CHAPTER

4

Conduction– Unsteady– State (Transient)

4.1. INTRODUCTION

If the temperature of a body does not *vary with time*, it is said to be in a *steay state*. But if there is an *abrupt change* in its surface temperature, it (body) attains an equilibrium temperature or a steady state after some period. During this period the temperature *varies with time* and the body is said to be in an *unsteady or transient state.* The term transient or unsteady designates a phenomenon which is time dependent. The steady state is thus the *limit* of transient temperature distribution for large values of time.

Conduction of heat in unsteady state refers to the transient conditions wherein the heat flow and the temperature distribution at any point of the system vary continuously with time. Transient conditions occur in:

(*i*) Cooling of I.C. engines;

(*ii*) Automobile engines;

(*iii*) Heating and cooling of metal billets;

(*iv*) Cooling and freezing of food;

(*v*) Heat treatment of metals by quenching;

(*vi*) Starting and stopping of various heat exchange units in power installation;

(*vii*) Brick burning;

(*viii*) Vulcanization of rubber etc.

The temperature field in any transient problem, in general, is given by

$$t = f(x, y, z, \tau)$$

During an unsteady state the change in temperature may follow a periodic or non-periodic variation.

(*i*) **Non-periodic variation.** In a non-periodic transient state, the temperature at any point within the system varies *non-linearly with time*.

Examples:

(*i*) Heating of an ingot in a furnace;

(*ii*) Cooling of bars, blanks and metal billets in steel works, etc.

Transient conditions occur in automobile engines.

(*ii*) Periodic variation. In a periodic transient state, *temperatures undergo periodic changes* (within the system) *which are either regular or irregular but definitely 'cyclic'*. A regular periodic variation is characterised by a harmonic sinusoidal or nonsinusoidal function, and irregular periodic variations by any function which is cyclic but not necessarily harmonic.

Examples: The temperature variations in

(*i*) cylinder of an I.C. engine;

(*ii*) building during a period of 24 hours;

(*iii*) surface of earth during a period of 24 hours;

(*iv*) heat processing of regenerators (whose packings are heated alternately by fuel gases and cooled by air) etc.

The transient heat conduction problems may be solved by the following methods :

(*i*) Analytical; (*ii*) Graphical;

(*iii*) Analogical; (*iv*) Numerical.

4.2. HEAT CONDUCTION IN SOLIDS HAVING INFINITE THERMAL CONDUCTIVITY (NEGLIGIBLE INTERNAL RESISTANCE) – LUMPED PARAMETER ANALYSIS

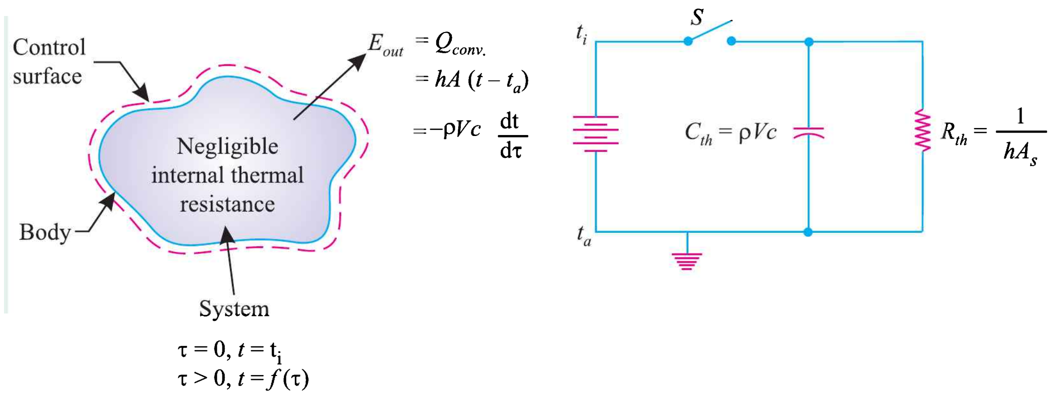
All solids have a finite thermal conductivity and there will be always a temperature gradient inside the solid whenever heat is added or removed. However, for solids of large thermal conductivity with surface areas that are large in proportion to their volume like plates and thin metallic wires, the internal resistance $\left(\frac{L}{kA}\right)$ can be assumed to be small or negligible in comparison with the convective resistance $\left(\frac{1}{hA}\right)$ at the surface. Typical examples of this type of heat flow are:

(*i*) Heat treatment of metals;

(*ii*) Time response of thermocouples and thermometers etc.

