Obviously, the midpoint of the boundary layer $\left(y = \frac{\delta}{2}\right)$ occurs at

$$\eta = y\sqrt{\frac{U}{vx}} = 2.5$$

The stream wise velocity component is obtained for,m the Blasius solution in tabular form (Refer table 7.1).

At, $$\eta = y\sqrt{\frac{U}{vx}} = 2.5 \text{ , we get } \frac{u}{U} = 0.736$$

or, $$u = 0.736\, U = 0.736 \times 1.8 = \mathbf{1.325\ m/s} \quad \textbf{(Ans.)}$$

(ii) **The maximum boundary layer thickness, δ_L :**

The maximum boundary layer thickness occurs at $x = 0.6$ m. Thus,

$$Re_L = \frac{\rho UL}{\mu} = \frac{1.205 \times 1.8 \times 0.6}{(0.06533/3600)} = 71713 \text{, hence flow in } \textit{laminar}.$$

The boundary layer thickness at the trailing edge,

$$\delta_L = \frac{5L}{\sqrt{Re_L}} = \frac{5 \times 0.6}{\sqrt{71713}} = 0.0112\text{m} \quad \text{or,} \quad \mathbf{11.2\ mm} \quad \textbf{(Ans.)}$$

(iii) **The maximum value of the normal component of velocity at the trailing edge, *v* :**

The maximum value of the normal component of velocity occurs at the outer edge of the boundary layer where $u = U$. Hence for, $\frac{u}{U} = 1$, we have

$$\frac{\upsilon}{U}\sqrt{Re_L} = 0.86 \qquad \text{(Refer Table 7.1)}$$

or, $$\upsilon = \frac{0.86U}{\sqrt{Re_L}} = \frac{0.86 \times 1.8}{\sqrt{71713}} = 0.00578 \text{ m} \quad \text{or,} \quad \mathbf{5.78\ m/s} \quad \textbf{(Ans.)}$$

7.2. LAMINAR TUBE FLOW

7.2.1. DEVELOPMENT OF BOUNDARY LAYER

In case of a pipe flow, the development of boundary layer proceeds in a fashion similar to that for, flow along a flat plate. A fluid of uniform velocity entering a tube is retarded near the walls and a boundary layer begins to develop as shown in Fig. 7.12 by dotted lines. The thickness of the boundary layer is limited to the pipe radius because of the flow being within a confined passage. Boundary layers from the pipe walls meet at the centre of the pipe and the entire flow acquires the characteristics of a boundary layer. Once the boundary layer thickness becomes equal to the radius of the tube there will not be any further change in the velocity distribution, this *invariant* velocity distribution is called *fully developed velocity profile i.e.*, **Poiseulle flow** (parabolic distribution).

According to Langhar (1942), the entrance length (L_e) is expressed as : $\frac{L_e}{D} = 0.0575\, Re$ where D represents the inside diameter of the pipe.

Laminar tube flow with a small disturbance.

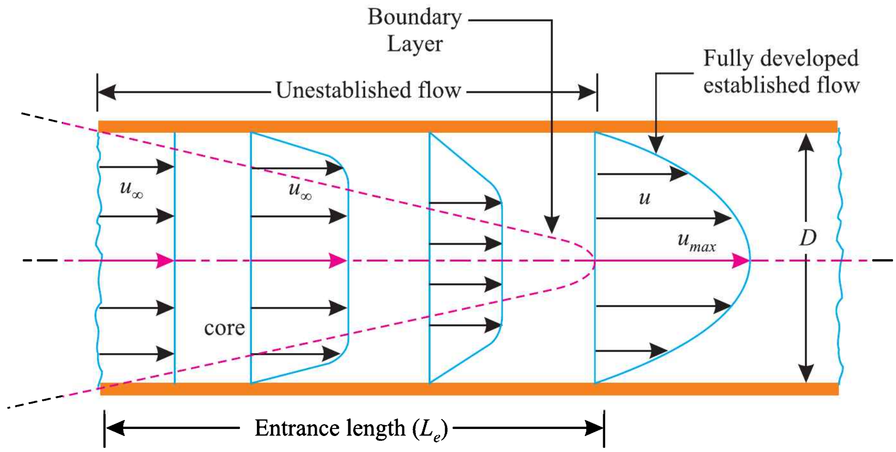


Fig. 7.12. The development of a laminar velocity profile in the intake region of a tube.

