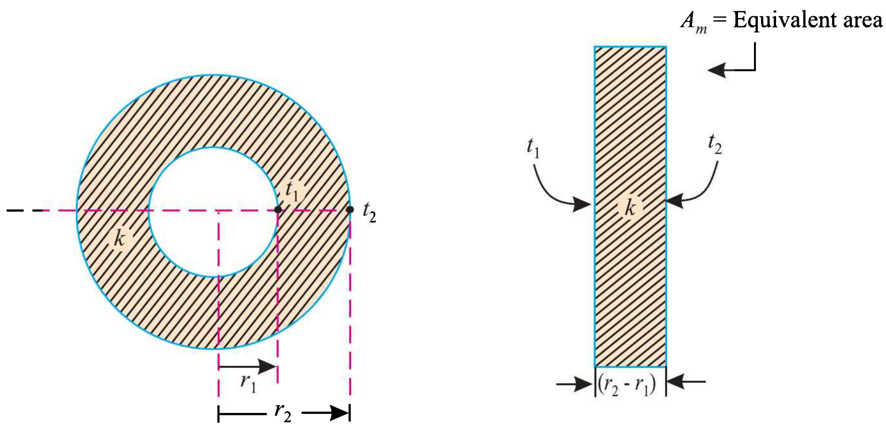
$$= -\frac{1}{\beta} + \sqrt{\left(t_1 + \frac{1}{\beta}\right)^2 - \frac{Q}{\beta k_0} \cdot \frac{\ln(r/r_1)}{\pi L}}$$

i.e., $$t = -\frac{1}{\beta} + \left[\left(t_1 + \frac{1}{\beta}\right)^2 - \frac{Q}{\beta k_0} \cdot \frac{\ln(r/r_1)}{\pi L}\right]^{\frac{1}{2}} \quad ...(2.66)$$

2.6.1.1. LOGARITHMIC MEAN AREA FOR THE HOLLOW CYLINDER

Invariably it is considered convenient to have an expression for the heat flow through a hollow cylinder of the same form as that for a plane wall. Then thickness will be equal to ($r_2 - r_1$) and the area A will be an equivalent area A_m as shown in the Fig. 2.46. Now, expressions for heat flow through the hollow cylinder and plane wall will be as follows :



(*a*) Hollow cylinder (*b*) Plane wall

Fig. 2.46.

$$Q = \frac{(t_1 - t_2)}{\frac{\ln(r_2/r_1)}{2\pi k L}} \quad \text{... Heat flow through cylinder.}$$

$$Q = \frac{(t_1 - t_2)}{\frac{(r_2 - r_1)}{k A_m}} \quad \text{... Heat flow through plane wall.}$$

A_m is so chosen that heat flow through cylinder and plane wall will be equal for the same thermal potential.

$$\therefore \quad \frac{(t_1 - t_2)}{\frac{\text{In}(r_2/r_1)}{2\pi k L}} = \frac{(t_1 - t_2)}{\frac{(r_2 - r_1)}{k A_m}}$$

or, $$\frac{\ln(r_2/r_1)}{2\pi k L} = \frac{(r_2 - r_1)}{k A_m}$$

or, $$A_m = \frac{2\pi L(r_2 - r_1)}{\ln(r_2/r_1)} = \frac{2\pi L r_2 - 2\pi L r_1}{\ln(2\pi L r_2 / 2\pi L r_1)}$$

or, $$A_m = \frac{A_0 - A_i}{\ln(A_0 - A_i)} \quad ...(2.67)$$

where A_i and A_o are inside and outside surface areas of the cylinder.

The expression is known as *logarithmic mean area* of the plane wall and the hollow cylinder. By the use of this expression a cylinder can be transformed into a plane wall and the problem can be solved easily.