The process in which the *internal resistance is assumed negligible in comparison with its surface resistance* is called the *Newtonian heating or cooling process*. The temperature, in this process, is considered to be uniform at a given time. Such an analysis is called *Lumped parameter analysis* because the whole solid, whose energy at any time is a *function of its temperature* and total heat capacity is treated as *one lump*.

Let us consider a body whose initial temperature is t_i throughout and which is placed suddenly in ambient air or any liquid at a constant temperature t_a as shown in Fig. 4.1(*a*). The transient response of the body can be determined by *relating its rate of change of internal energy with convective exchange at the surface*. That is:



(*a*) General system for unsteady heat conduction

(*b*) Equivalent thermal circuit for lumped capacitance solid

Fig. 4.1. Lumped heat capacity system.

$$Q = -\rho Vc\,\frac{dt}{d\tau} = hA_s\,(t - t_a) \qquad ...(4.1)$$

where,
- ρ = Density of solid, kg/m^3,
- V = Volume of the body, m^3,
- c = Specific heat of body, J/kg°C,
- h = Unit surface conductance, W/m^2°C,
- t = Temperature of the body at any time, °C,
- A_s = Surface area of the body, m^2,
- t_a = Ambient temperature, °C, and
- τ = Time, s.

After rearranging the eqn. (4.1), and integrating, we get

$$\int \frac{dt}{(t - t_a)} = -\frac{hA_s}{\rho Vc}\int d\tau \qquad ...(4.2)$$

or,
$$ln\,(t - t_a) = -\frac{hA_s}{\rho Vc}\tau + C_1 \qquad ...(4.3)$$

The boundary conditions are:

At $\tau = 0$, $\quad t = t_i$ (initial surface temperature)

$\therefore \quad C_1 = ln\,(t_i - t_a)$ [From eqn. (4.3)]

Hence
$$ln\,(t - t_a) = -\frac{hA_s}{\rho Vc}\tau + ln\,(t_i - t_a)$$
[Substituting the values in eqn. (4.3)]

or,
$$\frac{t - t_a}{t_i - t_a} = \frac{\theta}{\theta_i} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] \qquad ...(4.4)$$

Following points are worth noting:

1. Eqn. (4.4) gives the temperature distribution in the body for *Newtonian heating or cooling* and it indicates that temperature rises *exponentially* with time as shown in Fig. 4.2.

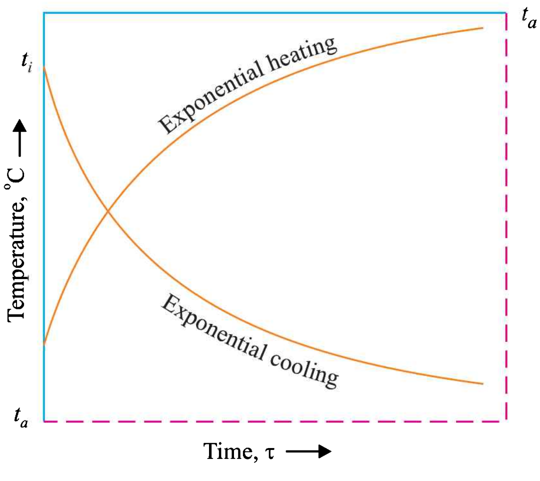


Fig. 4.2. Newtonian heating or cooling.

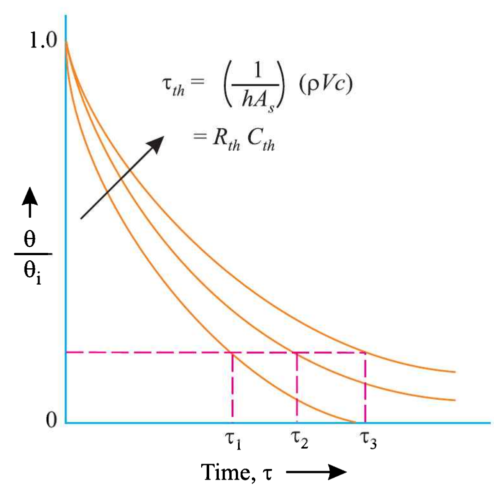


Fig. 4.3. Transient temperature response.