7.2.2 VELOCITY DISTRIBUTION

Fig. 7.13 shows a horizontal circular pipe of radius R, having laminar flow of fluid through it. Consider a small concentric cylinder (fluid element) of radius r and length dx as a free body.

If τ is the shear stress, the shear force F is given by

$$F = \tau \times 2\pi r \times dx$$

Let p be the intensity of pressure at left end and the intensity of pressure at the right end be $\left(p + \frac{\partial p}{\partial x} \cdot dx\right)$.

Thus the forces acting on the fluid element are :

1. The shear force, $\tau \times 2\pi r \times dx$ on the surface of fluid element.
2. The pressure force, $p \times \pi r^2$ on the left-end.
3. The pressure force, $\left(p + \frac{\partial p}{\partial x} \cdot dx\right) \pi r^2$ on the right end.

For steady flow, the net force on the cylinder must be *zero*.

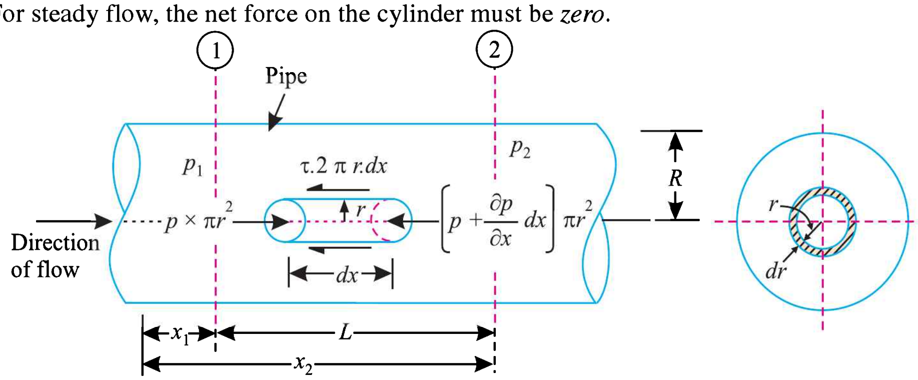


Fig. 7.13. Laminar flow through a circular pipe.

$$\left[p \times \pi r^2 - \left(p + \frac{\partial p}{\partial x} .dx\right)\pi r^2\right] - \tau \times 2\pi r \times dx = 0$$

or,
$$-\frac{\partial p}{\partial x} . dx \times \pi r^2 - \tau \times 2\pi r \times dx = 0$$

or, $$\tau = -\frac{\partial p}{\partial x} \cdot \frac{r}{2} \quad \text{...(7.95)}$$

– Equation (7.95) shows that flow will occur only if *pressure gradient exists in the direction of flow.*

The *negative sign shows that pressure decreases in the direction of flow.*

– Equation (7.95) indicates that the shear stress varies linearly across the section (see Fig. 7.14). Its value is zero at the centre of pipe ($r = 0$) and maximum at the pipe wall given by

$$\tau_0 = -\frac{\partial p}{\partial x}\left(\frac{R}{2}\right) \quad \text{...[7.95 (a)]}$$

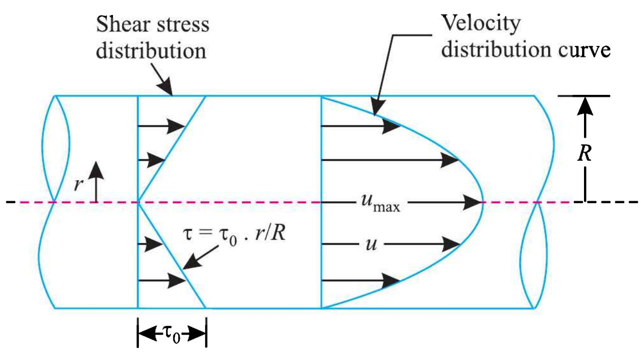


Fig. 7.14. Shear stress and velocity distribution across a section.

