12.3. SHAPE FACTOR ALGEBRA AND SALIENT FEATURES OF THE SHAPE FACTOR

In order to compute the shape factor for certain geometric arrangements for which shape factors or equations are not available, the concept of shape factor as fraction of intercepted energy, and reciprocity theorem can be used. The shape factors for these geometries can be derived in terms of *known shape factors of other geometries. The interrelation between various factors is called* **shape factor algebra.**

For the calculation of shape factors for specific geometries and for the analysis of radiant heat exchange between surfaces, the following facts and properties will be useful:

1. The shape factor is purely a function of geometric parameters only.
2. When two bodies are exchanging radiant energy with each other, the shape factor relation is given by the eqn. (12.12) *i.e.*,

$$A_1 F_{1-2} = A_2 F_{2-1}$$

In general, $A_i F_{i-j} = A_j F_{j-I}$ ...(Reciprocity theorem)

This reciprocal relation is particularly useful when one of the shape factors is *unity.*

3. When all the radiation emanating from a *convex surface* 1 is intercepted by the enclosing surface 2, the *shape factor of convex surface with respect to the enclosure F_{I-2} is unity.* Then in conformity with reciprocity theorem, the shape factor F_{2-1} is merely the ratio of areas.

i.e., when surface A_1 is *entirely convex*, say a sphere, completely enclosed by A_2, then according to reciprocity relation, we have

$$A_1 F_{1-2} = A_2 F_{2-1} \quad \text{and} \quad A_1 = A_2 F_{2-1}$$

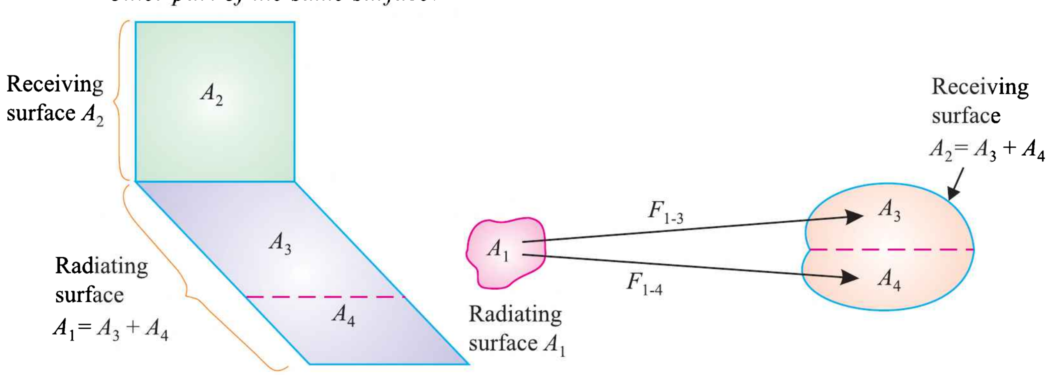
($\because$ $F_{1-2} = 1$, as surface 1 completely sees surface 2)

or $F_{2-1} = \dfrac{A_1}{A_2}$ (*i.e.*, ratio of areas), and $F_{2-1} + F_{2-2} = 1$

In this case, the black body radiation exchange is

$$Q_{12} = A_1 \sigma (T_1^4 - T_2^4)$$

4. A *concave surface* has a shape factor with itself because the radiant energy coming out from one part of the surface is intercepted by the another part of the same surface. *The shape factor of a surface with respect to itself is F_{I-I}.*
5. For *a flat or convex surface, the shape factor with respect to itself is zero* (*i.e.*, $F_{1-1} = 0$). This is due to the fact that for any part of flat or convex surface, one *cannot see/view any other part of the same surface.*



(*a*) (*b*)

Fig. 12.5. Relation between shape factors.

6. If two surfaces A_1 and A_2 are parallel and large, radiation occurs across the gap between them so that $A_1 = A_2$ and all radiation emitted by one falls on the other; then

$$F_{1-2} = F_{2-1} = 1$$

7. If one of the two surfaces (say A_i) is divided into sub-areas A_{i1}, A_{i2}, ... A_{in}, then

$$A_i F_{i-j} = \sum A_{in} F_{in-j} \quad ...(12.15)$$

Refer to Fig. 12.5 (a) : Radiating surface A_1 has been split up into areas A_3 and A_4; we have

$$A_1 F_{1-2} = A_3 F_{3-2} + A_4 F_{4-2}$$

Evidently, $F_{1-2} \neq F_{3-2} + F_{4-2}$

Thus if the radiant surface is subdivided, the shape factor for that surface with respect to the receiving surface is *not equal to the sum* of the individual shape factors.