2. The quantity $\frac{\rho Vc}{hA_s}$ has the dimensions of time and is called **thermal time constant**, denoted by τ_{th}. Its *value is indicative of the rate of response of a system to a sudden change in its environmental temperature i.e.*, how fast a body will response to a change in the environmental temperature.

$$\tau_{th} = \left(\frac{1}{hA_s}\right)(\rho Vc) = R_{th}\ C_{th}$$

where, $R_{th} = \left(\frac{1}{hA_s}\right)$ = Resistance to convection heat transfer, and

$C_{th}\ (= \rho V_c)$ = Lumped thermal capacitance of solid.

Fig. 4.3 shows that any increase in R_{th} or C_{th} will cause a solid to respond *more slowly* to changes in its *thermal environmental* and will *increase* the time required to attain the thermal equilibrium ($\theta = 0$).

Fig. 4.1(*b*) shows an analogus electric network for a *lumped heat capacity system*, in which $C_{th} = \rho Vc$ represents the *thermal capacity* of the system. The value of C_{th} can be obtained from the following thermal and electrical equations, by similarity.

$$Q = (\rho Vc)t = C_{th}.t \qquad \text{...Thermal equation.}$$

$$s = C.E \qquad \text{...Electrical equation.}$$

where, s = Capacitor charge,

C = Capacitance of the condenser, and

E = Voltage.

When the switch is *closed* [Fig. 4.1 (*b*)] the solid is charged to the temperature θ. On *opening* the switch, the thermal energy stored as C_{th} is dissipated through the thermal resistance $R_{th} = \left(\frac{1}{hA_s}\right)$ and the temperature of the body decays with time. From this analogy it is concluded that *RC* electrical circuits may be used to determine the transient behaviour of thermal systems.

The power on exponential, *i.e.*, $\frac{hA_s}{\rho Vc}\tau$ can be arranged in dimensionless form as follows.

$$\frac{hA_s}{\rho Vc}\tau = \left(\frac{hV}{kA_s}\right)\left(\frac{A_s^2 k}{\rho V^2 c}\tau\right) = \left(\frac{hL_c}{k}\right)\left(\frac{\alpha\tau}{L_c^2}\right) \quad ...(4.5)$$

where $\alpha = \left[\frac{k}{\rho c}\right]$ = Thermal diffusivity of the solid

$$L_c = \text{Characteristic length} = \frac{\text{Volume of the solid } (V)}{\text{Surface area of the solid } (A_s)}$$

The values of characteristic length (L_c), for simple geometric shapes, are given below:

$$\textit{Flat plate}: L_c = \frac{V}{A_s} = \frac{LBH}{2BH} = L/2 = \text{semi-thickness}$$

where L, B and H are thickness, width and height of the plate.

$$\textit{Cylinder (long)}: L_c = \frac{\pi R^2 L}{2\pi RL} = \frac{R}{2} \quad \text{where, } R = \text{radius of the cylinder.}$$

$$\textit{Sphere}: L_c = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} \quad \text{where, } R = \text{radius of the sphere.}$$

$$\textit{Cube}: L_c = \frac{L^3}{6L^2} = \frac{L}{6} \quad \text{where, } L = \text{Side of the cube.}$$

Further, from eqn. (4.5):

(*i*) The non-dimensional factor $\frac{hL_c}{k}$ is called the **Biot member B_i,**

i.e. $$B_i = \frac{hL_c}{k} = \text{Biot number.}$$

It gives an indication of the *ratio of internal* (*conduction*) *resistance to surface* (*convection*) *resistance*. When the value of B_i is small, it indicates that the system has a small internal (conduction) resistance, *i.e.*, relatively small temperature gradient or the existence of practically uniform temperature within the system. The convective resistance then predominates and the transient phenomenon is controlled by the convective heat exchange.

