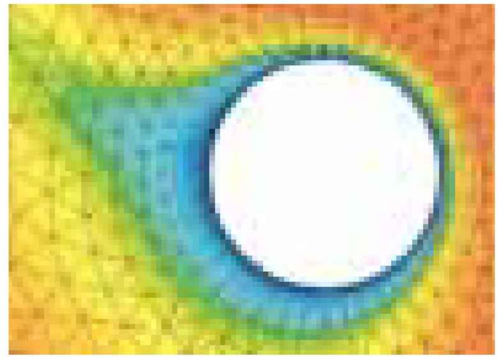


Fig. 7.7. Thermal boundary layer formed flow of warm fluid over a cool plate.

Figure 7.7. shows the shape of the thermal boundary layer when the free stream temperature t_∞ is above the plate surface temperature t_s.

The thermal boundary layer concept is analogous to hydrodynamic boundary layer; the parameters which affect their growth are however different. Whereas the velocity profile of the hydrodynamic boundary layer depends mainly on the fluid viscosity, the temperature profile of the thermal boundary layer depends upon the viscosity, velocity of flow, specific heat and thermal conductivity of the fluid. The relative magnitude of δ and δ_{th} are affected by the thermo-physical properties of the fluid; the governing parameter, however, is the non-dimensional Prandtel number, $Pr = \frac{\mu c_p}{k}$.



Thermal boundary layer at the coil surface.

(*i*) $\delta_{th} = \delta$ .... when $Pr = 1$; (*ii*) $\delta_{th} < \delta$ ....when $Pr > 1$

(*iii*) $\delta_{th} > \delta$ .... when $Pr < 1$.

7.1.6. ENERGY EQUATION OF THERMAL BOUNDARY LAYER OVER A FLAT PLATE

Figure 7.8 (*a*) shows a hot fluid flowing over a cool flat plate, and development of the thermal boundary layer. In order to derive an energy equation, consider control volume ($dx \times dy \times$ unit depth) in the boundary layer so that end effects are neglected. The enlarged view of this control volume is shown in Fig. 7.8 (*b*) in which the quantities of energy entering and leaving have been indicated.

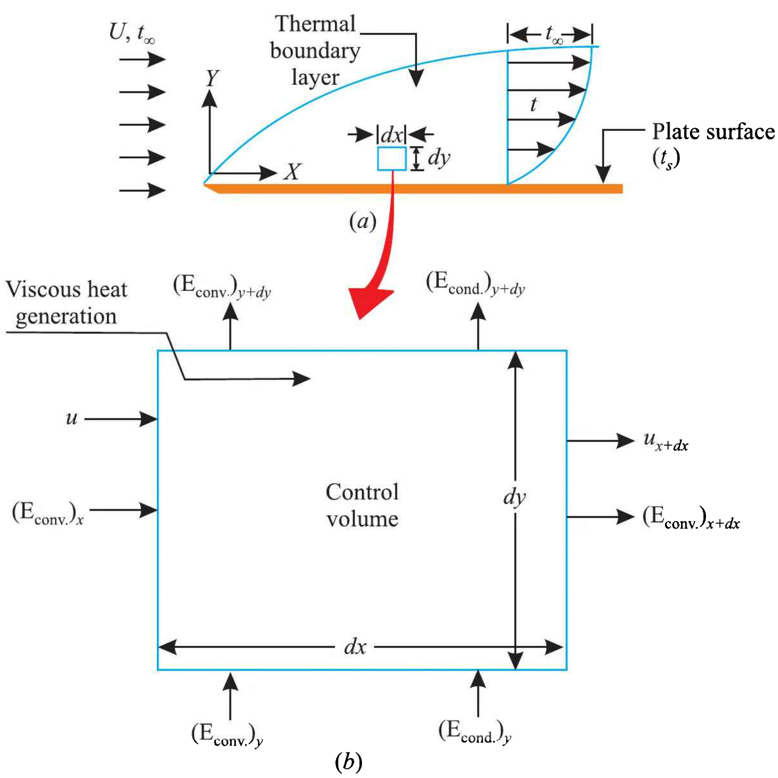


Fig. 7.8. Energies entering and leaving the control volume.

Involving *principle of conservation of energy for the steady state condition*, we have :

Heat energy convected ($E_{conv.}$) through the control volume in X and Y directions + heat energy conducted ($E_{cond.}$) through the control volume in X and Y directions + heat generated due to fluid friction (viscous heat generation) in the control volume = 0.

But as the rate of temperature change in the X-direction is small and can be neglected the conservation of energy becomes :

Heat energy convected in X and Y directions + heat energy conducted in Y-direction + viscous *heat generation* = 0.

or $$d(E_{conv.})_x + d(E_{conv.})_y + d(E_{cond.})_y + \text{viscous heat generation} = 0 \quad ...(7.41)$$

(*i*) (*ii*) (*iii*) (*iv*)

(*i*) **The Energy convected in X-direction :**

$$(E_{conv.})_x = \text{Mass} \times \text{specific heat} \times \text{temperature}$$
$$= [\rho u(dy \times 1)]\, c_p t = (\rho u\, dy)\, c_p t$$

$$(E_{conv.})_{x+dx} = \left[\rho\left(u + \frac{\partial u}{\partial x}dx\right)dy\right] c_p \left(t + \frac{\partial t}{\partial x}dx\right)$$
$$= \rho c_p\, dy\left[ut + u + \frac{\partial u}{\partial x}dx + t\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial x}\frac{\partial t}{\partial x}(dx)^2\right]$$
$$= \rho c_p\, dy\left[ut + u\frac{\partial t}{\partial x}dx + t\frac{\partial u}{\partial x}dx\right]$$

... neglecting the product of small quantities

∴ Net energy convected in X-direction,

$$d(E_{conv.})_x = (E_{conv.})_x - (E_{conv.})_{x+dx}$$
$$= (\rho u\, dy)\, c_p t - \left[\rho c_p dy\left\{ut + u\frac{\partial t}{\partial x}dx + t\frac{\partial u}{\partial x}dx\right\}\right]$$

or, $$d(E_{conv.})_x = -\rho c_p\left[u\frac{\partial t}{\partial x} + t\frac{\partial u}{\partial x}\right]dx\,dy \quad ...(7.42)$$

(*ii*) **The energy convected in Y-direction :**

The net energy convected in Y-direction,

$$d(E_{conv.})_y = (E_{conv.})_y - (E_{conv.})_{y+dy}$$
$$= (\rho\upsilon\, dx)\, c_p t - \left[\rho\left(\upsilon + \frac{\partial\upsilon}{\partial y}dy\right)dx\right] c_p\left(t + \frac{\partial t}{\partial y}dy\right)$$

or, $$d(E_{conv.})_y = -\rho c_p\left[\upsilon\frac{\partial t}{\partial y} + t\frac{\partial\upsilon}{\partial y}\right]dx\,dy \quad ...(7.43)$$

... neglecting the product of small quantities.