Refer to Fig. 12.5 (b): Receiving surface A_2 has been divided into subareas A_3 and A_4; we have

$$A_1 F_{1-2} = A_1 F_{1-3} + A_1 F_{1-4}$$

or $F_{1-2} = F_{1-3} + F_{1-4}$

Obviously the shape factor from a radiating surface to a subdivided receiving surface is simply the *sum of individual shape factors*.

8. Let us now take the case of an enclosure in which one surface is exchanging radiation with all the other surfaces in the enclosure *including itself*, if it *happens to be a concave surface*; this is because a concave surface can see/view another part of it (the shape factor of a convex surface with its enclosure is *always unity* because all the heat radiated from a convex surface is intercepted by its enclosure but *not vice- versa*).

If the enclosure comprises *n* surfaces, then the energy radiated from one surface is always intercepted by the other $(n - 1)$ surfaces, and the surface itself if it is a concave one. This is called **principle of conservation.**

Heat transfer equipment

$$F_{1-1} + F_{1-2} + F_{1-3} + \ldots\ldots + F_{1-n} = 1 = \sum_{i=1}^{n} F_{1-i} \qquad \ldots(12.16)$$

$$F_{2-1} + F_{2-2} + F_{2-3} + \ldots\ldots + F_{2-n} = 1$$

$$F_{n-1} + F_{n-2} + F_{n-3} + \ldots\ldots + F_{n-n} = 1$$

In general $\sum_{j=1}^{n} F_{i-j} = 1$ for $i = 1, 2, 3, \ldots\ldots\ldots n$

Example 12.1. *Assuming the sun to radiate as a black body, calculate its temperature from the data given below:*

The average radiant energy flux incident upon the earth's atmosphere (solar constant) = 1380 W/m²

Radius of the sun = 7.0 × 10⁸ m

Distance between the sun and the earth = 15 × 10¹⁰ m

Solution. *Given:* Solar constant = 1380 W/m²; r_s (radius of the sun) = 7×10^8 m

r (distance between the sun and the earth) = 15×10^{10} m

Surface temperature of the sun, *T* :

Refer Fig. 12.6. The heat flow from small area dA_1 (on the surface of sun) to the small area dA_2 (on the surface of the earth), is given by,

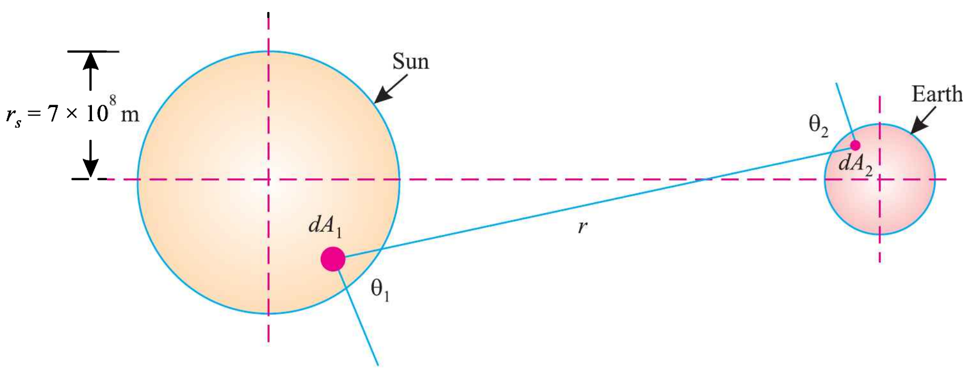


Fig. 12.6

$$dQ_{1-2} = \frac{I_{b1} \cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{r^2} \qquad \ldots[\text{Eqn. } (12.2)]$$

or, $$\frac{dQ_{1-2}}{dA_2 \cos\theta_2} = \frac{E_b}{\pi r^2} dA_1 \cos\theta_1 \qquad \left(\because I_b = \frac{E_b}{\pi}\right)$$

