$$\therefore \quad L = \frac{A}{\pi d N} = \frac{24.6}{\pi \times 0.025 \times 12} \; 26.1 \text{ m}$$

$$\text{The shell length} = \frac{26.1}{8} = \mathbf{3.26 \text{ m (Ans.)}}$$

10.7. HEAT EXCHANGER EFFECTIVENESS AND NUMBER OF TRANSFER UNITS (NTU)

A heat exchanger can be designed by the *LMTD* (logarithmic mean temperature difference) when *inlet and outlet conditions are specified.* However, when the problem is to determine the inlet or exit temperatures for a particular heat exchanger, the analysis is performed more easily, by using a method based on effectiveness of the heat exchanger (concept first proposed by Nusselt) and number of transfer units (*NTU*).

The **heat exchanger effectiveness** (ε) is defined as the *ratio of actual heat transfer to the maximum possible heat transfer.* Thus

$$\varepsilon = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}} = \frac{Q}{Q_{max}} \quad ...(10.35)$$

The actual heat transfer rate Q can be determined by writing an energy balance over either side of the heat exchanger.

$$Q = \dot{m}_h \, c_{ph} \, (t_{h1} - t_{h2}) = \dot{m}_c \, c_{pc} \, (t_{c2} - t_{c1}) \quad ...(10.36)$$

The product of mass flow rate and the specific heat, as a matter of convenience, is defined as the fluid capacity rate C :

$\dot{m}_h \, c_{ph}$ = C_h = Hot fluid capacity rate

$\dot{m}_c \, c_{pc}$ = C_c = Cold fluid capcity rate

C_{min} = The minimum fluid capacity rate (C_h or C_c)

C_{max} = The maximum fluid capacity rate (C_h or C_c).

The *maximum rate of heat transfer for parallel flow or counter-flow heat exchangers would occur if the outlet temperature of the fluid with smaller value of C_h or C_c i.e., C_{min} were to be equal to the inlet temperature of the other fluid.* The maximum possible temperature change can be achieved by *only one of fluids*, depending upon their heat capacity rates. This maximum change cannot be obtained by both the fluids except in the very special case of *equal heat capacity rates*. Thus :

$$Q_{max} = C_h \, (t_{h1} - t_{c1}) \text{ or } C_c \, (t_{h1} - t_{c1})$$

Q_{max} is the *minimum* of these two values, *i.e.*,

$$Q_{max} = C_{min} \, (t_{h1} - t_{c1}) \quad ...(10.37)$$

$$\therefore \quad \varepsilon = \frac{C_h \, (t_{h1} - t_{h2})}{C_{min} \, (t_{h1} - t_{c1})} = \frac{C_c \, (t_{c2} - t_{c1})}{C_{min} \, (t_{h1} - t_{c1})} \quad ...(10.38)$$

Once the effectiveness is known, the heat transfer rate can be very easily calculated by using the equation

$$Q = \varepsilon \, C_{min} \, (t_{h1} - t_{c1}) \quad ...(10.39)$$

Number of transfer units (NTU) method :

It is obvious from Eqn. (10.38) that effectiveness ε is a function of several variables and as such it is inconvenient to combine them in a graphical or tabular form. However, by compiling a non-dimensional grouping, ε can be expressed as a function of three non-dimensional parameters. This method is known as ***NTU method***. This method/approach *facilitates the comparison between the various types of heat exchangers* which may be used for a particular application. The effectiveness expresssions for the parallel flow and counter-flow cases can be derived as follows :

(*i*) **Effectiveness for the "Parallel-flow" heat exchanger :**

Refer Fig. 10.8. The heat exchange dQ through an area dA of the heat exchanger is given by

$$dQ = U.dA\,(t_h - t_c) \quad ...(i)$$

$$= -\,\dot{m}.c_{ph}.dt_h = \dot{m}_c.c_{pc}.dt_c$$

$$= -\,C_h.dt_h = C_c.dt_c \quad ...(ii)$$

From expression (*ii*), we have

$$dt_h = \frac{-\,dQ}{C_h} \quad \text{and} \quad dt_c = \frac{dQ}{C_c}$$

$$\therefore \quad d\,(t_h - t_c) = -\,dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Substituting the value of dQ from expression (*i*) and rearranging, we get

$$\frac{d\,(t_h - t_c)}{(t_h - t_c)} = -\,U.dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Upon integration, we get

$$ln\left[\frac{(t_{h2} - t_{c2})}{(t_{h1} - t_{c1})}\right] = -\,UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$ln\left[\frac{(t_{h2} - t_{c2})}{(t_{h1} - t_{c1})}\right] = -\,\frac{UA}{C_h}\left(1 + \frac{C_h}{C_c}\right)$$

or, $$\left(\frac{t_{h2} - t_{c2}}{t_{h1} - t_{c1}}\right) = \exp\,[-\,(UA/C_h)\,\{1 + (C_h/C_c)\}] \quad ...(10..40)$$