(*iii*) **The heat conduction in the Y-direction :**

$$d(E_{cond.})_y = (E_{cond.})_y - (E_{cond.})_{y+dy}$$
$$= -k\,(dx \times 1)\frac{\partial t}{\partial y} - \left[-k\,(dx \times 1)\left\{\frac{\partial t}{\partial y} + \frac{\partial}{\partial y}\left(\frac{\partial t}{\partial y}\right)\right\}dy\right]$$

or, $$d(E_{cond.})_y = k\frac{\partial^2 t}{\partial y^2}dx\,dy \quad ...(7.44)$$

***(iv)* Viscous heat generation :**

Owing to relative motion of fluid in the boundary layer (fluid on the top face of the control volume moves faster than fluid on bottom face), there will be viscous effects which will cause generation of heat.

Viscous heat generation = Viscous force (average) × distance travelled by the viscous force (this is determined by the relative velocity of fluid flow at the upper and lower faces of the element).

= [Shear stress (τ) × area upon which it acts)] × distance travelled

$$= \left[\mu \frac{\partial u}{\partial y} (dx \times 1) \right] \times \left(\frac{\partial u}{\partial y} dy \right)$$

or, Viscous heat generation $= \mu \left(\frac{\partial u}{\partial y} \right)^2 dx\, dy$ ...(7.45)

Substituting the values in eqn. (7.41), we get

$$-\rho c_p \left[\mu \frac{\partial t}{\partial x} + t \frac{\partial u}{\partial x} \right] dx dy + \left[-\rho c_p \left(\upsilon \frac{\partial t}{\partial y} + t \frac{\partial \upsilon}{\partial y} \right) \right] dx dy + \left[k \frac{\partial^2 t}{\partial y^2} dx dy \right] + \left[\mu \left(\frac{\partial u}{\partial y} \right)^2 dx dy \right] = 0$$

$$-\rho c_p \left[u \frac{\partial t}{\partial x} + \upsilon \frac{\partial t}{\partial y} + t \left(\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} \right) \right] dx dy + \left[k \frac{\partial^2 t}{\partial y^2} dx\, dy \right] + \left[\mu \left(\frac{\partial u}{\partial y} \right)^2 dx dy \right] = 0 \quad ...(7.46)$$

Form the continuity equation for two-dimensional flow, we have

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$; thus the eqn. (7.46) reduces to

$$u \frac{\partial t}{\partial x} + \upsilon \frac{\partial t}{\partial y} = \frac{k}{\rho c_p} \cdot \frac{\partial^2 t}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 = 0 \quad ...(7.47)$$

Equation (7.47) is the required *differential energy equation for flow past a flat plate*. If viscous heat generation is neglected [when the value of U is relatively low and difference of temperature between the free stream and the plate is small (of the order of 40°C)], the energy equation reduces to

$$u \frac{\partial t}{\partial x} + \upsilon \frac{\partial t}{\partial y} = \frac{k}{\rho c_p} \cdot \frac{\partial^2 t}{\partial y^2} = \alpha \frac{\partial^2 t}{\partial y^2} = 0 \qquad \left(\text{where } \alpha = \frac{k}{\rho c_p} \right) \quad ...(7.48)$$

It may be noted, the energy equation is similar to the momentum equation. Further the dimensions of kinematic viscosity v and thermal diffusivity α are the same.

The equation (7.48) has been derived with the following **assumptions :**

1. Steady incompressible flow.
2. The properties of the fluids evaluated at the film temperature $t_f = \frac{t_\infty - t_s}{2}$ are constant.
3. The body forces, viscous heating and conduction in the flow direction are negligible.

Pohlhausen solution for the 'Energy equation' :

By using the following variables the energy equation $\left[u \frac{\partial t}{\partial x} + \upsilon \frac{\partial t}{\partial x} = \alpha \frac{\partial^2 t}{\partial y^2} \right]$ can be recast into an *ordinary differential equation* as follows :

η (Stretching factor) $= y \sqrt{\frac{U}{vx}}$, ψ (stream function) $= \sqrt{vxU}\, f(\eta)$, and

Viscous heating of fluid dampers.

$$\theta = \frac{t_s - t}{t_s - t_\infty} = f(\eta) = f\left[y\sqrt{\frac{U}{\nu x}} \right] \quad ...(7.49)$$

Also, the values of the velocity components u and υ already calculated earlier are :

$$u = U\frac{df}{d\eta} \quad ...[\text{Eqn. } (7.15)]$$

$$\upsilon = \left[\frac{y}{2x}U\frac{df}{d\eta} - \frac{1}{2}\sqrt{\frac{U\nu}{x}}f(\eta) \right] \quad ...[\text{Eqn. } (7.19)]$$

Further, from temperature parameter θ (non-dimensional) defined above, we have

$$t = t_s + (t_\infty - t_s)\theta$$

$$\frac{\partial t}{\partial x} = (t_\infty - t_s)\frac{\partial \theta}{\partial x} = (t_\infty - t_s)\frac{\partial \theta}{\partial \eta} \times \frac{\partial \eta}{\partial x}$$

or,
$$\frac{\partial t}{\partial x} = (t_\infty - t_s)\left[-\frac{y}{2x^{3/2}}\sqrt{\frac{U}{\nu}} \right]\frac{\partial \theta}{\partial \eta} \quad ...(7.50)$$

and,
$$\frac{\partial t}{\partial y} = (t_\infty - t_s)\frac{\partial \theta}{\partial y} = (t_\infty - t_s)\frac{\partial \theta}{\partial \eta} \times \frac{\partial \eta}{\partial y}$$

or,
$$\frac{\partial t}{\partial y} = (t_\infty - t_s)\sqrt{\frac{U}{\nu x}}\frac{d\theta}{d\eta} \quad ...(7.51)$$

Also,
$$\frac{\partial^2 t}{\partial y^2} = \frac{\partial}{\partial y}\left[(t_\infty - t_s)\sqrt{\frac{U}{\nu x}}\frac{d\theta}{d\eta} \right]$$

$$= (t_\infty - t_s)\sqrt{\frac{U}{\nu x}}\frac{d}{d\eta}\left(\frac{d\theta}{d\eta} \right)\frac{d\eta}{dy}$$

$$= (t_\infty - t_s)\sqrt{\frac{U}{\nu x}}\frac{d^2\theta}{d\eta^2}\sqrt{\frac{U}{\nu x}}$$

or, $$\frac{\partial^2 t}{\partial y^2} = (t_\infty - t_s)\ \frac{U\, d^2\theta}{v\, x\, d\eta^2} \quad ...(7.52)$$