Integrating both sides, we get

$$\int \frac{dQ_{1-2}}{dA_2 \cos\theta_2} = \frac{E_b}{\pi r^2} \int dA_1 \cos\theta_1$$

$$\text{Also, solar constant} = \int \frac{dQ_{1-2}}{dA_2 \cos\theta_2}$$

$$\therefore \qquad \text{Solar constant} = \frac{E_b}{\pi r^2} \int dA_1 \cos\theta_1$$

$\int dA_1 \cos\theta_1 = A_1 = \pi r_s^2$, here θ_1 is taken as zero because all rays from the sun falling on the earth due to extremely long distance are considered parallel to each other; therefore, cos 0° = 1.

$$\therefore \qquad \text{Solar constant} = \frac{E_b}{\pi r^2} \times \pi r_s^2 = 1380$$

But, $E_b = \sigma T^4$

$$\therefore \qquad \frac{\sigma T^4}{\pi r^2} \times \pi r_s^2 = 1380$$

or, $$\sigma T^4 = 1380 \times \left(\frac{r}{r_s}\right)^2$$

or, $$5.67 \times \left(\frac{T}{100}\right)^4 = 1380 \times \left(\frac{15 \times 10^{10}}{7 \times 10^8}\right)^2$$

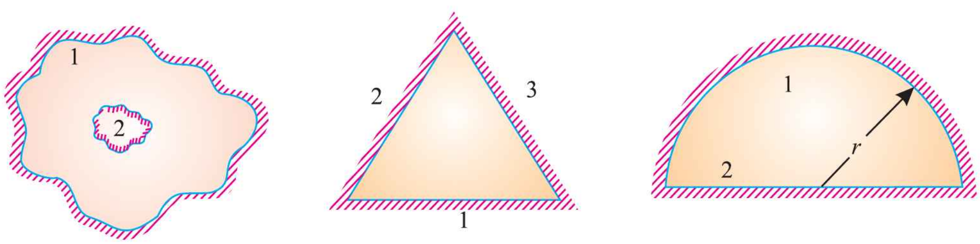
or, $$\left(\frac{T}{100}\right)^4 = 1117.58 \times 10^4 \text{ or } \frac{T}{100} = (1117.58 \times 10^4)^{1/4} = 57.82$$

or, T = **5782 K (Ans.)**

Example 12.2. *Calculate the shape factors for the configurations shown in the Fig. 12.7.*

Solution. The shape factors can be worked out by using *summation rule*, the *reciprocity theorem* and from the *inspection of geometry*.

(*i*) *A black body inside a black enclosure:*



A black body inside a black enclosure (*i*)

A tube with cross-section of an equilateral triangle (*ii*)

Hemispherical surface and a plane surface (*iii*)

Fig. 12.7

$F_{2-1} = 1$ ...Because all radiation emanating from the black surface is intercepted by the enclosing surface 1.

$F_{1-1} + F_{1-2} = 1$ ... By *summation rule* for radiation from surface 1

$A_1 F_{1-2} = A_2 F_{2-1}$ ... By *reciprocity theorem*

or, $$F_{1-2} = \frac{A_2}{A_1} F_{2-1}$$

$$\therefore \qquad F_{1-1} = 1 - F_{1-2} = 1 - \frac{A_2}{A_1} F_{2-1} = 1 - \frac{A_2}{A_1} \qquad (\because F_{2-1} = 1)$$

Hence, $$F_{1-1} = \mathbf{1 - \frac{A_2}{A_1}} \text{ (Ans.)}$$

(*ii*) *A tube with cross-section of an equilateral triangle:*

$$F_{1-1} + F_{1-2} + F_{1-3} = 1 \quad \text{... By summation rule}$$

$$F_{1-1} = 0 \quad \text{... Because the flat surface 1 \textit{cannot see itself.}}$$

$$\therefore \quad F_{1-2} + F_{1-3} = 1$$

$$F_{1-2} = F_{1-3} = \mathbf{0.5\ (Ans.)} \quad \text{... By symmetry}$$

Similarly, considering radiation from surface 2 :

