This value is *indeterminate*. The value of θ_m for such a case can be found by applying *L*' Hospital's rule :

$$\lim_{\theta_2 \to \theta_1} \frac{\theta_2 - \theta_1}{ln\,(\theta_2/\theta_1)} = \lim_{(\theta_2/\theta_1) \to 1} \frac{\theta_1 \left[\frac{\theta_2}{\theta_1} - 1\right]}{ln\,(\theta_2/\theta_1)}$$

Let $(\theta_2/\theta_1) = R$. Therefore, the above expresion can be written as

$$\lim_{R \to 1} \frac{\theta\,(R - 1)}{ln\,(R)}$$

Differentiating the numerator and denominator with respect to R and taking limits, we get

$$\lim_{(R \to 1)} \frac{\theta}{(1/R)} = \theta$$

Hence, when $\theta_1 = \theta_2$ eqn. (10.3) becomes

$$Q = UA\,\theta$$

θ_m *(LMTD) for a counter-flow unit is always greater than that for a parallel flow unit*; hence counter-flow heat exchanger can transfer *more* heat than parallel-flow one; in other words a counter-flow heat exchanger needs a *smaller heating surface for the same rate of heat transfer.* For this reason, the *counter-flow arrangement is usually used.*

When the temperature variations of the fluids are relatively small, then temperature variation curves are approximately straight lines and adequately accurate results are obtained by taking the *arithmatic mean temperature difference* (*AMTD*).

$$AMTD = \frac{t_{h1} + t_{h2}}{2} - \frac{t_{c1} + t_{c2}}{2} = \frac{(t_{h1} - tc_1) + (t_{h2} - tc_2)}{2} = \frac{\theta_1 + \theta_2}{2} \qquad ...(10.17)$$

However, practical considerations suggest that the logarithmic mean temperature difference (θ_m) should be invariably used when $\frac{\theta_1}{\theta_2} > 1.7$.

10.5. OVERALL HEAT TRANSFER COEFFICIENT

In a heat exchanger in which two fluids are separated by a **plane wall** as shown in the Fig. 10.11, the overall heat transfer coefficient is given by

$$U = \frac{1}{\frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_o}} \qquad ...(10.18)$$

If the fluids are separated by a **tubewall** as shown in Fig. 10.12 the overall heat transfer coefficient is given by,

Inner surface :

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i}{k} \ln\,(r_o/r_1) + (r_i/r_o) \times \frac{1}{h_o}} \qquad ...(10.19)$$

Outer surface :

$$U_0 = \frac{1}{(r_o/r_i)\frac{1}{h_c} + \frac{r_o}{k} \ln\,(r_o/r_i) + \frac{1}{h_o}} \qquad ...(10.20)$$

where,

$$U_i A_i = U_o A_o \qquad ...(10.21)$$

$$A_i = 2\,\pi r_i\,L; \qquad A_o = 2\pi r_o\,L$$

It may be noted that eqns. (10.20) and (10.21) are valid only for *clean and uncorroded surface*.

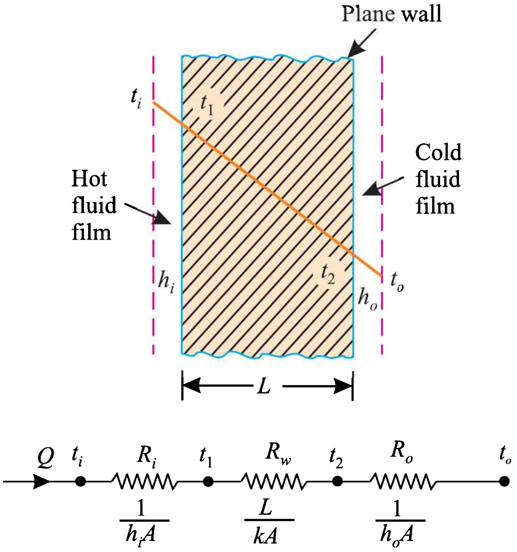

Fig. 10.11. Overall heat transfer coefficient of two fluids separated by a plane wall.