Inserting the above values in the energy equation, we get :

$$u\frac{\partial t}{\partial x} + \upsilon\frac{\partial t}{\partial y} = \alpha\frac{\partial^2 t}{\partial y^2}$$

$$U\frac{df}{d\eta}(t_\infty - t_s)\left[-\frac{y}{2x^{3/2}}\sqrt{\frac{U}{v}}\right]\frac{d\theta}{d\eta} + \left[\frac{y}{2x}U\frac{df}{d\eta} - \frac{1}{2}\sqrt{\frac{Uv}{x}}f(\eta)\right](t_\infty - t_s)\sqrt{\frac{U}{vx}}\frac{d\theta}{d\eta}$$

$$= \alpha(t_\infty - t_s)\frac{U}{vx}\frac{d^2\theta}{d\eta^2} \quad ...(7.53)$$

After simplification and arrangement of the above equation, we obtain

$$\frac{d^2\theta}{d\eta^2} + \frac{1}{2}\frac{v}{\alpha}f(\eta)\frac{d\theta}{d\eta} = 0$$

or, $$\frac{d^2\theta}{d\eta^2} + \frac{1}{2}Pr\, f(\eta)\frac{d\theta}{d\eta} = 0 \quad ...(7.54)$$

$$\left[\because\ pr\ (\text{Prandtl number}) = \frac{\nu}{\alpha}\right]$$

Thus the partial differential equation (7.48) has been converted into ordinary differential equation. The boundary conditions to be satisfied are:

At $t = t_s$, $y = 0$

At $t = t_\infty$, $y = \infty$

At $\eta = 0$ $\theta(\eta) = 0$
At $\eta = \infty$ $\theta(\eta) = 1$ } values in terms of new variable ...(7.55)

The solution obtained by Pohlhausen for energy equation is given by :

$$\theta(\eta) = \left(\frac{d\theta}{d\eta}\right)_{\eta=0}\int_0^\eta \exp\left[-\frac{Pr}{2}\int_0^\eta f(\eta)\, d\eta\right]d\eta \quad ...(7.56)$$

The factor $\left(\frac{d\theta}{d\eta}\right)_{\eta=0}$ represents the dimensionless slope of the temperature profile at the surface where $\eta = 0$; its value can be obtained by applying the boundary condition at $\eta = \infty$, $\theta(\eta) = 1$. Thus,

$$\left(\frac{d\theta}{d\eta}\right)_{\eta=0}\int_0^\infty \exp\left[-\frac{Pr}{2}\int_0^\infty f(\eta)\, d\eta\right]d\eta \quad ...(7.57)$$

Evidently the dimensionless slope is a function of Prandtl number and the calculations made by Prandtl gave the following result :

For $0.6 < Pr$ 15, $\left(\frac{d\theta}{d\eta}\right)_{\eta=0} = 0.332\,(Pr)^{1/3}$ ...(7.58)

Figure 7.9 shows the values of θ (dimensionless temperature distribution) plotted for various values of Pr (Prandtl number).

– The curve for $Pr = 0.7$ is typical for air and several other gases.
– The curve for $Pr = 1$ is the same as that of curve *I* in Fig. 7.4.
– These curves also enable us to determine the thickness of thermal boundary layer δ_{th} and

local average heat transfer coefficients $\bar{h}$.

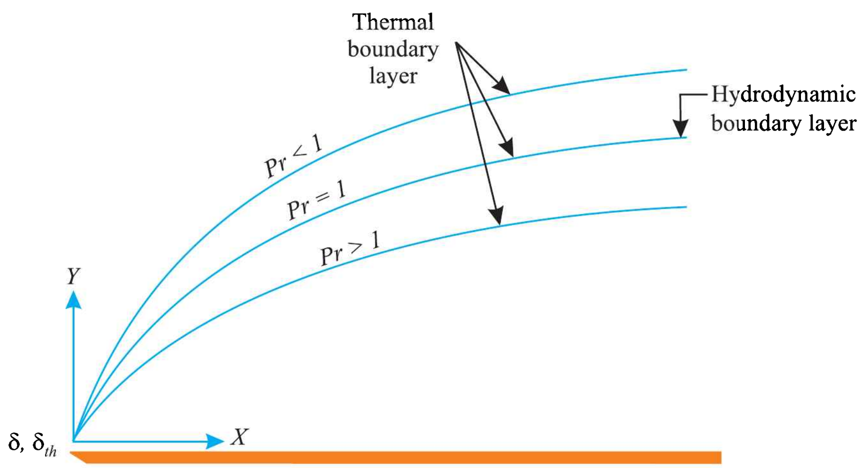


Fig. 7.9. Hydrodynamic and thermal boundary layers for different Prandtl numbers.

Thickness of thermal boundary layer, δ_{th} :

Case I. *When Pr = 1.*

$$\eta = \left[y\sqrt{\frac{U}{vx}} \right] \simeq 5.0 \text{ at } \theta = 0.99$$

Since $y = \delta_{th}$ at the outer edge of thermal boundary layer, therefore,

$$\delta_{th}\sqrt{\frac{U}{vx}} = 5.0$$

or,
$$\frac{\delta_{th}}{x} = \frac{5.0}{\sqrt{(xU)/v}} = \frac{5.0}{\sqrt{Re_x}} = \frac{\delta}{x} \quad \text{...(7.59)}$$

This equation shows that for $Pr = 1$, the thickness of thermal boundary layer, δ_{th} is *equal to* hydrodynamic boundary layer, δ.

Case II. *When Pr < 1.*

$$\eta\left[= y\sqrt{\frac{U}{vx}} \right] > 5.0 \text{ at } \theta = 0.99$$

$$\frac{\delta_{th}}{x} > \frac{5.0}{\sqrt{Re_x}} > \frac{\delta}{x} \quad \text{...(7.60)}$$

This equation shows that for $Pr < 1$, $\delta_{th} > \delta$.