From Newton's law of viscosity,

$$\tau = \mu \cdot \frac{du}{dy} \quad \text{...}(i)$$

In this equation, the distance y is measured from the boundary. The radial distance r is related to distance y by the relation

$$y = R - r \text{ or, } dy = -dr$$

The eqn. (i) becomes

$$\tau = -\mu \frac{du}{dr} \quad \text{...(7.96)}$$

Comparing two values of τ from eqns. 7.95 and 7.96, we have

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

or, $$du = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) r \cdot dr$$

Integrating the above equation *w.r.t.* 'r', we get

$$u = \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} r^2 + C \quad \text{...(7.97)}$$

where, C is the constant of integration and its value is obtained from the boundary condition :

At $r = R,\ u = 0$

$\therefore \qquad 0 = \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} R^2 + C \qquad$ or, $\qquad C = -\frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} R^2$

Substituting this value of C in eqn. (7.97), we get

$$u = \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} R^2$$

or,
$$u = -\frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} (R^2 - r^2) \qquad \text{...(7.98)}$$

Equation (7.98) shows that the velocity distribution curve is a *parabola* (see Fig. 7.14). The maximum velocity occurs at the centre and is given by

$$u_{max} = -\frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} R^2 \qquad \text{...(7.99)}$$

From eqns. (7.98) and (7.99), we have

$$u = u_{max} \left[1 - \left(\frac{r}{R} \right)^2 \right] \qquad \text{...(7.100)}$$

Eqn. 7.100 is the *most commonly used equation for, the velocity distribution for, laminar flow through pipes*. This equation can be used to calculate the discharge as follows :

The discharge through an elementary ring of thickness dr at radial distances r is given by

$$dQ = u \times 2\pi r \times dr$$

$$= u_{max} \left[1 - \left(\frac{r}{R} \right)^2 \right] 2\pi r.dr$$

Total discharge
$$Q = \int dQ$$

$$= \int_0^R u_{\max} \left[1 - \left(\frac{r}{R} \right)^2 \right] 2\pi r.dr$$

$$= 2\pi u_{max} \int_0^R \left(1 - \frac{r^3}{R^2} \right) dr$$

$$= 2\pi u_{max} \left[\frac{r^2}{2} - \frac{r^4}{4R^2} \right]_0^R = 2\pi\, u_{max} \left[\frac{R^2}{2} - \frac{R^4}{4R^2} \right]$$

$$= \frac{\pi}{2} u_{max} R^2$$

Average velocity of flow,
$$\bar{u} = \frac{Q}{A} = \frac{\frac{\pi}{2} u_{max} R^2}{\pi R^2} = \frac{u_{max}}{2} \qquad \text{...(7.101)}$$

Eqn. (7.101) shows that the *average velocity is one-half the maximum velocity*. Substituting the value of u_{max} from eqn. (7.99), we have

$$\bar{u} = \frac{1}{8\mu} \left[-\frac{\partial p}{\partial x} \right] R^2 \qquad \text{...(7.102)}$$

The pressure gradient $\frac{\partial p}{\partial x}$ is usually expressed in terms of a friction factor, f, defined as

$$-\frac{\partial p}{\partial x} = \frac{f}{D}\frac{\rho \bar{u}^2}{2} \quad ...(7.103)$$

where, $\frac{\rho \bar{u}^2}{2}$ is dynamic pressure of the mean flow and D is the tube diameter.

From eqns. (7.102) and (7.103), we get the friction factor, as a simple function of Reynolds number,

$$f = \frac{64}{(\rho D\bar{u}/\mu)} = \frac{64}{Re} \quad ...(7.104)$$

which is valid for, laminar tube flow, $Re < 2300$

Further, eqn. (7.102) can be written as,

$$-\partial p = \frac{8\mu\bar{u}}{R^2} \cdot \partial x$$

The pressure difference between two sections 1 and 2 at distances, x_1 and x_2 (see Fig. 7.13), is given by

$$-\int_{p_1}^{p_2} \partial p = \frac{8\mu\,\bar{u}}{R^2}\int_{x_1}^{x_2} \partial x$$

or,
$$(p_1 - p_2) = \frac{8\,\mu\bar{u}}{R^2}(x_2 - x_1) = \frac{8\mu\bar{u}\,L}{R^2} = \frac{32\,\mu\bar{u}\,L}{D^2} = \frac{128\,\mu\,Q\,L}{\pi D^4}$$