Consideration of fouling or scaling. In a heat exchanger, during normal operation the tube surface gets covered by deposits of ash, soot, dirt and scale etc. This *phenomenon of rust formation and deposition of fluid impurities is called* **fouling.** Due to these surface deposits the thermal resistance is increased and eventually the performance of the heat exchanger lowers. Since it is difficult to ascertain the thickness and thermal conductivity of the scale deposits, the effect of scale on heat flow is considered by specifying an equivalent *scale heat transfer coefficient* h_s. If h_{si} and h_{so} be the heat transfer coefficients for the scale deposited on the inside and outside surfaces respectively, then the thermal resistances to scale formation on the inside surface (R_{si}) and outside surface (R_{so}) are given by

$$R_{si} = \frac{1}{A_i\, h_{si}} \qquad \text{...(10.22)}$$

$$R_{so} = \frac{1}{A_o\, h_{so}} \qquad \text{...(10.23)}$$

The *reciprocal of scale heat transfer coefficient*, h_s is called the *fouling factor*, R_f. Thus

$$R_f = \frac{1}{h_s} \text{ m}^2\text{°C/W} \qquad \text{...(10.24)}$$

Fouling factors are determined experimentally by testing the heat exchanger in both the clean and dirty conditions. The fouling factor, R_f is thus defined as :

$$R_f \left(= \frac{1}{h_s} \right) = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \qquad \text{...(10.25)}$$

Some typical values (approximate) of R_f are given in table 10.1. Representative values of overall heat transfer coefficient, U is given in table 10.2.

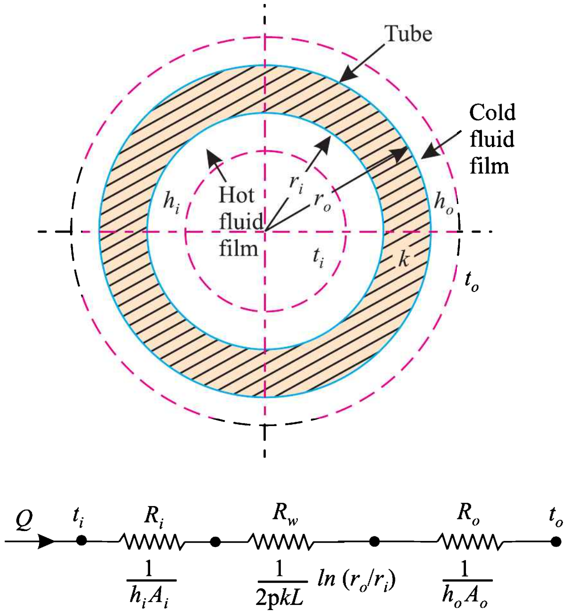

Fig. 10.12. Overall heat transfer coefficient of two fluids flowing inside and outside a tube.

The heat transfer, considering the thermal resistance due to scale formation, is given by :

$$Q = \frac{(t_i - t_o)}{\frac{1}{A_i\, h_i} + \frac{1}{A_i\, h_{si}} + \frac{1}{2\pi Lk} \ln (r_o/r_i) + \frac{1}{A_o\, h_{so}} + \frac{1}{A_o\, h_o}} \qquad \text{...(10.26)}$$

The overall heat transfer coefficients, U based on the inner and outer surfaces of the inner tube are given by,

$$U_i = \frac{1}{\frac{1}{h_i} + R_{f_i} + \frac{r_i}{k} \, ln \, (r_o/r_i) + (r_i/r_o) \, R_{f_o} + (r_i/r_o) \, \frac{1}{h_o}} \quad ...(10.27)$$

$$U_o = \frac{1}{(r_o/r_i) \, \frac{1}{h_i} + (r_o/r_i) \, R_{fi} + \frac{r_o}{k} \, ln \, (r_o/r_1) + R_{fo} + \frac{1}{h_o}} \quad ...(10.28)$$

In case the tube is thin walled and the thermal resistances due to tube wall thickness and scale formed are neglected, then the overall heat transfer coefficient based on outer surface is given by :

$$U_o = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} \quad ...(10.29)$$

When only fouling factors are neglected, we have

$$U_o = \frac{1}{(r_o/r_i) \, \frac{1}{h_i} + \frac{r_o}{k} \, ln \, (r_o/r_i) + \frac{1}{h_o}} \quad ...(10.30)$$

Points worth noting :

1. The overall heat transfer coefficient depends upon the following *factors* :
   (*i*) The flow rate,
   (*ii*) The properties of the fluid,
   (*iii*) The thickness of material,
   (*iv*) The surface condition of the tubes, and
   (*v*) The geometrical configuration of the heat exchanger.
2. The overall heat transfer coefficient U will generally decrease when any of the fluids (e.g. tars, oils or any of the gases) having low values of heat transfer coefficient, h flows on one side of the exchanger.
3. The highly conducting liquids such as water and liquid metals give much higher values of heat transfer coefficient, h and overall heat transfer coefficient, U. In case of boiling and condensation processes also, the values of U are high.
4. All the thermal resistances in the heat exchanger must be low for its efficient and effective design.