Case III. *When Pr >* 1.

$$\eta\left[= y\sqrt{\frac{U}{vx}} \right] > 5.0 \text{ at } \theta = 0.99$$

$$\frac{\delta_{th}}{x} < \frac{5.0}{\sqrt{Re_x}} < \frac{\delta}{x} \quad \text{...(7.61)}$$

This equation shows that for $Pr > 1$, $\delta_{th} < \delta$

Pohlhausen has suggested that the following relation is general may be assumed between the thermal and hydrodynamic boundary layers :

$$\delta_{th} = \frac{\delta}{(Pr)^{1/3}} \quad ...(7.62)$$

The local and average heat transfer coefficients :

At the surface of the plate, since there is no fluid motion and the heat transfer can occur only through conduction, the heat flux may be written as,

$$\frac{Q}{A} = h_x (t_s - t_\infty) = -k\left(\frac{\partial t}{\partial y}\right)_{y=0} \quad ...(7.63)$$

From the relation 7.63, we may develop $\left(\frac{\partial t}{\partial y}\right)_{y=0}$ (*i.e.*, surface temperature gradient) as,

$$\left(\frac{\partial t}{\partial y}\right)_{y=0} = -(t_s - t_\infty)\sqrt{\frac{U}{\nu x}} \times \left(\frac{\partial \theta}{\partial y}\right)_{\eta=0}$$

$$= -(t_s - t_\infty)\sqrt{\frac{U}{\nu x}} \times 0.332(Pr)^{1/3}$$

$$= -\frac{0.332}{x}(t_s - t_\infty)\sqrt{\frac{U}{\nu}}(Pr)^{1/3}$$

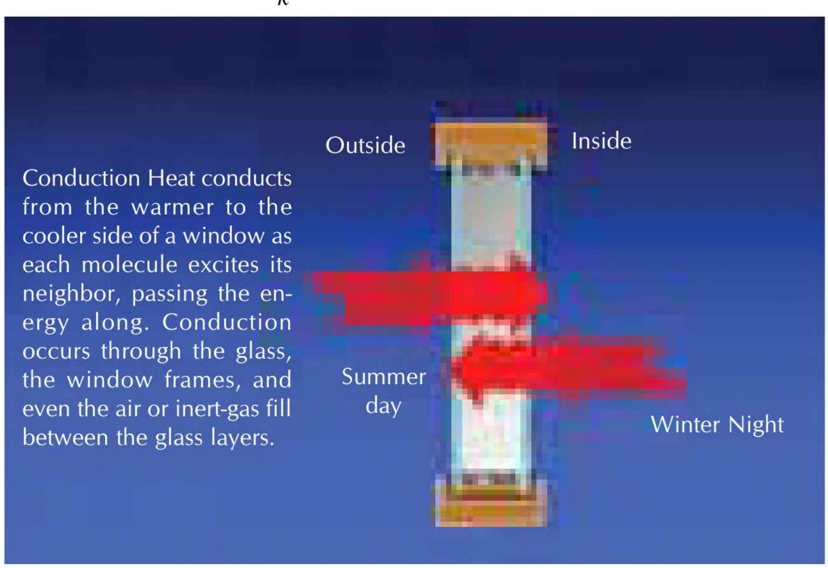
$$= -\frac{0.332}{x}(t_s - t_\infty)(Re_x)^{1/2}(Pr)^{1/3}$$

Substituting for $\left(\frac{\partial t}{\partial y}\right)_{y=0}$ in eqn. (7.63), we obtain

$$\frac{Q}{A} = h_x (t_s - t_\infty) = 0.332\frac{k}{x}(t_s - t_\infty)(Re_x)^{1/2}(Pr)^{1/3}$$

or,
$$h_x = 0.332\frac{k}{x}(Re_x)^{1/2}(Pr)^{1/3} \quad ...(7.64)$$

or,
$$Nu_x = \frac{h_x x}{k} = 0.332(Re_x)^{1/2}(Pr)^{1/3} \quad ...(7.65)$$



Conduction = Heat Flow through Materials

... (In non-dimensional form)

[where, h_x = Local convective heat transfer coefficient, and

Nu_x = Local value of Nusselt number (at a distance x from the leading edge of the plate,].

The *average* heat transfer coefficient is given by

$$\bar{h} = \frac{1}{L}\int_0^L h_x \,.\, dx = \frac{1}{L}\int_0^L 0.332\frac{k}{x}(Re_x)^{1/2}(Pr)^{1/3}\,dx$$

$$= \frac{1}{L}\int_0^L 0.332\,k\,(Pr)^{1/3}\sqrt{\left(\frac{U}{\nu}\right)}\,x^{-1/2}\,dx$$

or, $$\bar{h} = 0.664\left(\frac{k}{L}\right)(Re_L)^{1/2}(Pr)^{1/3} \qquad ...(7.66)$$

If we compare the eqns. (7.64) and (7.66), we find that

$$\bar{h} = 2h_x \qquad ...(7.67)$$

and $\overline{Nu}$ (average value of Nusselt number) $= \frac{\bar{h}L}{k} = 0.664\,(Re_L)^{1/2}\,(Pr)^{1/3}$ ...(7.68)

All the results in eqns. (7.64), (7.65) and (7.68) are valid for $Pr > 0.5$.

7.1.7. INTEGRAL ENERGY EQUATION (APPROXIMATE SOLUTION OF ENERGY EQUATION)

Consider a control volume shown in Fig. 7.10. *Assume* that ρ, c_p and k (thermo-plastic properties) of fluid remain constant within the operating range of the temperature, and the heating of the plate commences at a distance x_0 from the leading edge of the plate (so that the boundary layer initiates at $x = x_0$ and develops and grows beyond that). For *unit width* of the plate we have :

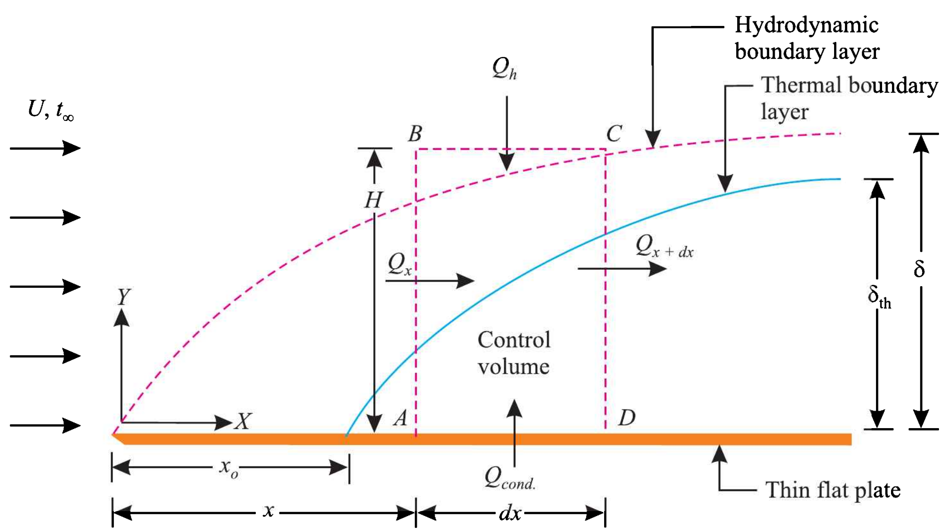


Fig. 7.10. Integral energy equation – control volume.