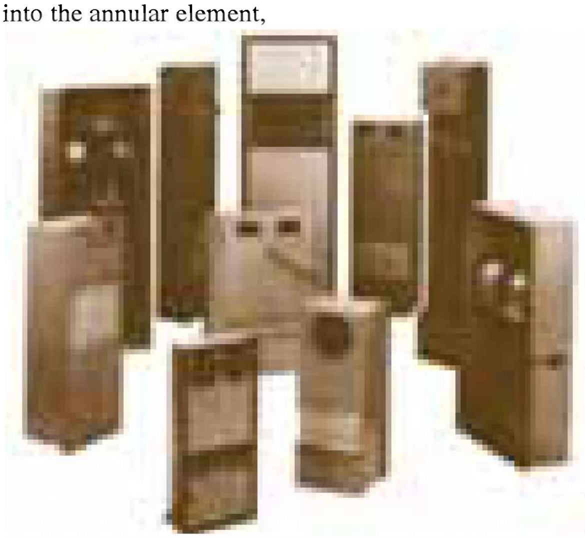
or,
$$\frac{p_1 - p_2}{w}\,(= h_L) = \frac{128\,\mu\,Q\,L}{w\pi D^4} \quad ...(7.105)$$

Obviously the head loss h_L over a length of pipe varies directly as the first power of the rate of discharge Q and inversely as the fourth power of the pipe diameter.

7.2.3. TEMPERATURE DISTRIBUTION

In order to estimate the distribution of temperature let us consider the flow of heat through an elementary ring of thickness dr and length dx as shown in Fig. 7.15. Considering the radial conduction (neglecting axial conduction) and axial enthalpy transport in the *annular element*, we have:

Heat conducted into the annular element,



Heat exchangers combine powerful forced convection cooling with the advantages of a closed loop system for contaminated environments.

$$Q_r = -k\,(2\pi r.dx)\,\frac{\partial t}{\partial r}$$

Heat conducted out of the annular element,

$$dQ_{r+dr} = -k\left[2\pi\,(r+dr)\,dx\,\frac{\partial}{\partial x}\left(t + \frac{\partial t}{\partial r}dr\right)\right]$$

Net heat convected out of the annular element,

$$dQ_{conv.} = \rho\,(2\pi r dr)\,u\,c_p\,\frac{\partial t}{\partial r}dx$$

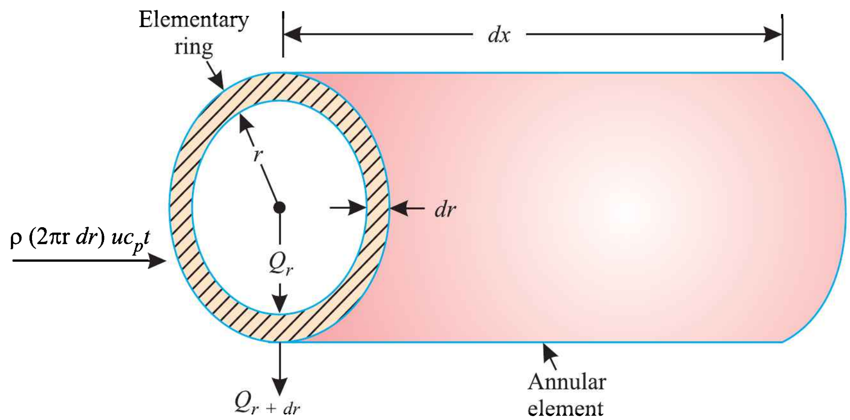


Fig. 7.15. Analysis of energy in the tube flow.

Considering energy balance on the annular element, we obtain

$$(\text{Heat conducted in})_{net} = (\text{Heat convected out})_{net}$$

$$dQ_r - dQ_{r+dr} = (dQ_{conv.})_{net}$$

$$-k\,(2\pi r.dx)\frac{\partial t}{\partial r} - \left[-k\left\{2\pi\,(r+dr)\,dx\frac{\partial}{\partial r}.\left(t+\frac{\partial t}{\partial r}.dr\right)\right\}\right] = \rho\,(2\pi r dr) u\,c_p\frac{\partial t}{\partial x}dx$$