Table 10.1. Fouling factors

S.No.	Fluid	Fouling factor, $R_f = \frac{1}{h_s}$ (m^2°C/W)
1.	Sea water	0.0001 (below 50°C) 0.0002 (above 50°C)
2.	Clean river and lake water	0.0002 – 0.0006
3.	Well water	0.0004
4.	Distilled water	0.0001
5.	Treated boiler feed water	0.0001 – 0.0002
6.	Worst water used in heat exchangers	< 0.0002
7.	Fuel oil and crude oil	0.0009
8.	Industrial liquids	0.0002

9.	Transformer or lubricating oil	0.0002
10.	Engine exhaust and fuel gases	0.002
11.	Steam (non-oil bearing)	0.0001
12.	Refrigerant liquids brine or oil-bearing	0.0002

Table 10.2. Representative values of overall heat transfer coefficient (U)

S.No.	*Fluid combination*	U ($W/m^2 °C$)
1.	Water to water	850 – 1170
2.	Water to oil	110 – 350
3.	Steam condensers (water in tubes)	1000 – 6000
4.	Alcohol condensers (water in tubes)	250 – 700
5.	Feed water heaters	110 – 8500
6.	Air-condensers	350 – 780
7.	Air to various gases	60 – 550
8.	Air to heavy tars and liquids	As low as 45
9.	Air to low viscosity liquids	As high as 600
10.	Finned-tube heat echanger (water in tubes, air in cross-flow)	25 – 50

Fouling processes :

1. Precipitation or crystallization fouling.
2. Sedimentation or particulate fouling.
3. Chemical reaction fouling or polymerisation.
4. Corrosion fouling.
5. Biological fouling.
6. Freeze fouling.

Parameters affecting fouling :

- Velocity
- Temperature
- Water chemistry
- Tube material.

Prevention of fouling :

The following methods may be used to keep fouling *minimum* :

1. Design of heat exchanger.
2. Treatment of process system.
3. By using cleaning system.

Properties to be considered for selection of materials for heat exchangers :

- Physical properties
- Mechanical properties
- Climatic properties
- Chemical environment

- Quality of surface finish
- Service life
- Freedom from noise
- Reliability.

Common failures in heat exchangers :

- Chocking of tubes either expected or extraordinary.
- Excessive transfer rates in heat exchanger.
- Increasing the pump pressure to maintain throughout.
- Failure to clean tubes at regularly scheduled intervals.
- Excessive temperatures in heat exchangers.
- Lack of control of heat exchangers atmosphere to retard scaling.
- Increased product temperature over a safe design limit.
- Unexpected radiation from refractory surfaces.
- Unequal heating around the circumference or along the length of tubes.

Example 10.1. *For what value of end temperature differences ratio* $\frac{\theta_1}{\theta_2}$, *is the arithmatic mean temperature difference 5 per cent higher than the log-mean temperature difference?*

Solution. The arithmetic mean temperature difference $(\bar{\theta})$ and log-mean temperature difference (θ_m) ratio may be written as

$$\frac{\bar{\theta}}{\theta_m} = \frac{\left(\frac{\theta_1 + \theta_2}{2}\right)}{\left[\frac{\theta_1 - \theta_2}{ln\,(\theta_1/\theta_2)}\right]} = \frac{(\theta_1 + \theta_2)}{2\,(\theta_1 - \theta_2)} \times ln\,(\theta_1/\theta_2)$$

It is given that $\bar{\theta}$ is to be 5 percent higher than θ_m

$$\therefore \qquad \frac{\bar{\theta}}{\theta_m} = 1.05 = \frac{(\theta_1/\theta_2) + 1}{2\,[(\theta_1/\theta_2) - 1]}\, ln\,(\theta_1/\theta_2)$$

$$\text{or,} \qquad \frac{(\theta_1/\theta_2) + 1}{(\theta_1/\theta_2) - 1}\, ln\,(\theta_1/\theta_2) = 2 \times 1.05 = 2.1$$

By hit and trial method, we get

$$\frac{\theta_1}{\theta_2} = \mathbf{2.2\ (Ans.)}$$

Thus the simple arithmetic mean temperature difference gives results to within 5 percent when end temperature differences vary by *no more than a factor of 2.2.*

Example 10.2. (*a*) *Derive an expression for the effectiveness of a parallel flow heat exchanger in terms of the number of transfer units, NTU, and the capacity ratio* C_{min}/C_{max}.

(*b*) *In a parallel flow double-pipe heat exchanger water flows through the inner pipe and is heated from 20°C to 70°C. Oil flowing through the annulus is cooled from 200°C to 100°C. It is desired to cool the oil to a lower exit temperature by increasing the length of the heat exchanger. Determine the minimum temperature to which the oil may be cooled.* **(U.P.S.C., 1995)**

Solution. (*a*) Refer Article 10.7.

(*b*) Using subscripts *h* and *c* for oil and water respectively, we have