Mass of fluid entering through face $AB = \int_0^H \rho u\,dy$ ...(7.69)

Mass of fluid leaving through face $CD = \int_0^H \rho u\,dy + \frac{\partial}{\partial x}\left[\int_0^H \rho u\,dy\right]dx$ ...(7.70)

$\therefore$ Mass of fluid entering the control volume through face BC

$$= \left[\int_0^H \rho u\, dy + \frac{\partial}{\partial x}\left\{ \int_0^H \rho u\, dy \right\} dx \right] - \int_0^H \rho u\, dy = \frac{\partial}{\partial x}\left[\int_0^H \rho u\, dy \right] dx \qquad ...(7.71)$$

Heat influx through the face *AB*,

$$Q_x = \text{Mass} \times \text{specific heat} \times \text{temperature}$$

or,

$$Q_x = \left(\int_0^H \rho u\, dy \right) \times cp \times t = \rho c_p \int_0^H ut\; dy \qquad ...(7.72)$$

Heat efflux through the face *CD*,

$$Q_{x+dx} = \int_0^H ut\; dy + \frac{\partial}{\partial x}\left[\rho c_p \int_0^H ut\, dy \right] dx \qquad ...(7.73)$$

Heat (energy) influx through the face *BC* (which is outside thermal boundary layer and there the temperature is constant at t_∞),

$$Q_h = \frac{\partial}{\partial x}\left[\int_0^H \rho u\; dy \right] dx.\, c_p\, t_\infty \qquad ...(7.74)$$

Heat conducted into the control volume through face *AD*,

$$Q_{cond.} = -kA\left[\frac{\partial t}{\partial y} \right]_{y=0} = -k\, dx \left(\frac{\partial t}{\partial y} \right)_{y=0} \qquad ...(7.75)$$

The energy balance for the element is given by

$$\rho c_p \int_0^H ut\, dy + \frac{\partial}{\partial x}\left[\rho c_p t_\infty \int_0^H u\, dy \right] dx + \left[-k\, dx \left(\frac{\partial t}{\partial y} \right)_{y=0} \right]$$

$$= \rho c_p \int_0^H ut\, dy + \frac{\partial}{\partial x}\left[\rho c_p \int_0^H ut\, dy \right] dx$$

After simplification and rearrangement, we have

$$\frac{d}{dx} \int_0^H (t_\infty - t) u\, dy = \frac{k}{\rho c_p}\left(\frac{\partial t}{\partial y} \right)_{y=0} = \alpha \left(\frac{\partial t}{\partial y} \right)_{y=0} \qquad ...(7.76)$$

Equation (7.76) is the integral equation for the boundary layer for constant properties and constant free stream temperature t_∞.

If the viscous work done within the element is considered, then eqn. (7.76) becomes

$$\frac{d}{dx} \int_0^H (t_\infty - t) u\, dy = \frac{\mu}{\rho c_p} \int_0^H \frac{\partial^2 u}{\partial y^2}\, dx\, dy = \alpha \left(\frac{\partial t}{\partial y} \right)_{y=0} \qquad ...(7.77)$$

[where $\frac{\mu}{\rho c_p} \int_0^H \frac{\partial^2 u}{\partial y^2}\, dx\, dy$ = Viscous work done within the element ...Eqn. (7.8)]

Usually the viscous dissipation term is very small and is neglected (and may be considered only when velocity of flow field becomes very large).

Expression for the convective heat transfer coefficient for laminar flow over a flat plate:

In order to derive an expression for convective heat transfer coefficient for laminar flow over a flat plate (that has an unheated starting length x_0), let us use *cubic velocity and temperature distributions* in the integral boundary layer energy equation as follows :

The cubic velocity profile within the boundary layer is of the form

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta} \right) - \frac{1}{2}\left(\frac{y}{\delta} \right)^3 \qquad ...[\text{Eqn. } (7.33)]$$

The conditions which are satisfied by the temperature distribution within the boundary layer are :

(*i*) At $y = 0$, $t = t_s$ (*ii*) At $y = 0$, $\frac{\partial^2 t}{\partial y^2} = 0$

(*iii*) At $y = \delta_{th}$, $t = t_\infty$ (*iv*) At $y = \delta_{th} = \dfrac{\partial t}{\partial y} = 0$

By using these boundary conditions for a cubic polynomial,

$$\frac{\theta}{\theta_\infty} = a + b\left(\frac{y}{\delta_{th}}\right) + c\left(\frac{y}{\delta_{th}}\right)^2 + d\left(\frac{y}{\delta_{th}}\right)^3 \quad ...(7.78)$$

The temperature distribution takes the following form

$$\frac{\theta}{\theta_\infty} = \frac{t - t_s}{t_\infty - t_s} = \frac{3}{2}\left(\frac{y}{\delta_{th}}\right) - \frac{1}{2}\left(\frac{y}{\delta_{th}}\right)^3 \quad ...(7.79)$$

By putting the proper values of velocity distribution and temperature distribution into the integral equation, we get

$$\alpha\left(\frac{dt}{dy}\right)_{y=0} = \frac{d}{dx}\int_0^H (t_\infty - t)\,u\,dy$$

$$= U(t_\infty - t_s)\frac{d}{dx}\int_0^H \frac{u}{U}\left(\frac{t_\infty - t}{t_\infty - t_s}\right)dy$$

$$= U(t_\infty - t_s)\frac{d}{dx}\int_0^H \frac{u}{U}\left[1 - \frac{t_\infty - t}{t_\infty - t_s}\right]$$

$$= U(t_\infty - t_s)\frac{d}{dx}\left[\int_0^H \left\{\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right\} \times \left\{1 - \frac{3}{2}\left(\frac{y}{\delta_{th}}\right) + \frac{1}{2}\left(\frac{y}{\delta_{th}}\right)^3\right\}dy\right]$$

$$= U(t_\infty - t_s)\frac{d}{dx}\left[\int_0^{\delta_{th}} \left\{\frac{3}{2\delta}y - \frac{9}{4\delta\,\delta_{th}}y^2 - \frac{1}{2\delta^3}y^3 + \left(\frac{3}{4\delta\,\delta_{th}^3} + \frac{3}{4\delta^3\,\delta_{th}}\right)y^4 - \frac{1}{4\delta^3\,\delta_{th}^3}y^6\right\}dy\right] \quad ...(7.80)$$

The upper limit has been changed to δ_{th} since $\delta_{th} < \delta$ for most of the gases (for $y > \delta_{th}$ integrand would be zero).