If, $\frac{A_0}{A_i} < 2$, then we can take,

$$A_{av.} = \frac{A_i + A_0}{2} \quad \text{which is within 4\% of } A_m \qquad (\text{where, } A_{av.} = \text{Average area})$$

Further, $$A_m = 2\pi r_m L = \frac{2\pi L(r_2 - r_1)}{\ln(r_2 / r_1)}$$

Obviously, *logarithmic mean radius* of the hollow cylinder is

$$r_m = \frac{(r_2 - r_1)}{\ln(r_2 / r_1)} \qquad ..(2.68)$$

2.6.2. HEAT CONDUCTION THROUGH A COMPOSITE CYLINDER

Consider flow of heat through a composite cylinder as shown in Fig. 2.47.

Let, t_{hf} = The temperature of the hot fluid flowing inside the cylinder,

t_{cf} = The temperature of the cold fluid (atmospheric air),

k_A = Thermal conductivity of the inside layer A,

k_B = Thermal conductivity of the outside layer B,

t_1, t_2, t_3 = Temperatures at the points 1, 2, and 3 (see Fig. 2.47)

L = Length of the composite cylinder, and

h_{hf}, h_{cf} = Inside and outside heat transfer coefficients.

The rate of heat transfer is given by

$$Q = h_{hf} . 2\pi r_1 . L (t_{hf} - t_1) = \frac{k_A . 2\pi L (t_1 - t_2)}{\ln(r_2 / r_1)}$$

$$= \frac{k_B . 2\pi L (t_2 - t_3)}{\ln(r_3 / r_2)} = h_{cf} . 2\pi r_3 . L (t_3 - t_{cf})$$

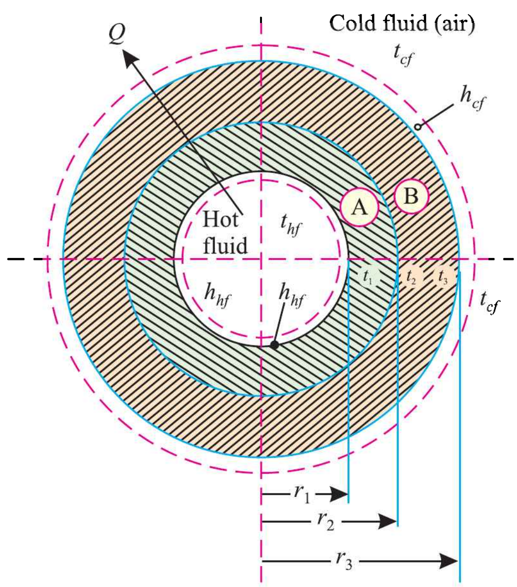


Fig. 2.47.

Rearranging the above expression, we get

$$t_{hf} - t_1 = \frac{Q}{h_{hf} . r_1 . 2\pi L} \quad ...(i)$$

$$t_1 - t_2 = \frac{Q}{\dfrac{k_A . 2\pi L}{\ln (r_2 / r_1)}} \quad ...(ii)$$

$$t_2 - t_3 = \frac{Q}{\dfrac{k_B . 2\pi L}{\ln (r_3 / r_2)}} \quad ...(iii)$$

$$t_3 - t_{cf} = \frac{Q}{h_{cf} . r_3 . 2\pi L} \quad ...(iv)$$

Adding (*i*), (*ii*), (*iii*) and (*iv*), we have

$$\frac{Q}{2\pi L}\left[\frac{1}{h_{hf} . r_1} + \frac{1}{\dfrac{k_A}{\ln (r_2 / r_1)}} + \frac{1}{\dfrac{k_B}{\ln (r_3 / r_2)}} + \frac{1}{h_{cf} . r_3}\right] = t_{hf} - t_{cf}$$

$\therefore$

$$Q = \frac{2\pi L (t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf} . r_1} + \dfrac{1}{\dfrac{k_A}{\ln (r_2 / r_1)}} + \dfrac{1}{\dfrac{k_B}{\ln (r_3 / r_2)}} + \dfrac{1}{h_{cf} . r_3}\right]}$$

or,

$$Q = \frac{2\pi L (t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf} . r_1} + \dfrac{\ln (r_2 / r_1)}{k_A} + \dfrac{\ln (r_3 / r_2)}{k_B} + \dfrac{1}{h_{cf} . r_3}\right]} \quad ...(2.69)$$