From eqn. (10.38), we have the expressions for effectiveness

$$\varepsilon = \frac{C_h\,(t_{h1} - t_{h2})}{C_{min}\,(t_{h1} - t_{c1})} = \frac{C_c\,(t_{c2} - t_{c1})}{C_{min}\,(t_{h1} - t_{c1})}$$

Hence, $$t_{h2} = t_{h1} - \frac{\varepsilon\,C_{min}\,(t_{h1} - t_{c1})}{C_h} \quad ...(10.41)$$

$$t_{c2} = t_{c1} + \frac{\varepsilon\,C_{min}\,(t_{h1} - t_{c1})}{C_c} \quad ...(10.42)$$

Eliminating t_{h2} and t_{c2} from eqn. (10.40) with the help of eqns. (10.41) and (10.42), we get

$$\frac{1}{(t_{h1} - t_{c1})}\left[(t_{h1} - t_{c1}) - \varepsilon\,C_{min}\,(t_{h1} - t_{c1})\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\right] = \exp\,[-\,(UA/C_h)\,\{1 + C_h/C_c\}]$$

or, $$1 - \varepsilon\,C_{min}\left(\frac{1}{C_h} + \frac{1}{C_c}\right) = \exp\,[-\,(UA/C_h)\,\{1 + C_h/C_c\}]$$

or, $$\varepsilon = \frac{1 - \exp\,[-\,(UA/C_h)\,\{1 + C_h/C_c\}]}{C_{min}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)} \quad ...(10.43)$$

If $C_c > C_h$ then $C_{min} = C_h$ and $C_{max} = C_c$, hence eqn. (10.43) becomes

$$\varepsilon = \frac{1 - \exp\,[-\,(UA/C_{min})\,\{1 + C_{min}/C_{max}\}]}{1 + (C_{min}/C_{max})} \quad ..(10.44)$$

If $C_c < C_h$ then $C_{min} = C_c$ and $C_{max} = C_h$, hence eqn. (10.43) becomes

$$\varepsilon = \frac{1 - \exp\,[-\,(UA/C_{max})\,\{1 + C_{max}/C_{min}\}]}{1 + (C_{min}/C_{max})} \quad ...(10.45)$$

By rearranging eqns. (10.44) and (10.45), we get a common equation

$$\varepsilon = \frac{1 - \exp [- (UA/C_{min}) \{1 + C_{min}/C_{max})\}]}{1 + (C_{min}/C_{max})}$$

where C_{min} and C_{max} represent the smaller and larger of the two heat capacities C_c and C_h.

- The grouping of the terms $(UA)/C_{min}$ is a dimensionless expression called the number of transfer units NTU; NTU is a *measure of effectiveness of the heat exchanger*.
- C_{min}/C_{max} is the second dimensionless parameter and is called the *capacity ratio R*.
- The last dimensionless parameter is the *flow arrangement*, *i.e.*, parallel flow, counter-flow, cross-flow and so on.

Thus the effectiveness of a parallel flow heat exchanger is given by

$$\varepsilon = \frac{1 - \exp [- NTU \ \{1 + (C_{min}/C_{max})\}]}{1 + (C_{min}/C_{max})} \quad ...(10.46)$$

or,

$$\varepsilon = \frac{1 - \exp [- NTU \ (1 + R)]}{1 + R} \quad ...[10.46 \ (a)]$$

***(ii)* "Counter-flow" heat exchanger :**

Refer Fig. 10.9. The heat exchange dQ through an area dA of the heat exchanger is given by

$$dQ = U.dA \ (t_h - t_c) \quad ...(i)$$

$$= - \dot{m} \ c_{ph} \ dt_h = - \ \dot{m} \ c_{pc} \ dt_c$$

$$= - C_h \ dt_h = - C_c \ dt_c \quad ...(ii)$$

From expression (*ii*), we have

$$dt_h = - \frac{dQ}{C_h} \text{ and } dt_c = - \frac{dQ}{C_c}$$

$\therefore$

$$d \ (t_h - t_c) = - dQ \left[\frac{1}{C_h} - \frac{1}{C_c}\right] = dQ \left[\frac{1}{C_c} - \frac{1}{C_h}\right]$$