After putting $\dfrac{\delta_{th}}{\delta} = r$ and carrying out the integration, eqn. (7.80) gets reduced to

$$\alpha\left(\frac{dt}{dy}\right)_{y=0} = U(t_\infty - t_s)\frac{d}{dx}\left[\delta\left(\frac{3}{20}r^2 - \frac{3}{280}r^4\right)\right] \quad ...(7.81)$$

Neglecting the term involving r^4 (because $\delta_{th} < \delta$, $r < 1$), we have

$$\alpha\left(\frac{dt}{dy}\right)_{y=0} = \frac{3}{20}U\,(t_\infty - t_s)\frac{d}{dx}(\delta r^2) \quad ...(7.82)$$

Further, from eqn. (7.77), we have

$$t = t_s + (t_\infty - t_s)\left[\frac{3}{2}\left(\frac{y}{\delta_{th}}\right) - \frac{1}{2}\left(\frac{y}{\delta_{th}}\right)^3\right]$$

or,
$$\frac{dt}{dy} = (t_\infty - t_s)\left[\frac{3}{2\delta_{th}} - \frac{3}{2\delta_{th}^3}y^2\right]$$

or,
$$\left(\frac{dt}{dy}\right)_{y=0} = \frac{3}{2}\left(\frac{t_\infty - t_s}{\delta_{th}}\right) = \frac{3}{2}\left[\frac{t_\infty - t_s}{r\delta}\right] \quad ...(7.83)$$

Upon substitution in eqn. (7.82), we get

$$\alpha \times \frac{3}{2}\left[\frac{t_\infty - t_s}{r\delta}\right] = \frac{3}{20}U(t_\infty - t_s)\frac{d}{dx}(\delta r^2)$$

or,
$$\alpha = \frac{2}{3}(r\delta) \times \frac{3}{20}U\frac{d}{dx}(\delta r^2)$$

$$= (r\delta)\frac{U}{10}\left(\delta \times 2r + \frac{dr}{dx} + r^2 \times \frac{d\delta}{dx}\right)$$

or,
$$\alpha = \frac{U}{10}\left[2\delta^2 r^2 \times \frac{dr}{dx} + \delta r^3 \times \frac{d\delta}{dx}\right] \quad ...(7.84)$$

Also, $\delta\frac{d\delta}{dx} = \frac{140}{13} \cdot \frac{\mu}{\rho U} = \frac{140}{13} \cdot \frac{v}{U}$ and $\delta^2 = \frac{280}{13} \times \frac{ux}{\rho U} = \frac{280\ vx}{13U}$ ...[Refer Art 7.5]

Putting these values in eqn. (7.84), we obtain

$$\alpha = \frac{U}{10}\left[2r^2 \times \frac{280\ vx}{13U} \times \frac{dr}{dx} + r^3 \times \frac{140\,v}{13U}\right]$$

or,
$$r^3 + 4r^2 \times \frac{dr}{dx} = \frac{13\alpha}{14v} \quad ...(7.85)$$

Using the equality $\frac{d}{dx}(r^3) = 3r^2\frac{dr}{dx}$, we can write eqn. (7.85) as

$$r^3 + 4xr^2\frac{dr}{dx} = \frac{13\alpha}{14v} \quad ...(7.86)$$

The general solution of the equation (which is a linear differential equation of the first order in r^3) is given by

$$r^3 = Cx^{-3/4} + \frac{13\alpha}{14v}$$

The value of the constant C can be evaluated from the following boundary conditions :

At $x = x_0$,
$$r^3 = \left(\frac{\delta_{th}}{\delta}\right)^3 = 0$$

Thus,
$$0 = Cx_0^{3/4} + \frac{13\alpha}{14v} \quad \text{or} \quad C = -\frac{13\alpha}{14v}x_0^{3/4}\text{, and}$$

$$r^3 = -\frac{13\alpha}{14v}x_0^{3/4}x^{-3/4} + \frac{13\alpha}{14v}$$

or,
$$r^3 = \frac{13\alpha}{14v}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right] \quad ...(7.87)$$

∴
$$r = \frac{\delta_{th}}{\delta} = \left(\frac{13}{14}\right)^{1/3}\left(\frac{\alpha}{v}\right)^{1/3}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]$$

$$\frac{\delta_{th}}{\delta} = \frac{0.975}{(Pr)^{1/3}}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3} \quad ...(7.88)$$

[$\because \frac{v}{\alpha} = Pr$ (Prandtl number)]

or,
$$\frac{\delta_{th}}{\delta} = \frac{0.975}{(Pr)^{1/3}} \quad ...(7.89)$$

... when the plate is heated over the entire length *i.e.*, $x_0 = 0$.

Local heat transfer coefficient h_x :

We know that, $$\frac{Q}{A} = h_x (t_s - t_\infty) = -k\left(\frac{dt}{dy}\right)_{y=0}$$

or, $$h_x = \frac{-k(dt/dy)_{y=0}}{t_s - t_\infty}$$

But, $$\left(\frac{dt}{dy}\right)_{y=0} = \frac{3}{2}\left(\frac{t_s - t_\infty}{\delta_{th}}\right) \quad \text{...[Eqn. (7.83)]}$$

$\therefore$ $$h_x = \frac{-k \times \frac{3}{2}\left(\frac{t_s - t_\infty}{\delta_{th}}\right)}{(t_s - t_\infty)} = \frac{3k}{2\delta_{th}} = \frac{3k}{2} \times \frac{1}{r\delta} \quad \text{...(7.90)}$$

Substituting $r = \frac{0.975}{(Pr)^{1/3}}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}$ and $\delta = \frac{4.64\,x}{\sqrt{Re_x}}$ in eqn. (7.90), we get

$$h_x = \frac{3k}{2} \times \frac{(Pr)^{1/3}}{0.975\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}} \times \frac{\sqrt{Re_x}}{4.64\,x}$$

or, $$h_x = 0.332\,\frac{k}{x}(Pr)^{1/3}\,(Re)^{1/2} \times \frac{1}{\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}} \quad \text{...(7.91)}$$

or, $$Nu_x = \frac{h_x x}{k} = \frac{0.332\,(Pr)^{1/3}\,(Re_x)^{1/2}}{\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}} \quad \text{...(7.92)}$$

When the plate is heated over the whole length *i.e.*, $x_0 = 0$, we have

$$h_x = 0.332\frac{k}{x}(Pr)^{1/3}\,(Re_x)^{1/2} \quad \text{...(7.93)}$$

and, $$Nu_x = 0.332\,(Pr)^{1/3}\,(Re_x)^{1/2} \quad \text{...(7.94)}$$

The above results are applicable for *laminar conditions only.*

Example 7.10. *Air at 20°C and at a pressure of 1 bar is flowing over a flat plate at a velocity of 3 m/s. If the plate is 280 mm wide and at 56°C, calculate the following quantities at x = 280 mm, given that properties of air at the bulk mean temperature* $\left(\frac{20 + 56}{2}\right)$ *= 38°C are :*

ρ = 1.1374 kg/m^3; k = 0.02732 W/m°C; c_p = 1.005 kJ/kgK; ν = 16.768 × 10^{-6} m^2/s; Pr = 0.7.