If $B_i < 0.1$, the lumped heat capacity approach can be used to advantage with simple shapes such as plates, cylinders, spheres and cubes. The error associated is around 5%.

(*ii*) The non-dimensional factor $\frac{\alpha\tau}{L_c^2}$ is called the **Fourier number, F_0.**

i.e. $$F_0 = \frac{\alpha\tau}{L_c^2} = \text{Fourier number}$$

It signifies the *degree of penetration of heating or cooling effect* through a solid.

Using non-dimensional terms, eqn. (4.4) takes the form of

$$\frac{\theta}{\theta_i} = \frac{t - t_a}{t_i - t_a} = e^{-B_i F_0} \quad ...(4.6)$$

The graphical representation of eqn. (4.5) for different solids (Infinite plates, infinite cylinders and infinite square rods and cubes and spheres) is shown in Fig. 4.4.

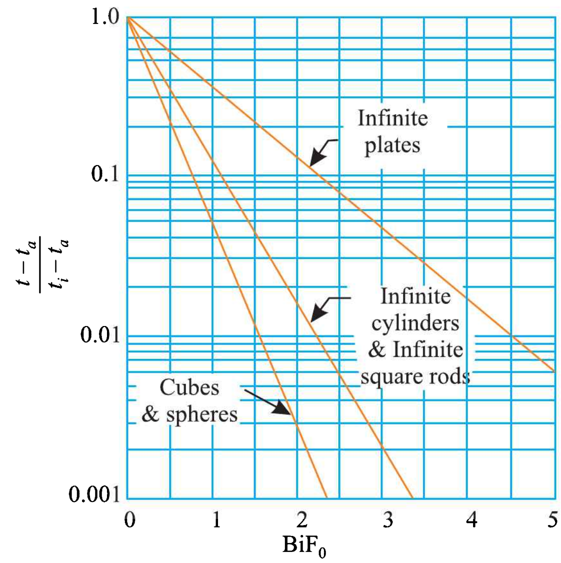


Fig. 4.4. Newtonian heating or cooling (for various solids)

***Instantaneous heat flow rate and total heat transfer*:**

The *instantaneous rate of heat flow* (Q_i) may be found as follows:

$$Q_i = \rho Vc \frac{dt}{d\tau} = \rho Vc \frac{d}{d\tau}\left[t_a + (t_i - t_a) \exp\left\{-\frac{hA_s}{\rho Vc}\tau\right\}\right]$$

or, $$Q_i = \rho Vc \left[(t_i - t_a)\left\{-\frac{hA_s}{\rho Vc}\right\} \exp\left\{-\frac{hA_s}{\rho Vc}\tau\right\}\right]$$

or, $$Q_i = -hA_s(t_i - t_a) \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] \quad ...(4.7)$$

or, $$Q_i = -hA_s(t_i - t_a)\, e^{-B_i F_0} \quad ...[4.7\ (a)]$$

The *total* or *cumulative heat transfer is*

$$Q' = \int_0^\tau Q_i\, d\tau$$

$$= \int_0^\tau -hA_s(t_i - t_a) \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] d\tau$$

$$= \left[-hA_s(t_i - t_a)\frac{\exp(-hA_s/\rho Vc)\tau}{-hA_s/\rho Vc}\right]_0^\tau$$

$$= \rho Vc(t_i - t_a)\left[\exp\left\{-\frac{hA_s}{\rho Vc}\tau\right\}\right]_0^\tau$$

or, $$Q' = \rho Vc(t_i - t_a)\left[\exp\left\{-\frac{hA_s}{\rho Vc}\right\} - 1\right] \quad ...(4.8)$$

$$Q' = \rho Vc(t_i - t_a)[e^{-B_i F_0} - 1] \quad ...[4.8\ (a)]$$

... in terms of non-dimensional B_i and F_0 number.

Example 4.1. *A 50 cm × 50 cm copper slab 6.25 mm thick has a uniform temperature of 300°C. Its temperature is suddenly lowered to 36°C. Calculate the time required for the plate to reach the temperature of 108°C.*

Take ρ = 9000 kg/m³; c = 0.38 kJ/kg°C; k = 370 W/m°C and h = 90 W/m²°C