Substituting the value of dQ from expression (*i*), we get,

$$\frac{d \ (t_h - t_c)}{t_h - t_c} = U \ dA \left[\frac{1}{C_c} - \frac{1}{C_h}\right]$$

Upon integration, we get

$$ln \left[\frac{t_{h2} - t_{c1}}{t_{h1} - t_{c2}}\right] = UA \left[\frac{1}{C_c} - \frac{1}{C_h}\right]$$

or,

$$ln \left[\frac{t_{h2} - t_{c1}}{t_{h1} - t_{c2}}\right] = \frac{UA}{C_c} \left[1 - \frac{C_c}{C_h}\right]$$

or,

$$\frac{t_{h2} - t_{c1}}{t_{h1} - t_{c2}} = \exp [(UA/C_c) \{1 - (C_c/C_h)\}] \quad ...(10.47)$$

From Eqn. (10.38), we have the expressions for effectiveness,

$$\varepsilon = \frac{C_h \ (t_{h1} - t_{h2})}{C_{min} \ (t_{h1} - t_{c1})} = \frac{C_c \ (t_{c2} - t_{c1})}{C_{min} \ (t_{h1} - t_{c1})}$$

Hence,

$$t_{h2} = t_{h1} - \frac{\varepsilon \ C_{min} \ (t_{h1} - t_{c1})}{C_h} \quad ...(iii)$$

$$t_{c2} = t_{c1} + \frac{\varepsilon \ C_{min} \ (t_{h1} - t_{c1})}{C_c} \quad ...(iv)$$

Substituting these values in eqn. (10.47), we get,

$$\frac{\left[t_{h1} - \dfrac{\varepsilon\, C_{min}\,(t_{h1} - t_{c1})}{C_h}\right] - t_{c1}}{t_{h1} - \left[t_{c1} + \dfrac{\varepsilon\, C_{min}\,(t_{h1} - t_{c1})}{C_c}\right]} = \exp\,[(UA/C_c)\,\{1 - (C_c/C_h)\}]$$

$$\frac{(t_{h1} - t_{c1})\left[1 - \dfrac{\varepsilon\,.\,C_{min}}{C_h}\right]}{(t_{h1} - t_{c1})\left[1 - \dfrac{\varepsilon\,.\,C_{min}}{C_c}\right]} = \exp\,[(UA/C_c)\,\{1 - (C_c/C_h)\}]$$

or, $$\frac{1 - \dfrac{\varepsilon\,.\,C_{min}}{C_h}}{1 - \dfrac{\varepsilon\,.\,C_{min}}{C_c}} = \exp\,[(UA/C_c)\,\{1 - (C_c/C_h)\}] \qquad ...(10.48)$$

Assume $C_c < C_h$, $C_c = C_{min}$ and $C_h = C_{max}$. Substituting these values is eqn. (10.48), we get,

$$\frac{1 - \dfrac{\varepsilon\,.\,C_{min}}{C_{max}}}{1 - \dfrac{\varepsilon\,.\,C_{min}}{C_{min}}} = \exp\,[(UA/C_{min})\,\{1 - (C_{min}/C_{max})\}]$$

or, $$\frac{1 - \dfrac{\varepsilon\,.\,C_{min}}{C_{max}}}{1 - \varepsilon} = \exp\,[(UA/C_{min})\,\{1 - (C_{min}/C_{max})\}]$$

or, $$1 - \frac{\varepsilon . C_{min}}{C_{max}} = \exp\,[(UA/C_{min})\,\{1 - (C_{min}/C_{max})\}] - \exp\,[(UA/C_{min})\,\{1 - (C_{min}/C_{max})\}]\,\varepsilon$$

or, $$1 - \exp\,[(UA/C_{min})\,\{1 - (C_{min}/C_{max})\}] = \varepsilon\left[\frac{C_{min}}{C_{max}} - \exp\left\{(UA/C_{min})\,(1 - C_{min}/C_{max})\right\}\right]$$

or, $$\varepsilon = \frac{1 - \exp\,[(UA/C_{min})\,\{1 - (C_{min}/C_{max})\}]}{\dfrac{C_{min}}{C_{max}} - \exp\,[(UA/C_{min})\,\{1 - (C_{min}/C_{max})\}]}$$

$$= \frac{\exp\,[(UA/C_{min})\,\{1 - (C_{min}/C_{max})\}] - 1}{\exp\,[(UA/C_{min})\,\{1 - (C_{min}/C_{max})\}] - \dfrac{C_{min}}{C_{max}}}$$

or, $$\varepsilon = \frac{1 - \exp\,[(-\,UA/C_{min})\,\{1 - (C_{min}/C_{max})\}]}{1 - \dfrac{C_{min}}{C_{max}}\exp\,[(-\,UA/C_{min})\,\{1 - (C_{min}/C_{max})\}]} \qquad ...(10.49)$$