$$-k\,(2\pi r.dx)\frac{\partial t}{\partial r} + k\,\{2\pi(r+dr)\}\,dx\left[\frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial r^2}dr\right] = \rho\,(2\pi r.dr) u\,c_p\frac{\partial t}{\partial x}dx$$

$$-k\,(2\pi r.dx)\frac{\partial t}{\partial r} + k\left(2\pi r\,dx.\frac{\partial t}{\partial r}\right) + k\left(2\pi r dx.\frac{\partial^2 t}{\partial r^2}\,dr\right) + k\,(2\pi\,dr\,dx)\frac{\partial t}{\partial r}$$

$$+\,k\left(2\pi dr\,dx.\frac{\partial t^2}{\partial r^2}dr\right) = \rho\,(2\pi r.dr) u\,c_p\frac{\partial t}{\partial x}dx$$

Neglecting second order terms, we get

$$k\left(\frac{\partial t}{\partial r} + r\frac{\partial^2 t}{\partial r^2}\right)dx.dr = \rho\,r\,u\,c_p\,\frac{\partial t}{\partial x}dx.dr$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) = u\frac{\rho c_p}{k}\frac{\partial t}{\partial x}$$

or,
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) = \frac{u}{\alpha}\frac{\partial t}{\partial x} \qquad ...(7.106)$$

Inserting the value of u from eqn. (7.100), we get,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial t}{\partial x}\,.\,u_{max}\left(1 - \frac{r^2}{R^2}\right) \qquad ...(7.107)$$

or, $$\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) = \frac{u_{max}}{\alpha}.\frac{\partial t}{\partial x}.\left(r - \frac{r^3}{R^2}\right) \quad ...(7.108)$$

Let us consider the case of uniform heat flux along the wall, where we can take $\frac{\partial t}{\partial x}$ as a constant. Integrating eqn. (7.108) we have

$$r\frac{\partial t}{\partial r} = \frac{1}{\alpha}\frac{\partial t}{\partial x}u_{max}.\left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right) + C_1$$

or, $$\frac{\partial t}{\partial r} = \frac{1}{\alpha}\frac{\partial t}{\partial x}u_{max}.\left(\frac{r}{2} - \frac{r^3}{4R^2}\right) + \frac{C_1}{r}$$

Integrating again, we have

$$t = \frac{1}{\alpha}\frac{\partial t}{\partial x}u_{max}\left(\frac{r^2}{4} - \frac{r^4}{16R^2}\right) + C_1\, ln\,(r) + C_2 \quad ...(7.109)$$

(where C_1 and C_2 are the constants of integration).

The boundary conditions are :

At $r = 0$, $\frac{\partial t}{\partial r} = 0$

At $r = R$, $t = t_s$

Applying the above boundary conditons, we get

$C_1 = 0$, $C_2 = t_s - \frac{1}{\alpha}\frac{\partial t}{\partial x}u_{max}\frac{3R^2}{16}$

Substituting the values of C_1 and C_2 in eqn. (7.109), we have

$$t = \frac{1}{\alpha}\frac{\partial t}{\partial x}u_{max}\left(\frac{r^2}{4} - \frac{r^4}{16R^2}\right) + \left[t_s - \frac{1}{\alpha}\frac{\partial t}{\partial r}u_{max}\frac{3R^2}{16}\right]$$

or, $$t_s - t = \frac{u_{max}}{\alpha}.\frac{\partial t}{\partial x}\left[\frac{3R^2}{16} - \frac{r^2}{4} + \frac{r^4}{16R^2}\right] \quad ...(7.110)$$

For determining the *heat transfer coefficient* for *fully developed pipe flow*, it is imperative to define a characteristic temperature of the fluid. It is the *bulk temperature* (t_b) or the *mixing up temperature* of the fluid which is an average taken so as to yield the total energy carried by the fluid and is defined as the *ratio of flux of enthalpy at a cross-section to the product of the mass flow rate and the specific heat of the fluid.* Thus,

$$t_b = \frac{\int_0^R \rho(2\pi r.dr)u\,c_p\,t}{\int_0^R \rho\,(2\pi r.dr)u\,c_p} \quad ...(7.111)$$