(*i*) *Boundary layer thickness,*
(*ii*) *Local friction coefficient,*
(*iii*) *Average friction coefficient,*
(*iv*) *Shearing stress due to friction,*
(*v*) *Thickness of the boundary layer,*
(*vi*) *Local convective heat transfer coefficient,*
(*vii*) *Average convective heat transfer coefficient,*
(*viii*) *Rate of heat transfer by convection,*
(*ix*) *Total drag force on the plate, and*
(*x*) *Total mass flow rate through the boundary.*

Solution. *Given* : $U = 3$m/s, $x = 280$ mm $= 0.28$ m, $\rho = 1.1374$ kg/m^3, $k = 0.02732$ W/m°C, $c_p =$ 1.005 kJ/kgK, $\nu = 16.768 \times 10^{-6}$ m^2/s.

Let us first ascertain the type of the flow, whether laminar or turbulent.

$$Re_x = \frac{Ux}{\nu} = \frac{3 \times 0.28}{16.768 \times 10^{-6}} = 5.0 \times 10^4$$

Since $Re_x < 5 \times 10^5$, hence flow in *laminar*.

(*i*) Boundary layer thickness at $x = 0.28$ m, δ :

$$\delta = \frac{5x}{\sqrt{Re_x}} \quad \text{...[Eqn. (7.22)]}$$

or, $$\delta = \frac{5 \times 0.28}{\sqrt{5 \times 10^4}} = 0.00626 \text{ m or } \mathbf{6.26\ mm\ (Ans.)}$$

(*ii*) Local friction coefficient, C_{fx} :

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} \quad \text{...[Eqn. (7.24)]}$$

or, $$C_{fx} = \frac{0.664}{\sqrt{5 \times 10^4}} = \mathbf{0.002969\ (Ans.)}$$

(*iii*) Average friction coefficient, $\overline{C}_f$:

$$\overline{C}_f = \frac{1.328}{\sqrt{Re_L}} \quad \text{...[Eqn. (7.25)]}$$

or, $$\overline{C}_f = \frac{1.328}{\sqrt{5 \times 10^4}} = \mathbf{0.005939\ (Ans.)} \quad \text{...}(\overline{C}_f = 2C_{fx})$$

(*iv*) Shearing stress due to friction, τ_0 :

$$\tau_0 = C_{fx} \times \frac{\rho U^2}{2} \quad \text{...[Eqn. (7.30)]}$$

$$= 0.002969 \times \frac{1.1374 \times 3^2}{2} = \mathbf{0.01519\ N/m^2\ (Ans.)}$$

(*v*) Thickness of thermal boundary layer, δ_{th} :

$$\delta_{th} = \frac{\delta}{(Pr)^{1/3}} \quad \text{...[Eqn. (7.62)}$$

$$= \frac{0.00626}{(0.7)^{1/3}} = 0.00705 \text{ , m or } \mathbf{7.05\ mm\ (Ans.)}$$

(*vi*) Local convective heat transfer coefficient, h_x :

$$h_x = 0.332 \frac{k}{x} (Re_x)^{1/2} (Pr)^{1/3} \quad \text{...[Eqn. (7.64)]}$$

$$= 0.332 \times \frac{0.02732}{0.28} \times (5 \times 10^4)^{1/2} \times (0.7)^{1/3}$$

$$= \mathbf{6.43\ W/m^2°C\ (Ans.)}$$

(*vii*) Average convective heat transfer coefficient, $\overline{h}$:

$$\overline{h} = 0.664 \left(\frac{k}{L}\right) (Re_L)^{1/2} (Pr)^{1/3} \quad \text{...[Eqn. (7.66)]}$$

$$= 0.664\left(\frac{0.02732}{0.28}\right)(5 \times 10^4)^{1/2}\ (0.7)^{1/3} = \mathbf{12.86\ W/m^2°C\ (Ans.)}$$

$$\dots(\bar{h} = 2h_x)$$

(*viii*) **Rate of heat transfer by convection, Q_{conv}.:**

$$Q_{conv.} = \bar{h}\, A_s (t_s - t_\infty)$$

$$= 12.85 \times (0.28 \times 0.28)\ (56 - 20) = \mathbf{36.29\ W\ (Ans.)}$$

(*ix*) **Total drag force on the plate, F_D :**

$$F_D = \tau_0 \times \text{ area of plate on one side upto 0.28 m}$$

$$= 0.01519 \times 0.28 \times 0.28 = \mathbf{0.00119\ N\ (Ans.)}$$

(*x*) **Total mass flow rate through the boundary, *m* :**

$$m = \frac{5}{8}\rho U\,(\delta_2 - \delta_1)$$

(where $\delta_1 = 0$ at $x = 0$ and $\delta_2 = \delta$ at $x = 0.28$ m)

$$= \frac{5}{8} \times 1.1374 \times\ 3\ (0.00626 - 0) = \mathbf{0.01335\ kg/s\ (Ans.)}$$

Example 7.11. *Air at atmospheric pressure and 200°C flows over a plate with a velocity of 5 m/s. The plate is 15 mm wide and is maintained at a temperature of 120°C. Calculate the thicknesses of hydrodynamic and thermal boundary layers and the local heat transfer coefficient at a distance of 0.5 m from the leading edge. Assume that flow is on one side of the plate.*

ρ = 0.815 kg/m^3; μ = 24.5 × 10^{-6} Ns/m^2; Pr = 0.7, k = 0.0364 W/m K. **(AMIE Summer, 1997)**

Solution. *Given* : U = 5 m/s; x = 0.5 m; ρ = 0.815 kg/m^3; μ = 24.5 × 10^{-6} Ns/m^2;

Pr = 0.7, k = 0.0364 W/m K.

Let us first ascertain the type of flow, whether laminar or tubulent.

$$Re_x = \frac{Ux}{\nu} = \frac{5 \times 0.5}{\mu/\rho} = \frac{5 \times 0.5}{(24.5 \times 10^{-6}/0.815)} = 83163$$

Since $Re_x < 5 \times 10^5$, hence flow is *laminar*.

Boundary layer thickness at x = 0.5 m, δ :

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 0.5}{\sqrt{83163}} = 8.669 \times 10^{-3}\ \text{m}$$

or **8.669 mm (Ans.)**

Thickness of thermal boundary layer, at x = 0.5 m, δ_{th}

$$\delta_{th} = \frac{\delta}{(Pr)^{1/3}} = \frac{8.669}{(0.7)^{1/3}} = \mathbf{9.763\ mm\ (Ans.)}$$

Local heat transfer coefficient, h_x :

$$h_x = 0.332 \times \frac{k}{x}\,(Re_x)^{1/2}\,(Pr)^{1/3} \quad \text{...[Eqn. (7.64)]}$$

$$= 0.332 \times \frac{0.0364}{0.5} \times\ (83163)^{1/2} \times (0.7)^{1/3} = \mathbf{6.189\ W/m^2K\ (Ans.)}$$

This tool can be used in a variety of heat transfer experiments, such as free convection, forced convection and fins.