$$F_{2-1} + F_{2-2} + F_{2-3} = 1$$

or, $$F_{2-1} + F_{2-3} = 1 \quad (\because F_{2-2} = 0)$$

or, $$F_{2-3} = 1 - F_{2-1}$$

$$A_1 F_{1-2} = A_2 F_{2-1} \quad \text{... By reciprocity theorem}$$

or, $$F_{2-1} = \frac{A_1}{A_2} F_{1-2} = F_{1-2} \quad (\because A_1 = A_2)$$

$$\therefore \quad F_{2-3} = 1 - F_{1-2} = 1 - 0.5 = \mathbf{0.5\ (Ans.)}$$

(*iii*) *Hemispherical surface and a plane surface:*

$$F_{1-1} + F_{1-2} = 1 \quad \text{... By summation rule}$$

$$A_1 F_{1-2} = A_2 F_{2-1} \quad \text{... By reciprocity theorem}$$

or, $$F_{1-2} = \frac{A_2}{A_1} F_{2-1}$$

But, $$F_{2-1} = 1$$ ... Because all radiation emanating from the black surface 2 are interecpted by the enclosing surface 1.

$$\therefore \quad F_{1-2} = \frac{A_2}{A_1} = \frac{\pi r^2}{2 \pi r^2} = \mathbf{0.5\ (Ans.)}$$

Thus in case of a hemispherical surface half the radiation falls on surface 2 and the other half is intercepted by the hemisphere itself.

Example 12.3. *Explain the meaning of the term geometric factor in relation to heat exchange by radiation. Derive an expression for the geometric factor F_{11} for the inside surface of a black hemispherical cavity of radius R with respect to itself.* **(U.P.S.C., 1994)**

Solution. • **Geometric factor** *is defined as the fraction of radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening reflection.*

- The *geometric factor* depends only on the *specific geometry of the emitter and the collection surfaces.*
- The geometric factor is represented by the symbol F_{i-j} which means the shape factor from a surface A_i to another surface A_j. Thus, the geometric factor F_{1-2} of surface A_1 to surface A_2 is

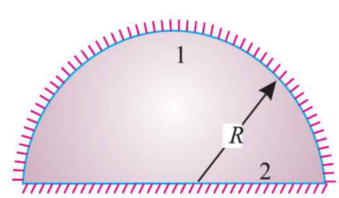


Fig. 12.8

$$F_{1-2} = \frac{\text{Direct radiation from surface 1 incident upon surface 2}}{\text{Total radiation from emitting surface}}$$

Geometric factor F_{1-1} for the inside surface of a black hemispherical cavity of radius R with respect to itself.

$$F_{1-1} = 1 - \frac{A_2}{A_1} = 1 - \frac{\pi R^2}{2\pi R^2} = 1 - \frac{1}{2} = \mathbf{0.5\ (Ans.)}$$

Example 12.4. *Derive expressions for shape factors of the cavities (each enclosed on its surface with a flat surface) shown in the Fig. 12.9. Also, calculate the net radiative heat transfer from the cavities, if h = 20 cm, d = 15 cm, temperature inside surface of each cavity = 400° C and the emissivity of each cavity surface is 0.8.*

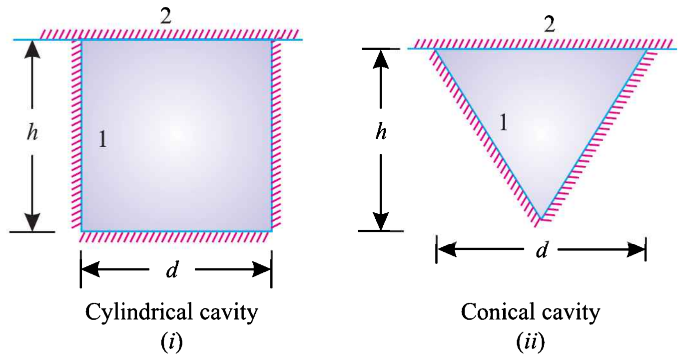


Fig. 12.9

Solution :

(i) Cylindrical cavity:

$F_{1-1} + F_{1-2} = 1$ ... By summation rule

or, $F_{1-1} = 1 - F_{1-2}$

Also, $F_{2-1} + F_{2-2} = 1$ ... By summation rule

$F_{2-2} = 0$ ... Being a flat surface (flat surface cannot see itself).

$F_{2-1} = 1$ ... Because all radiation emitted by the black surface 2 is intercepted by the enclosing surface 1.