For an incompressible fluid having constant density and specific heat

$$t_b = \frac{\int_0^R u\,t\,r\,dr}{\int_0^R u\,r\,dr} \quad ...(7.112)$$

The average/mean velocity ($\bar{u}$) also known as the *bulk mean velocity* is calculated from the following definition :

$$\bar{u} = \frac{1}{\pi R^2}\int_0^R 2\pi r.dr.u$$

or, $$\bar{u} = \frac{2}{R^2}\int_0^R ur.dr$$

Substituting this value of u in eqn. (7.112), we get

$$t_b = \frac{2}{\bar{u}R^2}\int_0^R u\,t\,r.dr \qquad ...(7.113)$$

Substituting the value of u from eqns. 7.100 and 7.101 and that of t from eqn. (7.110), we get

$$t_b = \frac{2}{\bar{u}R^2}\int_0^R 2\bar{u}\left[1-\frac{r^2}{R^2}\right]\left[t_s - \frac{u_{max}}{\alpha}.\frac{\partial t}{\partial x}\left\{\frac{3R^2}{16}-\frac{r^2}{4}+\frac{r^4}{16R^2}\right\}\right]rdr$$

$$= \frac{4}{R^2}\int_0^R\left[t_s\left(r-\frac{r^3}{R^2}\right)-\frac{u_{max}}{\alpha}.\frac{\partial t}{\partial x}\left\{\frac{3R^2 r}{16}-\frac{7}{16}r^3+\frac{5}{16}\frac{r^5}{R^2}-\frac{r^7}{16R^4}\right\}dr\right]$$

$$= \frac{4}{R^2}\left[t_s\left\{\frac{r^2}{2}-\frac{r^4}{R^2}\right\}\right]_0^R - \frac{4u_{max}}{\alpha R^2}.\frac{\partial t}{\partial x}\left[\frac{3R^2r^2}{32}-\frac{7}{16}\times\frac{r^4}{4}+\frac{5}{16}\times\frac{r^6}{6R^2}-\frac{r^8}{8\times 16R^4}\right]_0^R$$

$$= \frac{4}{R^2}\left[t_s\left\{\frac{R^2}{2}-\frac{R^4}{4}\right\}\right]^R - \frac{4u_{max}}{\alpha R^2}.\frac{\partial t}{\partial x}\left[\frac{3R^4}{32}-\frac{7R^4}{64}+\frac{5R^4}{96}-\frac{R^4}{128}\right]$$

$$= \frac{4}{R^2}\left(t_s\times\frac{R^2}{4}\right) - \frac{4u_{max}}{\alpha R^2}.\frac{\partial t}{\partial x}\times\frac{11}{96}R^4$$

or, $$t_b = t_s - \frac{11}{96}\frac{u_{max}}{\alpha}R^2\frac{\partial t}{\partial x} \qquad ...(7.114)$$

The *heat transfer coefficient* is calculated from the relation

$$h = \frac{Q}{A(t_s - t_b)} = \frac{kA\left(\frac{\partial t}{\partial r}\right)_{r=R}}{A(t_s - t_b)}$$

Patented forced convection system keeps the toaster cool and extends the life of critical components.

From eqn. (7.110), we have

$$\left(\frac{\partial t}{\partial r}\right)_{r=R} = -\frac{u_{max}}{\alpha}.\frac{\partial t}{\partial x}\left(-\frac{R}{2}+\frac{R}{4}\right)$$

$$\left(\frac{\partial t}{\partial r}\right)_{r=R} = \frac{u_{max}R}{4\alpha}.\frac{\partial t}{\partial x} \qquad ...(7.115)$$

$$\therefore \qquad h = \frac{k \times \frac{u_{max}R}{4\alpha}.\frac{\partial t}{\partial x}}{\frac{11}{96}\frac{u_{max}}{\alpha}.R^2.\frac{\partial t}{\partial x}} = \frac{24k}{11R} = \frac{48k}{11D} \qquad ...(7.116)$$

where D is the diameter of the tube.