Since $C_{min}/C_{max} = R$ and $UA/C_{min} = NTU$, therefore,

$$\varepsilon = \frac{1 - \exp\,[-\,NTU\,(1 - R)]}{1 - R\exp\,[-\,NTU\,(1 - R)]} \qquad ...(10.50)$$

We find that effectiveness of parallel flow and counter-flow heat exchangers is given by the following expressions :

$$(\varepsilon)_{parallel\ flow} = \frac{1 - \exp\,[-\,NTU\,(1 + R)]}{1 + R} \qquad ...(1)$$

$$(\varepsilon)_{counter\ flow} = \frac{1 - \exp\,[-\,NTU\,(1 - R)]}{1 - R\exp\,[-\,NTU\,(1 - R)]} \qquad ...(2)$$

where $R = (C_{min}/C_{max})$

Let us discuss *two limiting cases* of eqns. (1) and (2)

Case I : When $R \simeq 0$ **...Condensers and evaporators (boilers)**

By using the above case, we arrive at the following common expression for *parallel flow as well as counter-flow* heat exchangers

$$\varepsilon = 1 - exp\,(-\,NTU) \qquad ...(10.51)$$

Such cases are found in *condensers and evaporators in which one fluid remains at constant temperature throughout the exchanger*. Here $C_{max} = \infty$ and thus $R = \left(\frac{C_{\min}}{C_{\max}}\right) \simeq 0$.

Obviously, no matter how large the exchanger is or how large the overall transfer coefficient is the *maximum effectiveness for parallel flow heat exchanger is 50%. For counter-flow, this limit is 100%. For this reason, a counter flow is usually more advantageous for a gas turbine heat exchangers.*

Case II : When $R = 1$ ... **Typical regenerators**

(*i*) In case of *parallel flow* heat exchanger using $R = 1$, we get

$$\varepsilon = \frac{1 - \exp\,(-\,2NTU)}{2} \qquad ...(10.52)$$

(*ii*) In case of *counter-flow* heat exchanger using $R = 1$ we get an expression for effectiveness which is indeterminate. We can find the value of ε by applying *L*, Hospital's rule :

$$\lim_{R \to 1} = \frac{1 - \exp\,[-\,NTU\,(1 - R)]}{1 - R\exp\,[-\,NTU\,(1 - R)]}$$

$$\lim_{R \to 1} = \frac{\exp\,[NTU\,(1 - R)] - 1}{\exp\,[NTU\,(1 - R)] - R}$$

Differrentiating the numerator and the denomenator with respect to *R* and taking the limit, we get,

$$\lim_{R \to 1} = \frac{\exp\,[NTU\,(1 - R)\,(-\,NTU)]}{\exp\,[NTU\,(1 - R)]\,(-\,NTU) - 1} = \frac{NTU}{1 + NTU} \qquad ...(10.53)$$

The *NTU is a measure of the heat transfer size of the exchanger; the larger the value of NTU, the closer the heat exchanger approaches its thermodynamic limit.*

The effectiveness of various types of heat exchangers in the form of graphs (prepared by Kays and London) for values of $R\left(= \frac{C_{min}}{C_{max}}\right)$ and *NTU* are shown in Fig. 10.44 to 10.49.

10.8. PRESSURE DROP AND PUMPING POWER

An important consideration in heat exchanger design, besides the heat transfer requirements, is the *pressure drop pumping cost*. The heat exchanger size can be reduced by forcing the fluid through it at higher velocities thereby increasing the overall heat transfer coefficient. But due to higher velocities there will be larger pressure drops resulting in larger pumping costs. The smaller diameter pipe, for a given flow rate, may involve less initial capital cost but definitely higher pumping costs for the life of the exchanger.

We know that, $\Delta p \propto \dot{m}^2$ ...(*i*)

where, Δp = Pressure drop of an incompressible fluid flowing through the pipes, and

$\dot{m}$ = Mass flow rate.