$A_1 F_{1-2} = A_2 F_{2-1}$ ... By reciprocity theorem

or, $F_{1-2} = \frac{A_2}{A_1} F_{2-1} = \frac{A_2}{A_1}$

or, $F_{1-1} = 1 - F_{1-2} = 1 - \frac{A_2}{A_1}$

or, $$F_{1-1} = 1 - \frac{\frac{\pi}{4} d^2}{\frac{\pi}{4} d^2 + \pi dh} = 1 - \frac{d}{d + 4h} = \frac{d + 4h - d}{4h + d} = \mathbf{\frac{4h}{4h + d}} \quad \textbf{(Ans.)}$$

(*ii*) *Conical cavity:*

$F_{1-1} = 1 - \frac{A_2}{A_1}$ ... This relation (calculated above) is applicable in this case (and all such cases) also.

$$= 1 - \frac{\frac{\pi}{4} d^2}{\left[\frac{\pi d \times \text{slant height}}{2}\right]} = 1 - \frac{\frac{\pi}{4} d^2}{\frac{\pi d}{2} \times \left[\sqrt{h^2 + \left(\frac{d}{2}\right)^2}\right]}$$

or, $$F_{1-1} = 1 - \frac{\mathbf{d}}{\sqrt{\mathbf{4h^2 + d^2}}} \quad \textbf{(Ans.)}$$

Net radiative heat transfer:

The net radiative heat transfer from a cavity can be calculated by using the following formulae:

$$Q_1 = A_1 \varepsilon_1 \sigma T_1^4 \left[\frac{1 - F_{1-1}}{1 - (1 - \varepsilon_1) F_{1-1}}\right] \quad ...(12.17)$$

(*i*) *Cylindrical cavity:*

$$F_{1-1} = \frac{4h}{4h + d} = \frac{4 \times 0.2}{4 \times 0.2 + 0.15} = 0.842$$

$$Q_1 = \left[\frac{\pi}{4} \times (0.15)^2 + \pi \times 0.15 \times 0.2\right] \times 0.8 \times 5.67 \times \left[\frac{(400 + 273)}{100}\right]^4 \left[\frac{1 - 0.842}{1 - (1 - 0.8) \times 0.842}\right]$$

$$= 0.1119 \times 4.536 \times 2051.45 \times 0.19 = \mathbf{197.84\ W\ (Ans.)}$$

(*ii*) *Conical cavity:*

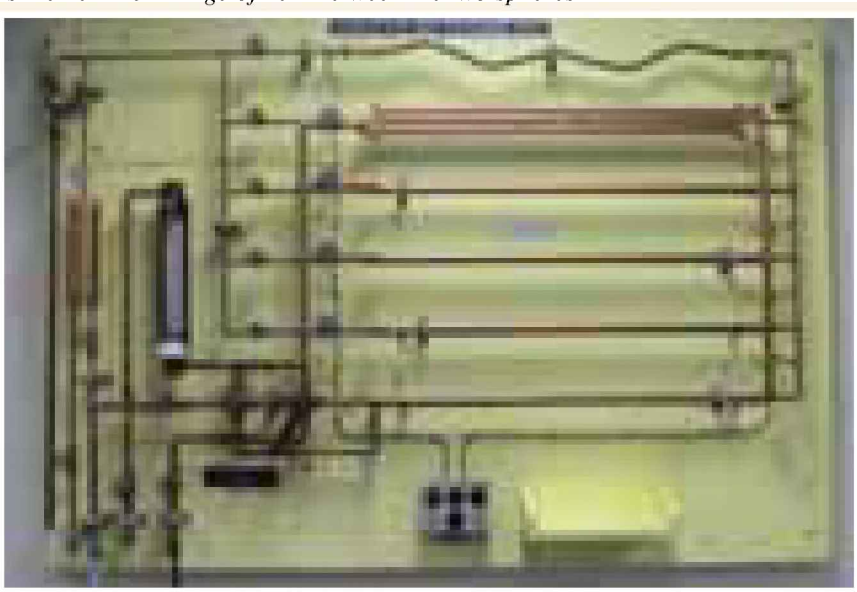
$$F_{1-1} = 1 - \frac{d}{\sqrt{4h^2 + d^2}} = 1 - \frac{0.15}{\sqrt{4 \times 0.2^2 + 0.15^2}} = 0.649$$

$$Q_1 = \frac{\pi \times 0.15}{2} \times \left[\sqrt{(0.2)^2 + \left(\frac{0.15}{2}\right)^2}\right] \times 0.8 \times 5.67 \times \left[\frac{400 + 273}{100}\right]^4 \left[\frac{1 - 0.649}{1 - (1 - 0.8 \times 0.649)}\right]$$

$$= 0.0503 \times 0.8 \times 5.67 \times 2051.45 \times 0.403 = \mathbf{188.63\ W\ (Ans.)}$$