The Nusselt number is given by

$$Nu = \frac{hD}{k} = \frac{48k}{11D} \times \frac{D}{k} = \frac{48}{11} = 4.364 \qquad ...(7.117)$$

This shows that the Nusselt number for the fully developed laminar tube flow is *constant* and is *independent of the Reynolds number and Prandtl number.*

The first analytical solution for laminar flow for *constant wall temperature* was formulated by Graetz in 1885. Since $\frac{\partial t}{\partial x}$ is not constant, therefore, the analysis of constant wall temperature is quite cumbersome. The final result comes out to be

$$Nu = \frac{hD}{k} = 3.65 \qquad ...(7.118)$$

Example 7.25. *For laminar flow in a circular tube of 120 mm radius, the velocity and temperature distribution are given by the relations :*

$$u = (2.7r - 3.2\,r^2)\ ; \qquad t = 85\,(1 - 2.2r)°C$$

where the distance r is measured from the tube surface. Calculate the following :

(*i*) *The average velocity and the mean bulk temperature of the fluid;*

(*ii*) *The heat transfer coefficient based on the bulk mean temperature if the tube surface is maintained at a constant uniform temperature of 90°C and there occurs a heat loss of 1000 kJ/h per metre length of the tube.*

Solution. *Given :* $u = (2.7\,r - 3.2\,r^2)$ ... Velocity distribution

$t = 85\,(1 - 2.2\,r)°C$ ... Temperature distribution.

(*i*) Average velocity ($\bar{u}$) and mean bulk temperature (t_b) :

The average velocity is obtained by equating the volumetric flow to the integrated flow through an elementary ring of radius r and thickness dr.

i.e.,

$$\bar{u}\,\pi R^2 = \int_0^R u\,(2\pi r)\,dr$$

$$\bar{u} = \frac{2}{R^2}\int_0^R u\,r\,dr \qquad ...(i)$$

$$= \frac{2}{R^2}\int_0^R (2.7r - 3.2r^2)\,rdr$$

$$= \frac{2}{R^2}\left[2.7 \times \frac{r^3}{3} - 3.2\,\frac{r^4}{4}\right]_0^R$$

or, $$\bar{u} = \frac{2}{R^2}[0.9\,R^3 - 0.8\,R^4] = 1.8\,R - 1.6\,R^2$$

Substituting $R = 0.12$ m, we have

$$\bar{u} = 1.8 \times 0.12 - 1.6 \times 0.12^2 = \mathbf{0.193\ m/s} \quad \textbf{(Ans.)}$$

The mean bulk temperature is given by

$$t_b = \frac{\int_0^R u\,t\,r\,dr}{\int_0^R u\,r\,dr}$$

Now, $$\int_0^R u\,t\,r\,dr = \int_0^R (2.7\,r - 3.2\,r^2) \times 85\,(1 - 2.2\,r)\,r\,dr$$

$$= 85\int_0^R (2.7\,r^2 - 3.2\,r^3)(1 - 2.2\,r)\,dr$$

$$= 85\int_0^R (2.7\,r^2 - 5.94\,r^3 - 3.2\,r^3 + 7.04\,r^4)\,dr$$

$$= 85\left[2.7 \times \frac{r^3}{3} - 9.14 \times \frac{r^4}{4} + 7.04 \times \frac{r^5}{5}\right]_0^R$$

$$= 85\,(0.9\,R^3 - 2.285\,R^4 + 1.408\,R^5)$$

$\therefore$ $$t_b = \frac{85\,(0.9\,R^3 - 2.285\,R^4 + 1.408\,R^5)}{(\bar{u}\,R^2/2)}$$

$$\left[\because \int_0^R ur\,dx = \frac{\bar{u}\,R^2}{2}\right.$$ ...From eqn. (*i*)]

or, $$t_b = \frac{170\,(0.9\,R - 2.285\,R^2 + 1.408\,R^3)}{\bar{u}}$$

Substituting, $R = 0.12$ m and $\bar{u} = 0.193$ m/s, we get

$$t_b = \frac{170\,(0.9 \times 0.12 - 2.285 \times 0.12^2 + 1.408 \times 0.12^3)}{0.193} = \mathbf{68.29°C} \quad \textbf{(Ans.)}$$

(*ii*) Heat transfer coefficient, h :

$$Q = hA\,(t_s - t_b)$$

where, $$Q = 1000 \text{ kJ/h per metre } = \frac{1000 \times 1000}{3600} = 277.77 \text{ J/s}\ ;\ t_s = 90°C$$ ...Given