If there are '*n*' concentric cylinders, then

$$Q = \frac{2\pi L (t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf} . r_1} + \displaystyle\sum_{n=1}^{n=n} \frac{1}{k_n} \ln \{r_{(n+1)} / r_n\} + \dfrac{1}{h_{cf} . r_{(n+1)}}\right]} \quad ...(2.70)$$

If inside and outside heat transfer coefficients are not considered then the above equation can be written as

$$Q = \frac{2\pi L [t_1 - t_{(n+1)}]}{\displaystyle\sum_{n=1}^{n=n} \frac{1}{k_n} \ln [r_{(n+1)} / r_n]} \quad ...(2.71)$$

Example 2.35. *A thick walled tube of stainless steel with 20 mm inner diameter and 40 mm outer diameter is convered with a 30 mm layer of asbestos insulation (k = 0.2 W/m°C). If the inside wall temperature of the pipe is maintained at 600°C and the outside insulation at 1000°C, calculate the heat loss per metre of length.* **(AMIE Summer, 1997)**

Solution. Refer to Fig. 2.48.

Given:

$$r_1 = \frac{20}{2} = 10 \text{ mm} = 0.01 \text{ m}$$

$$r_2 = \frac{40}{2} = 20 \text{ mm}$$

$= 0.02 \text{ m}$

$r_3 = 20 + 30 = 50 \text{ mm}$

$= 0.05 \text{m}$

$t_1 = 600°\text{C}$

$t_3 = 1000°\text{C}$

$k_B = 0.2 \text{ W/m°C}$

Heat transfer per metre of a length, Q/L:

$$Q = \frac{2\pi L(t_1 - t_3)}{\frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B}}$$

Since the thermal conductivity of stainless steel is not given, therefore, neglecting the resistance offered by stainless steel to heat transfer across the tube, we have

$$\frac{Q}{L} = \frac{2\pi(t_1 - t_3)}{\frac{\ln(r_3/r_2)}{k_B}} = \frac{2\pi(600 - 1000)}{\frac{\ln(0.05/0.02)}{0.2}}$$

$= -$**548.57 W/m (Ans.)**

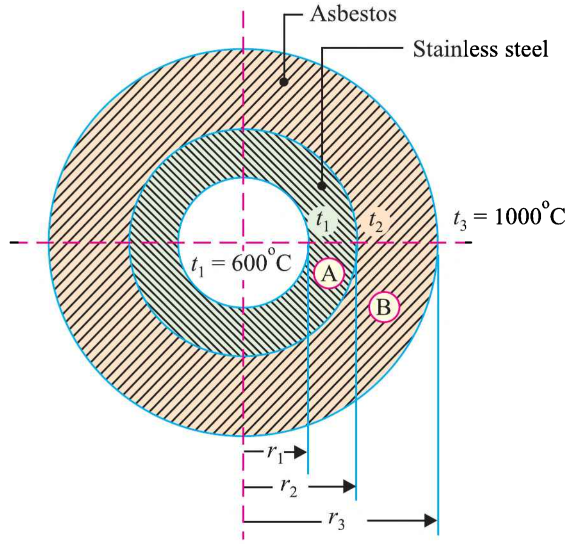


Fig. 2.48.

Negative sign indicates that the heat transfer takes place *radially inward.*

Example 2.36. *A steel pipe with 50 mm OD is covered with a 6.4 mm asbestos insulation [k = 0.166 W/mK] followed by a 25 mm layer of fiber-glass insulation [k = 0.0485 W/mK]. The pipe wall temperature is 393 K and the outside insulation temperature is 311 K. Calculate the interface temperature between the asbestos and fiber-glass.*

(AMIE Summer, 1998)

Stove for heating.

Solution.