Example 12.5. *A small sphere (outside diameter = 60 mm) with a surface temperature of 300° C is located at the geometric centre of a large sphere (inside diameter = 360 mm) with an inner surface temperature of 15° C. Calculate how much of emission from the inner surface of the large sphere is incident upon the outer surface of the small sphere; assume that both sides approach black body behaviour.*

What is the net interchange of heat between the two spheres?



Double pipe heat exchanger (steam)

Solution. *Given:* r_1 (small sphere) $= \frac{60}{2} = 30$ mm $= 0.03$ m; r_2 (large sphere) $= \frac{360}{2} = 180$ mm $= 0.18$ m.

Since all the radiation being emitted by the small sphere is incident upon and absorbed by the inner surface of the large sphere, therefore, configuration factor between 1 and 2 is $F_{1-2} = 1$.

Now, $A_1 F_{1-2} = A_2 F_{2-1}$ ... Reciprocity theorem

or, $4 \pi r_1^2 \times F_{1-2} = 4 \pi r_2^2 \times F_{2-1}$

$$\therefore \quad F_{2-1} = F_{1-2} \times \frac{4 \pi r_1^2}{4 \pi r_2^2} = 1 \times \frac{r_1^2}{r_2^2} = \left(\frac{0.03}{0.18}\right)^2 = \mathbf{0.0278\ (Ans.)}$$

Thus 2.78% of the emission from the inner surface of the large sphere is incident upon the small sphere and absorbed by it.

Also, $F_{2-1} + F_{2-2} = 1$ ... From energy balance for the large sphere.

or, $F_{2-2} = 1 - F_{2-1} = 1 - 0.0278 = 0.9722$

Thus, 97.22% of emission from the large sphere is absorbed by the inner surface of the sphere itself.

∴ The net interchange of heat between the two spheres is,

$$Q_{net} = F_{1-2} A_1 \sigma (T_1^4 - T_2^4)$$

$$= 1 \times (4\pi \times 0.03^2) \times 5.67 \left[\left(\frac{300 + 273}{100}\right)^4 - \left(\frac{15 + 273}{100}\right)^4\right]$$

$$= 0.0113 \times 5.67 \times 1009.2 = \mathbf{64.66\ W\ (Ans.)}$$

Example 12.6. *A 70 mm thick metal plate with a circular hole of 35 mm diameter along the thickness is maintained at a uniform temperature 250° C. Find the loss of energy to the surroundings at 27° C, assuming the two ends of the hole to be as parallel discs and the metallic surfaces and surroundings have black body characteristics.*

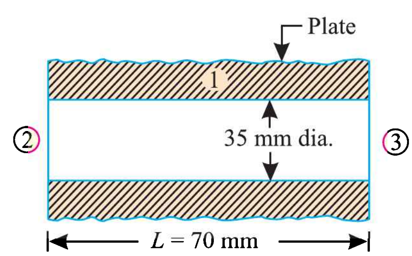


Fig. 12.10

Solution. *Given:* $r_2 = (r_3) = \frac{35}{2} = 17.5$ mm $= 0.0175$ m, $L = 70$ mm $= 0.07$ m, $T_1 = 250 + 273 = 523$ K

$T_{surr.} = 27 + 273 = 300$ K.

Refer Fig. 12.10. Let suffix 1 designate the cavity and the suffices 2 and 3 denote the two ends of the 35 mm dia. hole which are behaving as discs. Thus,

$$\frac{L}{r_2} = \frac{0.07}{0.0175} = 4; \frac{r_3}{L} = \frac{0.0175}{0.07} = 0.25$$

With reference to Fig. 12.3, the configuration factor, F_{2-3} is 0.065

Now, $F_{2-1} + F_{2-2} + F_{2-3} = 1$ ... By summation rule

But, $F_{2-2} = 0$

∴ $F_{2-1} = 1 - F_{2-3} = 1 - 0.065 = 0.935$

Also, $A_1 F_{1-2} = A_2 F_{2-1}$ ... By reciprocating theorem