$\therefore$ $$277.77 = h \times (2\pi \times 0.12 \times 1)\,(90 - 68.29)$$

or, $$h = \frac{277.77}{(2\pi \times 0.12 \times 1)\,(90 - 68.29)} = \mathbf{16.97\ W/m^2\,°C} \quad \textbf{(Ans.)}$$

Example 7.26. *Lubricating oil at a temperature of 60°C enters 1 cm diameter tube with a velocity of 3 m/s. The tube surface is maintained at 40°C. Assuming that the oil has the following average properties calculate the tube length required to cool the oil to 45°C.*

$$\rho = 865 \text{ kg/m}^3;\ k = 0.14 \text{ W/m K};\ c_p = 1.78 \text{ kJ/kg°C}.$$

Assume flow to be laminar (and fully developed)

$$\overline{Nu} = 3.657$$ **(AMIE Summer, 1997)**

Solution. *Given* : $t_i = 60°\,C$, $t_o = 45°C$; $D = 1\text{cm} = 0.01\text{m}$; $U = 3$ m/s, $t_s = 40°C$;

$\rho = 865$ kg/m^3; $k = 0.14$ W/m K; $c_p = 1.78$ kJ/kg°C.

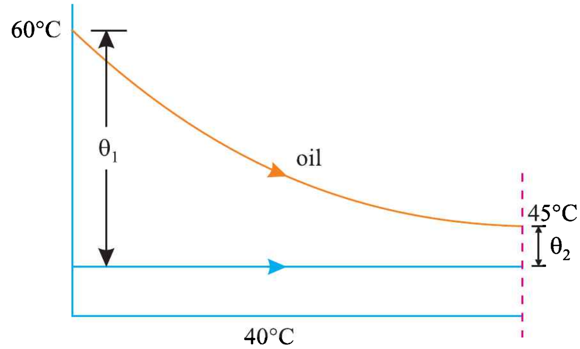


Fig. 7.16.

Length required, L :

$$Q = m\,c_p\,(t_i - t_0)$$
$$= (\rho A_f U)\,c_p\,(t_i - t_0)$$

(where U = average velocity, A_f = flow area)

$$= \left(\rho \frac{\pi}{4} D^2 U\right) c_p\,(t_i - t_0)$$

$$= (865 \times \frac{\pi}{4} \times 0.01^2 \times 3) \times 1.78 \times 10^3\,(60 - 45) = 5441.7\text{ W}$$

Also, $Q = \bar{h} A \theta_m$

where, A = heat transfer area $= \pi DL$, and

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} = \frac{(60-40)-(45-40)}{ln\left[\frac{(60-40)}{(45-40)}\right]} = \frac{15}{1.386} = 10.82°C$$

$$\overline{Nu} = \frac{\bar{h} D}{k} = 3.657 \qquad \text{...(Given)}$$

$$\bar{h} = \frac{3.657\,k}{D} = \frac{3.657 \times 0.140}{0.01} = 51.2\ \text{W/m}^2\text{K}$$

Now, $Q = 5441.7 = 51.2 \times \pi DL \times 10.82$

$\therefore$ $$L = \frac{5441.7}{51.2 \times \pi \times 0.01 \times 10.82} = \mathbf{312.7\ m} \quad \textbf{(Ans.)}$$

Example 7.27. *When 0.5 kg of water per minute is passed through a tube of 20 mm diameter, it is found to be heated from 20°C to 50°C. The heating is accomplished by condensing steam on the surface of the tube and subsequently the surface temperature of the tube is maintained at 85°C.Determine the length of the tube required for fully developed flow.*

Take the thermo-physical properties of water at 60°C as :

ρ = 983.2 kg/m^3, c_p = 4.178 kJ/kgK, k = 0.659 W/m°C, ν = 0.478 × 10^{-6} m^2/s

Solution. *Given :* m = 0.5 kg/min, D = 20 mm = 0.02 m, t_i = 20°C, t_o = 50°C

Length of the tube required for fully developed flow, L :

The mean film temperature, $t_f = \frac{1}{2}\left(85 + \frac{20+50}{2}\right) = 60°C$

Let us first determine the type of the flow.

$$m = \rho A \bar{u} = 983.2 \times \frac{\pi}{4} \times (0.02)^2 \times \bar{u} = \frac{0.5}{60}\ (\text{kg/s})$$