Example 7.12. *Air at atmospheric pressure and 40° C flows with a velocity of U = 5 m/s over a 2 m long flat plate whose surface is kept at a uniform temperature of 120°C. Determine the average heat transfer coefficient over the 2 m length of the plate. Also find out the rate of heat transfer between the plate and the air per 1 m width of the plate. [Air at 1 atm. and 80°C, $\nu = 2.107 \times 10^{-5}$ m²/s, k = 0.03025 W/mK ; Pr = 0.6965]* **(AMIE Winter, 1998)**

Solution. *Given* : *Given* $t_\infty = 40°C$; $U = 5$m/s; $L = 2$m, $t_s = 120°C$; $B = 1$ m

The properties of air at mean bulk temperature of $\left(\frac{120 + 40}{2}\right) = 80°C$ are :

$$\nu = 2.107 \times 10^{-5} \text{ m}^2/\text{s};\ k = 0.03025 \text{ W/m K};\ Pr = 0.6965.$$

Average heat transfer coefficient, $\bar{h}$:

$$Re = \frac{UL}{\nu} = \frac{5 \times 2}{2.107 \times 10^{-5}} = 4.746 \times 10^5$$

Assuming $Re_{cr} = 5 \times 10^5$, the flow is *laminar.*

Using *exact solution*, the average Nusselt number is given by

$$\overline{Nu} = 0.664\ (Re_L)^{1/2}\ (Pr)^{1/3} \qquad \text{...[Eqn. (7.68)]}$$

or, $$\frac{\bar{h}L}{k} = 0.664\ (4.746 \times 10^5)^{1/2}\ (0.6965)^{1/3} = 405.48$$

∴ $$\bar{h} = \frac{k}{L} \times 405.48 = \frac{0.03025}{2} \times 4054.48 = \mathbf{6.133\ W/m^3K} \quad \textbf{(Ans.)}$$

Rate of heat transfer, Q :

$$Q = \bar{h}\, A_s\, (t_s - t_\infty)$$
$$= 6.133 \times (2 \times 1)\ (120 - 40) = \mathbf{981.28\ W} \quad \textbf{(Ans.)}$$

Example 7.13. *Air at 27°C and 1 bar flows over a plate at a speed of 2 m/s.*

(*i*) *Calculate the boundary layer thickness at 400 mm from the leading edge of the plate. Find the mass flow rate per unit width of the plate.*

For air $\mu = 19.8 \times 10^{-6}$ kg/ms at 27°C.

(*ii*) *If the plate is maintained at 60°C, calculate the heat transferred per hour.*

The properties of air at mean temperature of (27 + 60)/2 = 43.5° C are given below :

$$\nu = 17.36 \times 10^{-6} \text{ m}^2/\text{s};\ k = 0.02749 \text{ W/m°C}$$

$$c_p = 1006 \text{ J/kg K};\ R = 287 \text{ Nm/kg m K};\ Pr = 0.7$$ **(M.U.)**

Solution. *Given : t* = 27°C; *p* = 1 bar, *U* = 2 m/s; *x* = 400 mm = 0.4 m

(*i*) Boundary layer thickness, δ :

$$\rho = \frac{p}{RT} = \frac{1 \times 10^5}{287 \times (27 + 273)} = 1.16\, kg/\text{m}^3$$

$$Re_x = \frac{\rho LU}{\mu} = \frac{1.16 \times 0.4 \times 2}{19.8 \times 10^{-6}} = 46869$$

Boundary layer thickness, $$\delta = \frac{4.64\,x}{\sqrt{Re_x}} \qquad \text{...[Eqn.(7.36)]}$$

or, $$\delta = \frac{4.64 \times 0.4}{\sqrt{46869}} = 0.00857 \text{ m or } \mathbf{8.57\ mm} \quad \textbf{(Ans.)}$$

The mass flow rate per metre width is given by,

$$m_x = \int_0^{\delta} (dy \times 1) u \,.\, \rho = \int_0^{\delta} \rho u \; dy$$

Now, $$u = U\left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]$$ ...*assumed*

$\therefore$ $$m_x = \int_0^{\delta} \rho U \left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right] dy$$

$$= \rho U \left[\frac{3}{4}\left(\frac{y^2}{\delta}\right) - \frac{1}{8}\left(\frac{y^4}{\delta^3}\right)\right]_0^{\delta} = \frac{5}{8}\rho U \delta$$

$$= \frac{5}{8} \times 1.16 \times 2 \times 0.00857 = \mathbf{0.01242\,kg/s} \quad \textbf{(Ans.)}$$

Note. If the mass added in the boundary is to be calculated when the fluid moves from x_1 to x_2 along the main flow direction, then it is given by

$$\Delta m = \frac{5}{8}\rho U\,(\delta_2 - \delta_1)$$

where δ_1 and δ_2 are the boundary layer thicknesses at x_1 and x_2 .

(*ii*) Heat transferred per hour, *Q* :

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.664\, Re^{1/2}\, Pr^{1/3}$$

$$\bar{h} = \frac{k}{L} = 0.664\, Re^{1/2}\, Pr^{1/3}$$

$$= \frac{0.02749}{0.4} \times 0.664 \times (46869)^{1/2} \times (0.7)^{1/3} = 8.77\, W/m^2\,°C$$

$$Q = \bar{h} A (t_s - t_\infty)$$

$$= 8.77 \times (0.4 \times 1)\,(60 - 27) = 115.76 \text{ J/s}$$

$$= \frac{115.76 \times 3600}{1000} = \mathbf{416.74\,kJ/h} \quad \textbf{(Ans.)}$$

Example 7.14. *Air at 1 bar and at a temperature of 30°C (μ = 0.06717 kg/hm) flows at a speed of 1.2 m/s over a flat plate. Determine the boundary layer thickness at distance of 250 mm and 500 mm from the leading edge of the plate. Also, calculate the mass entrainment between these two sections. Assume the parabolic velocity distribution as :* $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$.

Solution. *Given :* t_∞ = 30°C, μ = 0.06717 kg/hm, U = 1.2 m/s

Boundary layer thicknesses :

The density of air, $$\rho = \frac{p}{RT} = \frac{1 \times 10^5}{287 \times (30 + 273)} = 1.15 \text{ kg/m}^3$$

At x = 250 *mm* = 0.25 *m*

Reynolds number $$Re_x = \frac{\rho U x}{\mu} = \frac{1.15 \times 1.2 \times 0.25 \times 3600}{0.06717} = 18490$$

$\therefore$ Boundary layer thickness, $$\delta_1 = \frac{4.64\,x}{\sqrt{Re_x}}$$ ...[Eqn. (7.36)]

or, $$\delta_1 = \frac{4.64 \times 0.25}{\sqrt{18490}} = 0.00853 \text{ m or } \mathbf{8.53\ mm} \quad \textbf{(Ans.)}$$