Given : $r_1 = \frac{50}{2} = 25 \text{ mm} = 0.025 \text{ m};$

$r_2 = r_1 + 6.4 = 25 + 6.4$

$= 31.4 \text{ mm or } 0.0314 \text{ m};$

$r_3 = r_2 + 25 = 31.4 + 25$

$= 56.4 \text{ mm} = 0.0564 \text{ m};$

$T_1 = 393 \text{ K}; T_3 = 311 \text{ K}$

$k_A = 0.166 \text{ W/mK};$

$k_B = 0.0485 \text{ W/mK}.$

Interface temperature between the asbestos and fiber-glass, t_2 :

We know that, $$Q = \frac{2\pi L(T_1 - T_3)}{\frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B}}$$

$$\frac{Q}{L} = \frac{2\pi(T_1 - T_3)}{\dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B}}$$

$$= \frac{2\pi(393 - 311)}{\dfrac{\ln(0.0314/0.025)}{0.166} + \dfrac{\ln(0.0564/0.0314)}{0.0485}}$$

$$= \frac{515.22}{1.373 + 12.075} = 38.31\,\text{W/m}$$

Also, $$\frac{Q}{L} = \frac{2\pi(T_1 - T_2)}{\dfrac{\ln(r_2/r_1)}{k_A}}$$

or, $$38.31 = \frac{2\pi(393 - T_2)}{\left[\dfrac{\ln(0.0314/0.025)}{0.166}\right]}$$

$$38.31 = \frac{2\pi(393 - T_2)}{1.373}$$

∴ $$T_2 = 393 - \frac{38.31 \times 1.373}{2\pi} = 384.6\,\text{K}$$

or, $t_2 = 384.6 - 273 =$ **111.6° C** **(Ans.)**

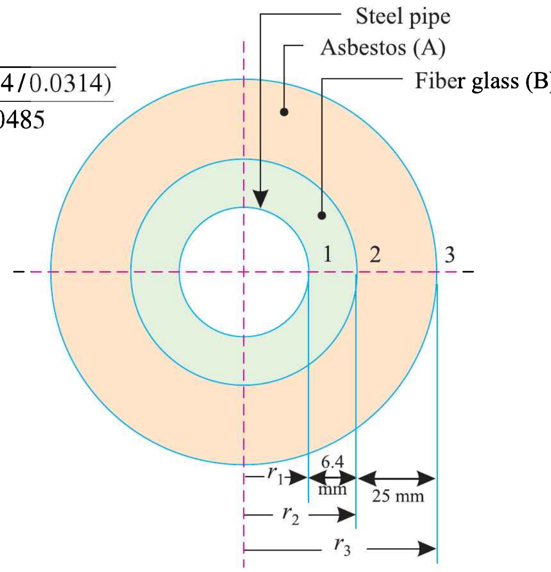


Fig. 2.49.

Example 2.37. *A gas filled tube has 2 mm inside diameter and 25 cm length. The gas is heated by an electrical wire of diameter 50 microns (0.05 mm) located along the axis of the tube. Current and voltage drop across the heating element are 0.5 amps and 4 volts, respectively. If the measured wire and inside tube wall temperatures are 175°C and 150°C respectively, find the thermal conductivity of the gas filling the tube.* **(GATE, 1998)**

Solution. *Given* : Inside radius of the tube, r_t = 2mm

Length of the tube, L = 25 cm = 0.25 m

Radius of the electric wire, r_w = 0.025 mm

Inside tube temperature, t_t = 150°C

Wire temperature, t_w = 175°C

Current through the element = 0.5 A

Voltage across the element = 4 V

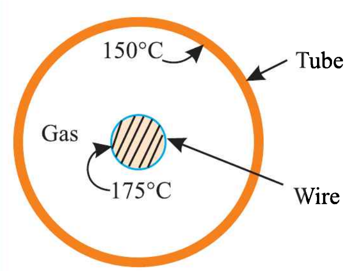


Fig. 2.50.

Thermal conductivity of the gas, *k* :

Heat transferred through a cylinder,

$$Q = \frac{2\pi k L(t_w - t_t)}{\ln(r_t/r_w)}$$

$$= \frac{2\pi k \times 0.25(175 - 150)}{\ln(1/0.025)} = 10.645\ \text{kW} \quad ...(i)$$

Also, $Q = VI = 4 \times 0.5 = 2.0$ W ...(*ii*)

From (*i*) and (*ii*), we get

$10.645\,k = 2.0$

or, $k =$ **0.188 W/m°C.** **(Ans.)**